Astro 350

Lecture 20

March 7, 2022

Announcements:

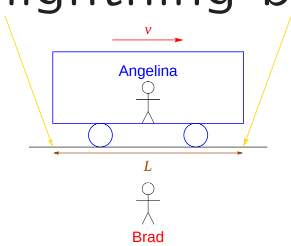
- **Discussion 5 due Wednesday**
- no HW this week, but:

  Paper Topic and Abstract due Friday

  info on Canvas

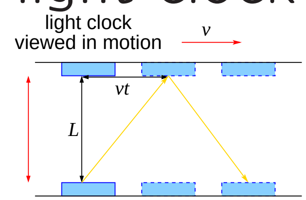
Last time: special relativity and time

lightning bolt experiment



Q: lesson?

light clock experiment



Q: lesson?

Lightning bolt and train:
disagreement about whether events are simultaneous

$\rightarrow$ "at the same time" is not a universal concept

$\rightarrow$ "relativity of simultaneity"

$\rightarrow$ "universal time" doesn't exist, depends on motion

Moving clock: disagreement about tick duration

- moving clock appears to run slowly (tick lasts longer)
- "time dilation"
- time does not "flow" at universal rate, depends on motion
- a clock moving at speed v appears to tick at rate

$$(\Delta t)_{\text{moving}} = \frac{(\Delta t)_{\text{rest}}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

percieved time depends on state of motion of observer

Relativity and Lengths

Turn light clock on side, use as yardstick:
at rest, clock length L_{rest}
shine light, front-to back roundtrip
$\rightarrow$ travel time $t_{\text{rest}} = 2L_{\text{rest}}/c$

HW6: bystander times light pulse
finds clock length to be

$$L_{\text{moving}} = \sqrt{1 - \frac{v^2}{c^2}} L_{\text{rest}} \quad (1)$$

actQ: lesson?

Length Contraction

★ moving yardsticks don't appear to have same length as yardsticks at rest

★ moving yardsticks appears shorter

★ moving objects appear shorter in direction of motion!

★ space depends on state of motion! not universal!

The Special Theory of Relativity

Einstein 1905: realized all of the above

Implications deep and wide
Newton's laws of motion built upon absolute space, time
$\Rightarrow$ no longer valid! have to be rebuilt
to respect principle of relativity

Einstein in 1905 also:
Revamped Newton's laws of motion $\rightarrow$ special relativity

- describes motions at any speed
- includes all wierdo time, space effects
- tested many ways many times, always found to agree with experiment
- e.g., particle accelerators (Fermilab, the LHC) wouldn't work if we did not use relativity!

Energy in Relativity

Einstein revised expressions for energy:
a particle of mass m
moving with speed v relative to an observer
has **energy** (according to observer)

$$E = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{2}$$

Enormously important formula

Q: at which v *is* E *smallest? largest?*
Q: what is E *when* $v = 0$*? What does this mean?*

Mass is Energy

$$E = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{3}$$

- if $v = 0$, particle at rest
  yet $E = mc^2$ not zero!
  represents energy due to mere *existence* of particle
  i.e., just due to presence of mass!
  $E_{\text{rest}} = mc^2$ is *rest mass energy*
- energy output if mass m totally converted
  to some other energy form
- can be enormous: $m_{\text{donut}}c^2 = 4 \times 10^{15}$ Joules $= 1$ Mton TNT

So: mass is a form of energy!

Q: so what prevents donuts from exploding?
Q: when and where is *mass converted to energy?*

Mass $\Leftrightarrow$ Energy Interconversion

Implies: can convert mass to energy, *and* energy to mass
in fact–happens every day at Fermilab and the LHC!

energy $\to$ matter `www: LHC collision event`
2 particles with huge KE $\to$ many particles

but: if mass stays as mass, energy remains "stored"
and harmless (but tasty: mmmmm, donuts...)

∞

Kinetic Energy Generalized

Eintsein: energy of particle with mass m and speed v

$$E = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{4}$$

- if $v \neq 0$, then $E > mc^2$:
  speeding up a particle gives it more energy
  extra amount above mc^2 is kinetic energy (Einstein's version)

- if $v \ll c$ but $v \neq 0$, can show (HW 6)

$$E \approx mc^2 + \frac{1}{2}mv^2 \tag{5}$$

$$= \text{(rest energy)} + \text{(Newtonian kinetic energy)} \tag{6}$$

- Einstein sez: particles have *same* KE as Newtonian
  if the motion speeds slow compared to c

Q: Why did this conclusion have *to be true?*

In general (haven't proven this, but can):
At slow speeds ($\ll c$):
Special Relativity $\rightarrow$ Galilean/Newtonian physics

Had to be true!

- recall: a theory has to explain *all* data
- Newton was wildly successful – explained all available data until new measurements involving fast speeds (i.e., light)
- new theory (Special Relativity) must explain new data but also must explain all old data!

$\Rightarrow$ so SR must agree with Newton where Newton was successful

The beauty and power of Relativity:

$\star$ *does* give back Newtonian results at slow speeds
"inherits successes of Newton"
$\star$ explains conditions when Newton does and does not work!
$\star$ provides larger, more complete picture!

Cosmology theory will have to respect these ideas of causality!

Acceleration to Lightspeed?

relativistic energy of particle with speed v

$$E = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{7}$$

let's try to accelerate a particle
from rest to $v \rightarrow c$!

what is energy "cost" to do this?

The Futility of Acceleration to Lightspeed

energy cost to accelerate from rest to speed v:

$$E = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{8}$$

as $v \to c$, we find $E \to \infty$!

Interpretation:
can speed up a mass m, but as speed gets close to c...

- to accelerate the particle a little faster takes more and more energy
- and to get particle to go at c requires *infinite* energy physically unachievable, impossible!
- matter (objects with mass) cannot reach speed of light

Q: what if massless particle, $m = 0$?

Solve E equation for v:

$$v = c\sqrt{1 - \left(\frac{mc^2}{E}\right)^2} \tag{9}$$

if $m = 0$, then $v = c$!

- to move at c, must be massless!
- light particles (photons) are massless!

technical note:

- more general formula works for all particles with mass m and momentum p:

$$E = \sqrt{(mc^2)^2 + (cp)^2} \tag{10}$$

- massive particles have relativistic momentum $p = mv/\sqrt{1 - v^2/c^2}$
- massless particles have $E = cp$
- all particles have $v/c = cp/E \leq 1$

The Cosmic Speed Limit and Causality

all particles (massive or massless) have $v/c \leq 1$
i.e., always have $v \leq c$

speed of light is universal speed limit

particles & information cannot travel faster than c
profound implications for cause & effect ("**causality**")

- an event can only affect future happenings
  which can be reached by light signal from the event
- this sets "region of future influence" *by* an event

Q: what past occurences can influence an event?

An event can only be affected by past events
which could have sent light signals to it
$\rightarrow$ this defines "region of past influence" *on* an event

What's more: events so far apart in space
that they *cannot* be connected with a light signal
are *unaffected* by each other
Q: specifically, what's an event not *affected by your finger snap here and now?*

$\Rightarrow$ key Relativity result/outlook:

- information cannot travel instantaneously
- actions are "local" in the sense that
  effects transmitted over finite distance in finite time

Special Relativity Executive Summary

⋆ Special Relativity:
includes high-speed motions (near c), doesn't include gravity

⋆ Space & Time
apparent distances, time intervals, simultaneity
not universal but depend on relative motion

⋆ Energy & Mass
can be converted into each other, mass is form of energy

⋆ Cause & effect ("causality")

- information cannot travel instantaneously
- actions are "local" in the sense that
  effects transmitted over finite distance in finite time

What About Gravity?

Special relativity beautifully accommodates light
(and all of electricity & magnetism)
but ignores gravity

How to include? consider Newton gravity force law

$$F_{\text{grav}} = \frac{GMm}{R^2} \tag{11}$$

gravity force due to mass M depends on present distance R
and spreads over all space ($F \neq 0$ for any $R < \infty$)

Einstein sez: this is totally illegal! an unmitigated disaster!
Q: why? what's the problem?

Newton: mass M exerts force on *any* mass m
determined by *present* distance R

$$F_{\text{grav}} = \frac{GMm}{R^2} \tag{12}$$

but if M moves $\rightarrow$ R changes
Newton's gravity law then implies that
  $\rightarrow$ gravity force changes instantaneously over all space!
  no signal–including gravity–can move faster than c!

Big Al concludes: *verboten! gotta be wrong!*
major changes needed!

The Equivalence Principle Revisited

How to go about revising gravity? Where to start?

Recall Galileo atop the Tower of Pisa:
gravity $\rightarrow$ all objects move (accelerate) the same way in free fall
regardless of object mass, shape, composition not new result,
but different explanations...

Newton sez:
it just so happens that gravitational mass
the way objects "feel" or "couple to" gravity $F_{\mathrm{grav}} = m_{\mathrm{grav}} g$
is always exactly the same as inertial mass
the way objects resist acceleration $a = F/m_{\mathrm{inert}}$

Einstein sez:
too amazing to be a coincidence, must be deeper...

Einstein's Equivalence Principle

Einstein notes:
Gravity causes acceleration, but in "democratic" way:
all objects accelerate the same

Einstein's Equivalence Principle:
in a closed room, no experiment can distinguish
(non-gravitational) acceleration from gravity
Note similar "feel" to Einstein's Relativity Principle

But note: acceleration is aspect of motion
relates to objects' travel through space and time
$\rightarrow$ gravity=acceleration equivalence will have impact
(i.e., bizarreness) on space and time

Director's Cut Extras

Extra for the Technorati: Invariants and the Interval

Different observers (typically) disagree about space, time
Is there *anything* they *do* agree about? yes!

Recall: relativistic $\vec{p}$, $E = \sqrt{(mc^2)^2 + (c|\vec{p}|)^2}$: observer-dependent
but: $(mc^2)^2 = E^2 - (c|\vec{p}|)^2$ same for everyone
$\rightarrow$ "invariant" quantity! everyone agrees on its value!

Another key example: two events and their "distance"

- *no* general agreement on separation in time Δt or in space $\Delta \vec{x}$
- *but* everyone agree on the value of $(\Delta s)^2 \equiv (c\Delta t)^2 - |\Delta \vec{x}|^2$, the "interval"

relativity built on relationships among invariant quantities
and how to connect these to experiences of observers

For the Technorati: More on the Interval

for two nearby events: different observers with different motions

- *disagree* on event separations in time Δt and space Δx
- but *agree* on **interval** Δs
  which each observer calculates from his/her Δt, Δx:

$$(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2 \tag{13}$$

Example: observer A snaps fingers twice
all while bystander B sees A move at speed v

Interval according to A
all observers perceive self at rest
$\rightarrow$ $(\Delta x)_A = 0$ and $(\Delta t)_A =$ time between snaps, and
calculates interval $(\Delta s)_A = c(\Delta t)_A$
$\rightarrow$ interval is $c\times$ time diff for observer located at both events!

Interval according to B

bystander B sees A moving at speed v

$\rightarrow$ in time interval $(\Delta t)_B$ sees A move dist $(\Delta x)_B = v(\Delta t)_B$

calculates interval

$$(\Delta s)_B = \sqrt{(c\Delta t)_B^2 - (\Delta x)_B^2} = (c\Delta t)_B \sqrt{1 - \frac{(\Delta x)_B^2}{(c\Delta t)_B^2}} = (c\Delta t)_B \sqrt{1 - \frac{v^2}{c^2}} \tag{14}$$

Invariance

But interval is invariant, so $(\Delta s)_A = (\Delta s)_B$ and thus

$$(\Delta t)_{\text{rest}} = (\Delta t)_A = (\Delta t)_B \sqrt{1 - \frac{v^2}{c^2}} = (\Delta t)_{\text{moving}} \sqrt{1 - \frac{v^2}{c^2}} \tag{15}$$

we recover the time dilation formula!