Astro 350

Lecture 6

Jan 31, 2022

Announcements:

- **Discussion Question 2** posted on Canvas
  due next Wednesday
- **Homework 2** due Friday

Last time:

Galileo shows Ptolemaic system fails, triumph of heliocentrism

Also: free body and free fall *Q: what are these?*

Newton: laws of motion

Isaac Newton 1643-1727

Why Kepler's laws for planets?
Are they special?
Can we understand using general rules for all motion?

New concepts

⋆ **mass**: "amount of stuff"
measure in kg $\rightarrow$ 1 kg of anything has the same mass

• **force**: push or pull on object
can have more that one acting, in different directions

• **net force**: *total* of all forces acting.
if forces unbalanced, net force is present

Newton's Laws of Motion

motion & forces linked

Newton I. "Inertia"

- an object at rest stays at rest if no net force acts on it
- an moving object goes in straight line w/ const speed
  if no forces act on it

i.e., "free body" as per Galileo

so we say: objects have "*inertia*" or "*momentum*"
$\Rightarrow$ will keep their state of motion (i.e., velocity)
unless and until a net force acts

Newton II: "$F = ma$"

- a *net force* acting on an object causes it to *accelerate*
- $a \propto F$ and $a \propto 1/m$ *Q: examples?*
  so $a \propto F/m$, or $\boxed{F = ma}$

Examples:

- ball on table, at rest *Q: how many forces? net force?*
- circular motion: speed const, yet force applied *Q: what's up?*

diagram: circular motion: velocity, force, force-free path

2nd Law a mathematical machine which predicts future!

Q: how? where's the fortunetelling in $F = ma$*?*

Q: what information needed to do this?

Fortunetelling (and Archæology!) with Newton II

input: at initial time, need to know/specify

- object mass m
- all of forces acting on m

$\Rightarrow$ find *net force* F

turn the math crank: $a = F/m$

$\rightarrow$ find *acceleration* = change in velocity

$\rightarrow$ use this to find new position, new velocity
at at moment a little later

$\rightarrow$ at new time and position, find new net force

...lather, rinse, repeat

Result: find particle path in future!

But also: can mathematically "run the move backwards"
and predict the past history as well!

Newton III: "Action-Reaction"

a rule about how forces behave
between two objects

when 2 bodies interact = exert forces on each other:

the **force** exerted by object 1 on object 2
is **equal and opposite** to
the **force** exerted by object 2 on object 1

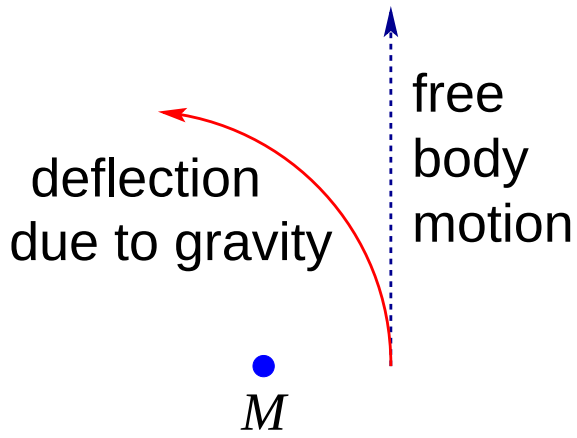
Q: application–you standing still
Q: Jump shot

Explaining Kepler

Kepler I: planets move in *ellipse*
this is *curved path*
direction of motion changing

So: velocity changes
$\rightarrow$ planets **accelerate**

therefore – need force:**gravity**



Universal Gravitation

Newton's Theory: combined all of the following ideas

- gravity is a **force**
  the force of gravity on you is your **weight**

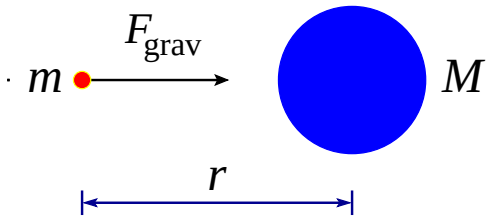
- gravity acts beyond earth

- gravity directed on line connecting centers of bodies

- gravity strength decreases with distance

∞ - all objects with mass are sources of gravity
  $\Rightarrow$ everything attracts everything else

Can summarize mathematically compact way:
for 2 bodies, masses m_1, m_2
centers separated by distance R



gravitational force proportionalities:

- $F_{\text{grav}} \propto m_1$
- $F_{\text{grav}} \propto m_2$
- $F_{\text{grav}} \propto 1/R^2$

Q: how to summarize this in one equation?

The Law of Universal Gravitation

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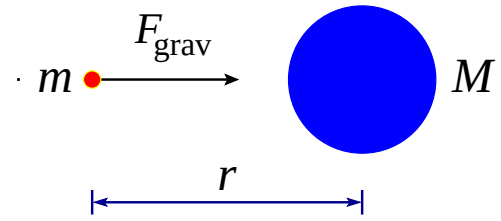

together: Newton's **Universal Law of Gravitation**

$$F_{\text{grav}} = G\frac{m_1 m_2}{R^2} \tag{1}$$

where G is just a fixed, constant number, same always:

$$G = 6.7 \times 10^{-11}\ \frac{\text{m}^3}{\text{kg s}^2} \tag{2}$$

- *Q: how is equation similar/different from list on previous slide?*

Living with Universal Gravity

$$F_{\text{grav}} = G\frac{m_1 m_2}{R^2} \tag{3}$$

- gravity force $F \propto 1/R^2$: *"inverse square law"*
- the force of gravity on an object is the object's *weight*

Q: why "universal"?

Q: would life change for us if Earth's mass doubled? how?

Q: would life change for us if Earth's radius was doubled? how?

Q: would life change for us if Earth's center hollowed out by bulldozing the material to compact it around center? how? what would the life be like at the hollow center?

if Earth's radius doubled (while keeping same mass)
$\Rightarrow$ Force on you = your weight would be $1/2^2 = 1/4$ as strong,
i.e., 4 times weaker

why? $F(R) \propto 1/R^2$ for any R
so: compare at $R = R_e$ (normal earth radius) and $R = 2R_e$ (bloated Earth)

proportional means that

$$\frac{F(2R_e)}{F(R_e)} = \frac{1/(2R_e)^2}{1/R_e^2} = \frac{1/(4R_e^2)}{1/R_e^2} = \frac{R_e^2}{4R_e^2} = \frac{1}{4} \qquad (4)$$

Gravity and Planet Motion

Newton II: input is force, output is motion
For planets around Sun, force is gravity (*free fall!*)

So: What is motion when $F = F_{\text{grav}} = G\frac{m_{\text{Sun}} m_{\text{planet}}}{R^2}$?
Now just a math problem: *diagram: sun, planet orbit,* $\vec{v}$, $\vec{F}$

Newton II + Gravity: properties of predicted orbits

- orbit is ellipse, with sun at one focus
- equal areas in equal times
- $a^3_{\text{AU}} = P^2_{\text{yr}}$
  actually better, more info: $a^3 = GM_{\text{Sun}}P^2/4\pi^2$
  Q: why is this better?

So: *Newton's laws + gravity force → Kepler's laws !*
theory agrees with observation!

Q: effect on planets of Sun's mass doubled?
Q: effect on planets of Sun's size doubled?

Note: only force on planet is gravity: free fall

$$m_{\text{planet}} a_{\text{planet}} = F_{\text{planet}} = G\frac{m_{\text{planet}} M_{\text{Sun}}}{R^2} \quad (5)$$

$$a_{\text{planet}} = G\frac{M_{\text{Sun}}}{R^2} \quad (6)$$

free fall acceleration only depends on Sun mass M_{Sun}
and Sun-planet orbit distance R

$\rightarrow$ **independent** of planet mass or size!

$\rightarrow$ at same R, all objects accelerate same way

$\Rightarrow$ equivalence principle pops out of Newton gravity! Woo hoo!

"Turning the Dials"

- double Sun's mass $\rightarrow$ double acceleration $\rightarrow$ faster orbits
- double Sun's size $\rightarrow$ same mass $\rightarrow$ no change in orbits

Testing Newton's Gravity

Moons of Jupiter: orbits obey Kepler's laws
  but in Newton's more general form, use M_{Jupiter}
$\rightarrow$ Jupiter's gravity works like Sun's, Earth's!

1830's: Uranus observed orbit did *not* follow predictions
of Newtonian solar system model
the death Newton's gravity?

Remember: have to agree with **all** data, not just some
even one clear failure is enough to kill theory
e.g., Kepler and Mars: just a small discrepancy from circular
  but still had to throw out circular orbits

maybe...but also: maybe have not included all sources of gravity
maybe gravity from unknown mass causes U's deviations

Poll: Uranus Discrepancy

1830's Problem: *measured* Uranus orbit doesn't match predictions of Newtonian Gravity *theory*

Vote your conscience!

Which seems more likely to you?

A Newton's gravity theory *correct*, but not all gravity sources had been included

B Newton's gravity theory *incorrect* (or at least incomplete)

Q: what experiment/observation would tell which is right?

19th Century Dark Matter?

to save Newton's gravity: need unseen massive object
  could have been called "dark matter"!
whose gravity is altering Uranus' motion

based on Uranus' known path, can use Newton's gravity
  to predict where the "dark matter" should be!

$\Rightarrow$ Crucial experiment: go look there with telescope!

this was done, in 1846...

Newton Triumphs

planet **Neptune** found at "dark matter" position
discovery *predicted* by Newton's gravity!

other more recent tests:

Binary Stars: two stars orbiting each other
move in ellipses, obey Kepler's laws
`www: binary star orbit`
$\rightarrow$ Gravity theory works outside solar system