Astro 350

Lecture 4

Jan 26, 2022

Announcements:

- **Discussion Question 1** posted on Canvas
  due **today** before midnight – great answers already!
- **Homework 1** due this Friday
- for number crunching I suggest a spreadsheet or small program
  easier to check your work
- Office Hours: after class today, or by appointment
- please turn on video if you are comfortable doing so

Last time: began shift to modern cosmology

note: wide range of ancient cosmologies existed around the world including some heliocentric ideas in India, and the Islamic world

Two Competing Worldviews

- ancient Greek geocentric cosmology
  cluminating in Ptolemaic system
  Q: key ingredients?
  Q: why would anyone believe this?
  bear in mind: we have been steeped in heliocentrism

- Copernicus & heliocentric cosmology
  Q: how does it explain retrograde motion?

Nicolaus Copernicus 1473–1543 Polish

adopted *heliocentric* cosmological model:

Note: motivation was *not* Ptolemaic disagreement with data but rather aesthetic – i.e., intuitive sense of beauty

`www: Copernican model`

- Mercury & Venus closer to Sun ⇒ always seen near Sun
- earth spins ⇒ daily motion of celestial objects
- earth orbits sun ⇒ apparent sun motion in ecliptic
- retrograde motion: during earth–planet passing

`www: retro animation`

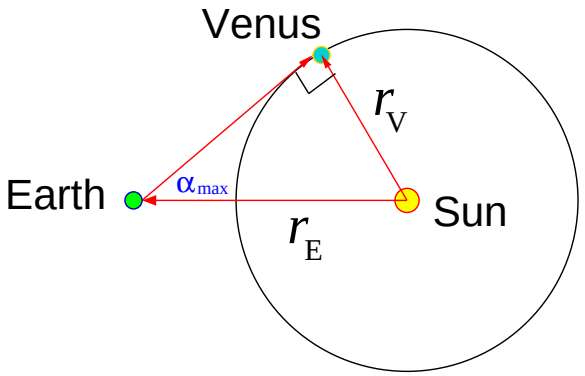
But: have to explain *all* data

Q: how would Nick account for non-observation of star parallax?

Copernicus Bonus: calculated relative distances of planets!

recall: Venus never seen too far from Sun
largest angle from Sun: maximum "elongation" $\alpha_{\rm Venus} = 46\,{\rm deg}$

in heliocentric model:
max elongation when slightline is
tangent to Venus orbit
from diagram: $\sin\alpha_{\rm max} = r_{\rm V}/r_{\rm E}$
and so $r_V = r_{\rm E}\sin\alpha_{\rm max} = 0.72\,r_{\rm E}$



New unit of distance/length:
"astronomical unit" = average Earth-Sun distance

$1\ {\rm au} \equiv r_{\rm E} = 1.50 \times 10^8\ {\rm km} = 93\ {\rm million\ miles}$

- Earth (average) orbit radius: 1 au
- Venus orbit: 0.72 au

Copernicus: What's New and What's Not

- planets still on spheres
- Copernicus still used epicycles!
- predictions *not* better than in Ptolemy's model
  $\rightarrow$ geometrically equivalent *Q: meaning?*
- Copernicus' model not generally accepted
  and Ptolemaic–Copernican disagreement though to be metaphysical, *unanswerable* question

Q: so how do we decide which is right?

Tycho Brahe 1546-1601: Danish Astronomy Extraordinare

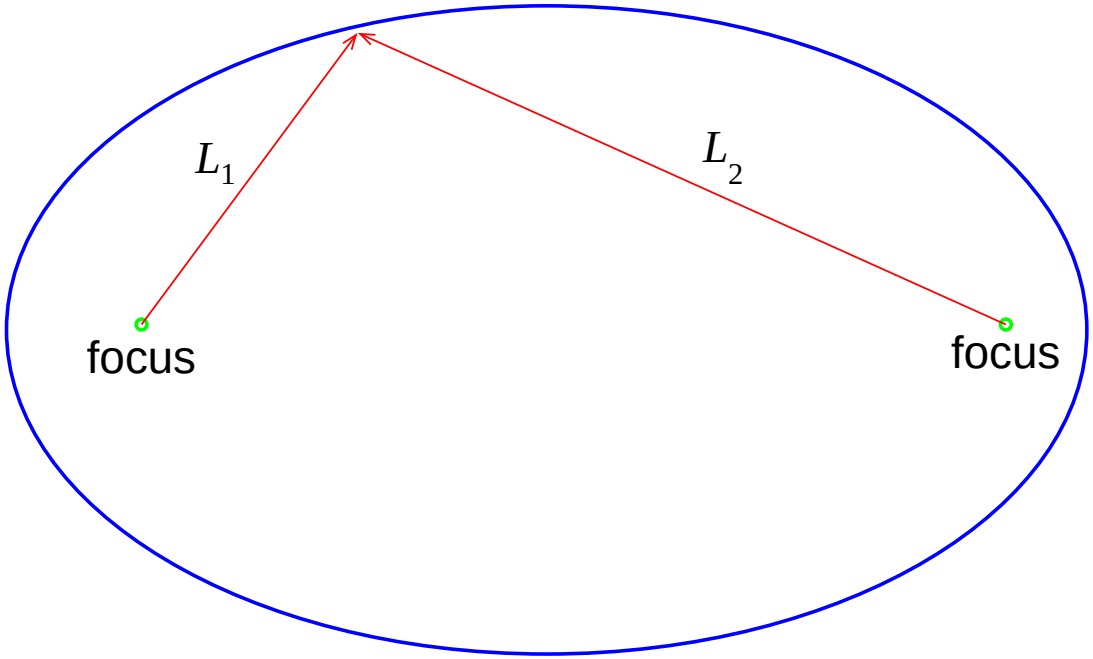
in youth: observed "nova stella" (supernova) `www: Tycho sketch`
→ change observed in heavens → corruptible!
observed Sun, Moon, planets for 20 years: careful, accurate data
but not a good number cruncher
→ like any good professor: made grad student do the work!

Johannes Kepler 1571–1630: Harmony of the Worlds

Analyzed Tycho's data for **20 years**(!), especially Mars motions
used heliocentric model with circles
but observations didn't quite agree
a small error (few arc min!) remained...took seriously
→ after trial & error:
completely & accurately described planet orbits

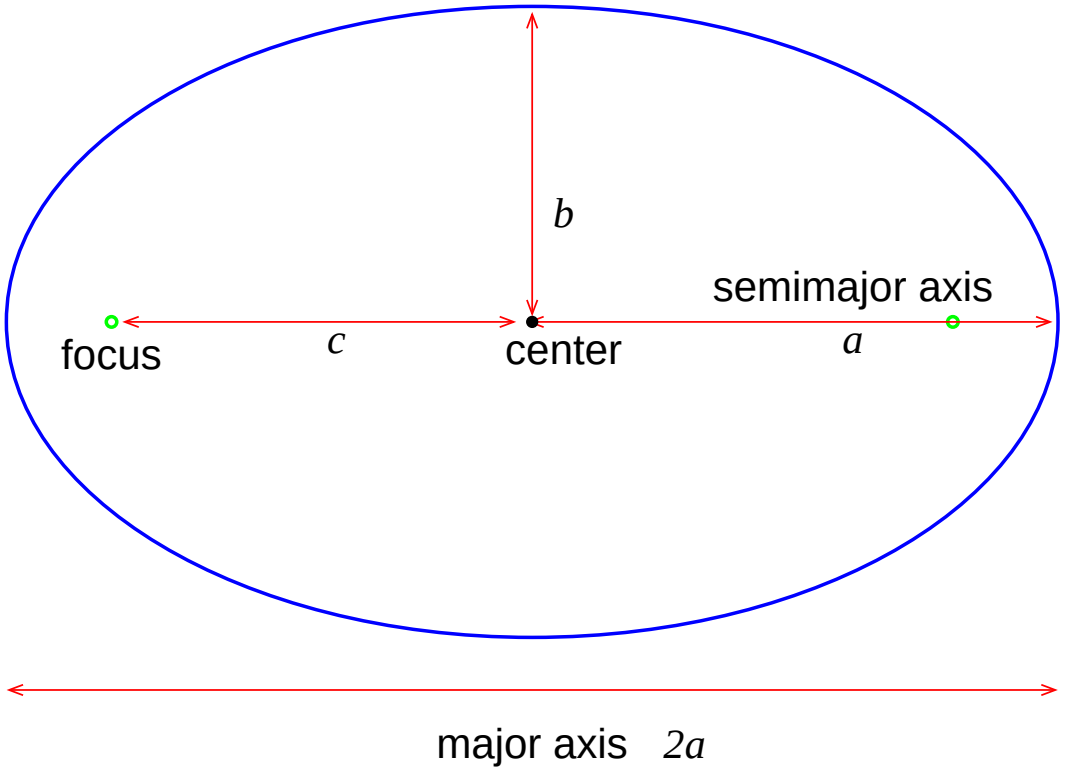
Kepler I: Law of Ellipses

each planet's orbit is an **ellipse**
with the **sun at one focus**



$$L_1 + L_2 = constant$$

Ellipse Anatomy



- two foci
- semi-major axis $\boxed{a}$
- focal length c
- semi-minor axis
  $b = \sqrt{a^2 - c^2}$

any ellipse fully characterized by:

∞ a and eccentricity $\boxed{e = c/a}$

Q: what do we get for $e = 0$*?* $e = 1$*?* `www: eccentricity demo`

Kepler I: Law of Ellipses

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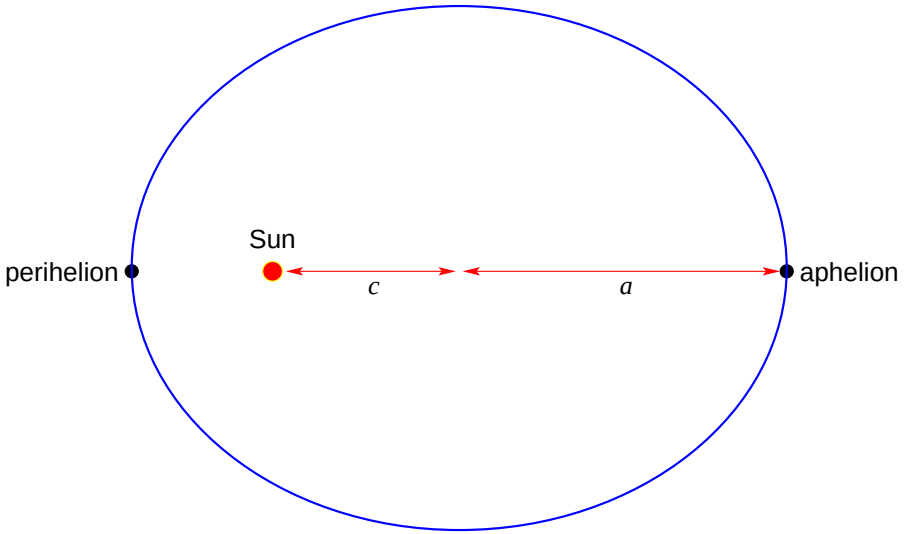


Orbit anatomy

aphelion: *farthest* point from Sun

perihelion: *closest* point to Sun

Note: Kepler I only gives orbit *shape*

but says *nothing* about how orbit evolves in time

$\rightarrow$ need more info to fully describe orbit, hence...

Kepler II: Law of Equal Areas

a straight line from the planet to the sun
sweeps out equal areas in equal times

diagram: sketch areas

note that this amounts to telling about speed of planet
Q: where fastest? slowest?
`www: area animation`

Q: This still doesn't fully characterize an orbits–why not?

Kepler I gives orbit shape in space
Kepler II gives orbit evolution over time

but haven't yet connected the two:
how does spatial character (e.g., semimajor axis a)
relate to time character (e.g., period P)?

Need one last law...

Kepler III: The Mighty Equation

period P and semi-major axis a are related:

$$P^2 \propto a^3$$

$\Rightarrow P^2/a^3 = const$, holds for all planets, with same constant

and since must hold for Earth:

$$P^2_{\text{yrs}} = a^3_{\text{AU}} \tag{1}$$

Q: ok for earth?

where P written in years, a in AU

Very powerful! e.g.:

Asteroids exist with orbits inside 1 AU (and some cross 1 AU!!)

`www: inner solar system objects--in real time!`

Poll: Kepler III

Kepler III: $P_{\text{yrs}}^2 = a_{\text{AU}}^3$

Consider an asteroid with an orbit entirely outside 1 AU

Is its period longer or shorter than a year?

A $P > 1$ yr, no matter eccentricity e

B $P < 1$ yr, no matter what e

C can't answer without knowing e

Kudos to Kepler

Several points worth noting...

⋆ An amazing discovery–mathematics underlies the workings of the cosmos!

⋆ Orbits have a simple geometry
...but *not* simplest: ellipse not circle

⋆ Kepler's laws remain (almost) perfectly accurate to this day–indeed, in slightly generalized form will show up in many (most!) situations where motions are controlled by gravity

⋆ Yet note what we still don't have:
an understanding of *why* Kepler's laws hold
→ that is, what is the *mechanism* that makes
planets move this way
...for that, need to wait for Kepler's successors...

Galileo Galilei: Astronomer

First to use telescope in Astronomy

`www: Galileo shows scope to Duke`

contributions:

- mountains on the Moon
- moons of Jupiter
- sunspots

These are bad for Ptolemy (but maybe not deadly) *Q: how?*

Crucial, decisive experiment:

- phases of Venus

`www: Venus phase animation`

observations contradicted Aristotle
supported Copernicus
"paradigm shift" (Kuhn)
radical change in outlook/conceptual framework

Note: Galileo put on trial, forced to recant heliocentrism

- his work, Copernicus, Kepler banned until 1832
- official semi-apology ("mistakes were made") 1992

complex: crackdown as much political as theological
shows view of the world people had

1. really not at all obvious that sun at center
2. the paradigm shift difficult, challenged outlook

The Science of Motion

Description of Motion

want precise language not just for planets but all objects

Speed: rate of motion

$$\text{speed} = \frac{\text{change in distance}}{\text{change in time}}$$

mathematically: $v = d/t$ (more technically $v = dx/dt$)

so: $\boxed{d = vt}$ distance traveled = speed $\times$ travel time

Fine Print: valid when speed constant = not changing

Velocity: both speed and direction of travel

ex: if 10 mi East in 1/2 hour,

velocity = 10/(1/2)= 20 mph East

Q: can two objects have same speed, different velocity?

Q: does car speedometer really measure speed or velocity?

Q: turn corner in car, speedometer pegged at 20mph–whassup?

Acceleration: *change* in speed *or* direction of motion

speed up rate or slow down rate

ex: slam on gas, brakes in car

Q: what kind(s) of motion(s) have zero *acceleration?*

intuitively: acceleration is rate of speeding up
or slowing down

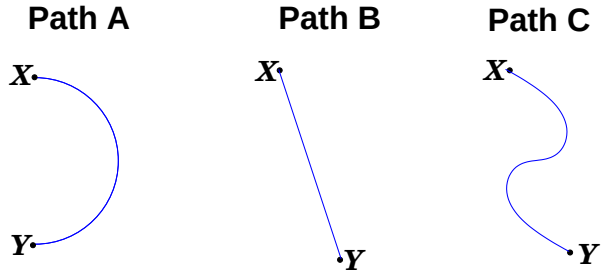
sometimes useful to distinguish:

- **ac**celeration = speeding up
- **de**celeration = slowing down

Poll: Acceleration

young James T. Kirk (remake version) drives from point X to Y
his motorcycle speedometer readings are unknown
maybe constant, maybe not

In which case(s) is it **certain** he accelerated?



A Path C only

B Paths A and C

C Paths A, B, and C

D if speed kept constant, all paths can be unaccelerated

Galileo: Physicist

studied motion of objects on earth two important cases:

Special Motion I: **"Free Body"**
moving with *no* external influences
(including friction, gravity)
$\rightarrow$ moves in straight line, constant speed $\rightarrow$ **constant velocity**

Galileo finds this is the "**natural motion**"
of an object – keeps constant speed & direction
unless something happens to change this

Contrary to Aristotle: natural motion is to come to rest
Q: Why did Aristotle think this?

Special Motion II: **“Free Fall”**
motion due to **gravity only**

`www: Tower of Pisa`

Demo: Pisa: heavy, light objects
Demo: Pisa: ball, paper sheet

Q: in free fall, is velocity constant?

even if fall in straight line, speed changes
$\rightarrow$ gravity causes **acceleration**
$\rightarrow$ *same* acceleration for all objects
independent of size, mass

Einstein called this independence the "equivalence principle"
crucial in his invention of General Relativity

Note: Galileo *describes* motion (mathematically)
but to *explain* with a theory fell to...

Isaac Newton 1643-1727

Why Kepler's laws for planets?
Are they special?
Can we understand using general rules for all motion?

New concepts

⋆ **mass**: "amount of stuff"
measure in kg $\rightarrow$ 1 kg of anything has the same mass

• **force**: push or pull on object
can have more that one acting, in different directions

• **net force**: *total* of all forces acting.
if forces unbalanced, net force is present

Newton's Laws of Motion

motion & forces linked

Newton I. "Inertia"

- an object at rest stays at rest if no net force acts on it
- an moving object goes in straight line w/ const speed
  if no forces act on it

i.e., "free body" as per Galileo

so we say: objects have "*interia*" or "*momentum*"
$\Rightarrow$ will keep their state of motion (i.e., velocity)
unless and until a net force acts

Newton II: "$F = ma$"

- a *net force* acting on an object causes it to *accelerate*
- $a \propto F$ and $a \propto 1/m$ *Q: examples?*
  so $a \propto F/m$, or $\boxed{F = ma}$

Examples:

- ball on table, at rest *Q: how many forces? net force?*
- circular motion: speed const, yet force applied *Q: what's up?*

diagram: circular motion: velocity, force, force-free path

2nd Law a mathematical machine which predicts future!

Q: how? where's the fortunetelling in $F = ma$?

Q: what information needed to do this?

Fortunetelling (and Archæology!) with Newton II

input: at initial time, need to know/specify

- object mass m
- all of forces acting on m

$\Rightarrow$ find *net force* F

turn the math crank: $a = F/m$

$\rightarrow$ find *acceleration* = change in velocity

$\rightarrow$ use this to find new position, new velocity
at at moment a little later

$\rightarrow$ at new time and position, find new net force

...lather, rinse, repeat

Result: find particle path in future!

But also: can mathematically "run the move backwards"
and predict the past history as well!

Newton III: "Action-Reaction"

a rule about how forces behave
between two objects

if 2 bodies interact:
the **force** exerted by object 1 on object 2
  is **equal and opposite** to
the **force** exerted by object 2 on object 1

Q: application–you standing still
Q: Jump shot