Astro 350

Lecture 19

March 4, 2022

Announcements:

- **Homework 5 due Friday**
- **Discussion 5 Due next Wednesday**
- no HW next week, but:
  Paper Topic and Abstract due Friday
  info on Canvas

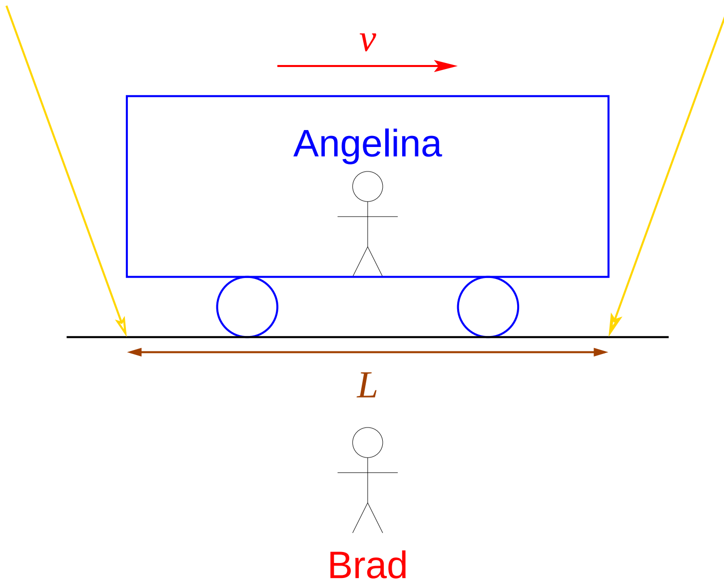
Last time: began Relativity

Q: what's the principle of relativity?

Q: what's an inertial reference frame? why important?

Q: what's does the Michelson-Morely experiment teach us?

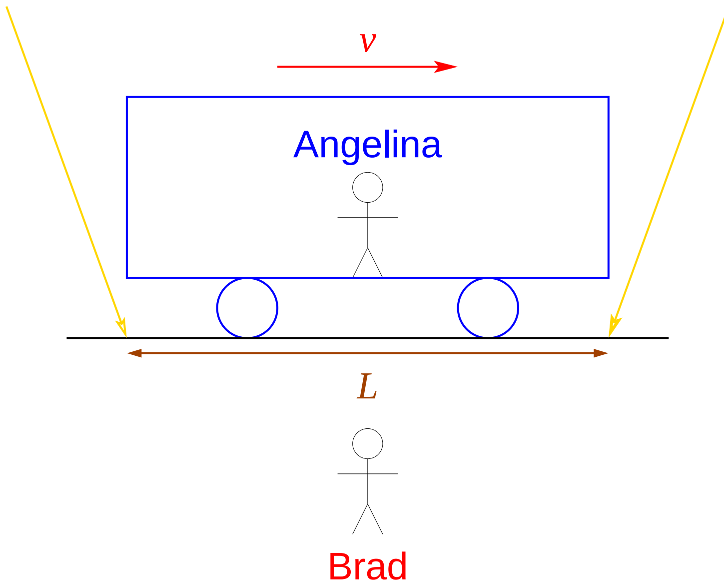
iClicker Poll Twofer: Train in a Thunderstorm



Does bystander Brad think flashes are simultaneous?

A yes

B no



Does Angelina in car midpoint think flashes simultaneous?

A yes

B no

The Relativity of Simultaneity

bystander Brad: two flashes each travel same distance $L/2$
so take same time $t = L/2c$
$\Rightarrow$ Brad sez: they're so totally simultaneous, dude!

passenger Angelina: train motion carries her
toward front flash, away from back flash
$\Rightarrow$ sees front flash first, then back flash later
But she thinks flashes traveled each same distance $L/2$
so concludes they took same travel time $L/2c$
$\Rightarrow$ Angelina sez: you lying dog, they're totally *not* simultaneous!

Q: so who's right?
Q: what's the larger lesson?

Who's right? Neither lying (about this), but disagree

- "simultaneous" is not a universally agreed condition
  relativity of simultaneity
- observers with different motion perceive time differently

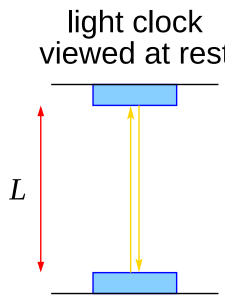
More Philosophical Commentary on Time

Strange things are afoot at the Circle-K.
– Cosmologist Ted "Theodore" Logan

Mirrors as Clocks

build "clock" in train car, height L

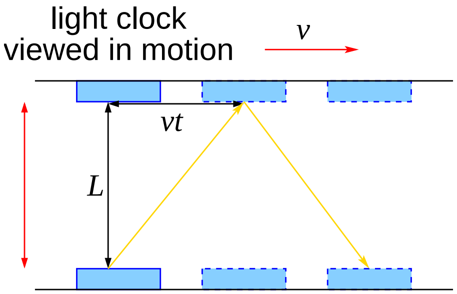
- mirrors on floor, ceiling reflect light up & down
- one "tick" per light bounce



in train frame: clock at rest (so $x = constant = 0$)

- light pathlength $d = \sqrt{x^2 + y^2} = y = L$
- tick duration $(\Delta t)_{\mathrm{rest}} = L/c$ (since $d = c\Delta t$)

Light clock: seen in motion



in trackside frame, train moving at speed v

- light zigzag due to mirror motion $\rightarrow$ path longer!
  Q: why? what will this mean?

in trackside frame, train moving at speed v

- during tick time $(\Delta t)_{\text{moving}}$ horizontal motion $x = v(\Delta t)_{\text{moving}}$
- light pathlength

$$d = \sqrt{x^2 + y^2} = \sqrt{v^2(\Delta t)^2_{\text{moving}} + L^2} \tag{1}$$

- tick duration $(\Delta t)_{\text{moving}} = d/c$, which means

$$\begin{aligned} d^2 = c^2(\Delta t)^2_{\text{moving}} &= v^2(\Delta t)^2_{\text{moving}} + L^2 \qquad (2) \\ (c^2 - v^2)(\Delta t)^2_{\text{moving}} &= L^2 \qquad (3) \end{aligned}$$

which gives

$$\begin{aligned} (\Delta t)_{\text{moving}} &= \frac{L}{\sqrt{c^2 - v^2}} = \frac{c}{\sqrt{c^2 - v^2}} \, \frac{L}{c} \qquad (4) \\ &= \frac{(\Delta t)_{\text{rest}}}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (5) \end{aligned}$$

Q: which means? and is bizarre because?

time dilation

we find

$$(\Delta t)_{\text{moving}} = \frac{(\Delta t)_{\text{rest}}}{\sqrt{1-\frac{v^2}{c^2}}} \quad (6)$$

$$= (\text{number} > 1) \times (\Delta t)_{\text{rest}} \quad (7)$$

$$> (\Delta t)_{\text{rest}} \quad (8)$$

- ⋆ moving clocks don't appear to keep same time as clocks at rest! Namely,
- ⋆ moving clock appears to have longer ticks!
- ⋆ moving clocks appear to run slow!
- ⋆ time depends on state of motion! not universal!

Train: disagreement about whether events are simultaneous

$\rightarrow$ "at the same time" is not a universal concept

$\rightarrow$ "relativity of simultaneity"

$\rightarrow$ "universal time" doesn't exist, depends on motion

Moving clock: disagreement about tick duration

- moving clock appears to run slowly (tick lasts longer)
- "time dilation"
- time does not "flow" at universal rate, depends on motion
- prediction: a clock moving at speed v will tick at rate

$$(\Delta t)_{\text{moving}} = \frac{(\Delta t)_{\text{rest}}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Q: who's right in all these disagreements?

Everybody's right! that is,

- all observations correctly reported
- real problem is deeper:
  Aristotelian notion of universal space, time are *invalid*

⋆ Space, time depend on state of motion of observers
⋆ but *no* observer *ever* sees her/himself as the wierdo
  i.e., your (nearby, at rest relative to you) clocks & yardsticks
  never appear weird to you
⋆ bizarreness only can arise when looking at things moving fast
  and/or at a distance

More later on what we *can* all agree on...

Note: time dilation guessed from "thought experiments"
(Einstein: "Gedankenexperiment")
Q: how would we test this in the real world?

Time Dilation in the Real World

high energy particles from space ("cosmic rays")
collide with Earth's atmosphere
produce unstable particles: muons μ seen at ground
in lab, at rest: muons decay after $t_{\text{decay}} = 2 \times 10^{-6}$ sec

at top of atmosphere: height $h = 10$ km
muons born with speed $v = 0.9999c$

Do they reach the ground?
Earth bystander: moving μ needs travel time
$t_{\text{moving}} = h/v = 33 \times 10^{-6}$ sec $> t_{\text{decay}} \rightarrow$ too long?!
should never see muons on ground! but we see plenty!
Why? muon "feels" at rest, and trip takes only
$t_{\text{rest}} = \sqrt{1 - v^2/c^2}\; t_{\text{moving}} = 0.5 \times 10^{-6}$ sec $< t_{\text{decay}}$
$\Rightarrow$ muon survive trip due to time dilation!

Relativity and Lengths

Turn light clock on side, use as yardstick:
at rest, clock length $L_{\rm rest}$
shine light, front-to back roundtrip
$\rightarrow$ travel time $t_{\rm rest} = 2L_{\rm rest}/c$

HW6: bystander times light pulse
finds clock length to be

$$L_{\rm moving} = \sqrt{1 - \frac{v^2}{c^2}} L_{\rm rest} \tag{9}$$

Q: lesson?

Length Contraction

⋆ moving yardsticks don't appear to have same length as yardsticks at rest

⋆ moving yardsticks appears shorter

⋆ moving objects appear shorter in direction of motion!

⋆ space depends on state of motion! not universal!

iClicker Poll: Special Relativity

Which of these can *all* observers agree on
regardless of their state of motion?

I. Albert Einstein was born 133 years ago

II. Chambana and Chicago are 133 miles apart

III. radio signals from spacecraft move at speed c

A I only

B II only

C III only

D all of I, II, and II

E none of I, II, and II

The Special Theory of Relativity

Einstein 1905: realized all of the above

Implications deep and wide
Newton's laws of motion built upon absolute space, time
$\Rightarrow$ no longer valid! have to be rebuilt
to respect principle of relativity

Einstein in 1905 also:
Revamped Newton's laws of motion $\rightarrow$ special relativity

- describes motions at any speed
- includes all wierdo time, space effects
- tested many ways many times, always found to agree with experiment
- e.g., particle accelerators (Fermilab, the LHC) wouldn't work if we did not use relativity!

Energy in Relativity

Einstein revised expressions for energy:
a particle of mass m with speed v has (total) energy

$$E = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{10}$$

Enormously important formula

Q: what is E when $v = 0$? What does this mean?
Q: what is E when $v \ll c$ but not zero? What does this mean?
Q: what is E when $v \to c$? What does this mean?

Mass is Energy

$$E = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{11}$$

- if $v = 0$, particle at rest
  yet $E = mc^2$ not zero!
  represents energy due to mere *existence* of particle
  i.e., just due to presence of mass!
  $E_{\text{rest}} = mc^2$ is *rest mass energy*
- energy output if mass m totally converted
  to some other energy form
- can be enormous: $m_{\text{donut}}c^2 = 4 \times 10^{15}$ Joules $= 1$ Mton TNT

So: mass is a form of energy!

Q: so what prevents donuts from exploding?

Q: when and where is *mass converted to energy?*

Mass $\Leftrightarrow$ Energy Interconversion

Implies: can convert mass to energy, *and* energy to mass
in fact–happens every day at Fermilab and the LHC!

energy $\to$ matter `www: LHC collision event`
2 particles with huge KE $\to$ many particles

but: if mass stays as mass, energy remains "stored"
and harmless (but tasty: mmmmm, donuts...)