Astro 350

Lecture 23

March 21, 2022

Announcements:

- **Discussion 6 due Wednesday**
- **Homework 6 due Friday**

Before break: the General Theory of Relativity
  a theory linking space, time, gravity, and matter

This week: General Relativity illustrated–*black holes*

Thereafter: General Relativity illustrated–*the Universe*

## Anachronistic News Network
### Point⇔Counterpoint: Gravity

Moderator–G. Galilei: The discussion today–what is gravity? Ike, what say you?

I. Newton: Gal, gravity is but one example of a **force**–a particularly beautiful one, to be sure (did I mention I invented it?) but a force like any other (e.g., the electrostatic force of that Frenchman, Coulomb).

Galilei: Let me stop you right there, Ike. My work in Pisa has shown that all objects fall at the same rate, regardless of their mass, shape, or composition. If you understood my work, you'd realize that that gravity is a uniquely special force!

Newton: There you go again, Gal. If you'd let me finish, I was going to explain. Any object has an **inertial mass**, which determines its response to acceleration–making massive objects harder to speed up: $a \propto F/m_{\text{intertial}}$. But an object also has **gravitational mass** which sets the strength of its coupling to the gravitational field–making massive objects heavier: $F_{\text{grav}} = m_{\text{grav}}\, g$. Now it so happens that an object's inertial mass is the *same* as ist gravitational mass. This particular accident leads to the result you observed and I explained.

A. Einstein: I have to jump in here. We know that objects fall the same way, regardless of any of their properties. This is the **equivalence principle**. Gravity is a unique interaction in being so democratic in its influence. Ike would have us believe that this is just a curious coincidence. But since falling is so universal, its origin is obviously best found in the very structure of space and time, not in some miraculous force. This is the heart of General Relativity (which by the way behaves the same as Newton's theory when gravity is weak). GR says that spacetime is "curved" and that motion in a gravity field ("falling") is a response to this curvature, much as a ball rolling on a dimpled, rubber sheet.

A. Einstein: I might add that this curvature has many consequences, one of which is that even light rays should be bent by gravity. Which by the way I predicted and was confirmed in observations during the 1919 solar eclipse. In your face, Ike!

Galileo: Ouch–that's gotta sting! More when we come back after these important messages from our sponsors...

# Black Holes

Laplace (1790's):

escape velocity $v_{\text{esc}} = \sqrt{2GM/R}$

What if star has mass $M$ and radius $R < 2GM/c^2$?

then $v_{\text{esc}} > c$ !

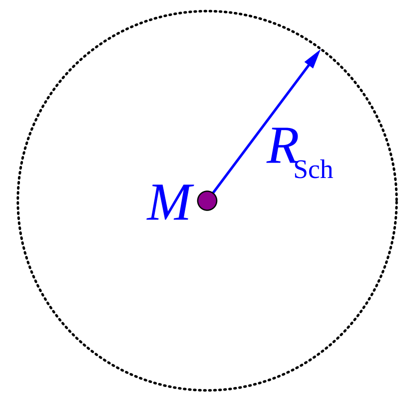
light cannot escape! $\rightarrow$ black hole

Wrong argument (Newtonian gravitation) ...but right answer!

General relativity predicts existence of black holes
and their properties

# Black Hole Properties

*any* object of *any* mass $M$ can (in principle)
become a black hole!
here: non-spinning case




**size: Schwarzschild radius**

$$R_{\rm Sch} = \frac{2GM}{c^2}$$

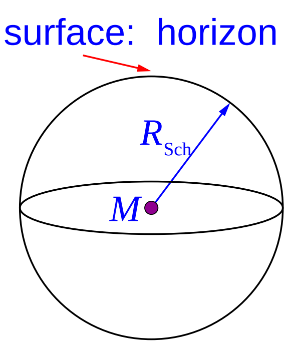
radius also provides BH "recipe":

- *crush* object $M$ *smaller than* $R_{\rm Sch}$ $\rightarrow$ *get BH!*
- example: for mass of Sun $R_{\rm Sch} = 2GM_{\odot}/c^2 = 3.0$ km
  but actual $R_{\odot} = 7 \times 10^6$ km
  $\rightarrow$ the Sun is not a black hole! (whew!)
- for mass of Earth: $R_{\rm Sch} \approx 1$ cm!

# The Black Hole Horizon

Why call $R_{\text{Sch}}$ the BH radius? nothing is there!

True, but: $R_{\text{Sch}}$ marks "point of no return"
**horizon**: surface enclosing the BH
i.e., horizon is surface of sphere w/ radius $R_{\text{Sch}}$




horizon is one-way "membrane"
*once inside $r \leq R_{\text{Sch}}$ nothing can escape...even light!*
cosmic roach motel!

Hence:
no light escapes → black
but nothing else moves as fast → nothing else escapes → hole

# Life Near a Black Hole

Experiment: lower astronaut (Jodie) near $R_{\rm Sch}$
we are at mission control, far away ($r_{\rm us} \gg R_{\rm Sch}$)
communicate w/ light signals

when viewing photons (or clock ticks)
emitted at $r_{\rm em}$, observed at $r_{\rm obs}$
general rule:

$$\frac{\Delta t_{\rm obs}}{\Delta t_{\rm em}} = \frac{\lambda_{\rm obs}}{\lambda_{\rm em}} = \sqrt{\frac{1 - R_{\rm Sch}/r_{\rm obs}}{1 - R_{\rm Sch}/r_{\rm em}}} \tag{1}$$

**What do we see?**

obs=us: $r_{\rm obs} \rightarrow \infty$; em=Jodie: $r_{\rm em} > R_{\rm Sch}$

- Jodie's watch: $\Delta t_{\rm obs}/\Delta t_{\rm em} = 1/\sqrt{1 - R_{\rm Sch}/r_{\rm em}} > 1$
  $\rightarrow \Delta t_{\rm obs} > \Delta t_{\rm em}$! appears to tick slow! time dilation!
- wavelengths: $\lambda_{\rm obs} > \lambda_{\rm em}$! redshift!

*Q: and Jodie?*

## What do we see?

intuitively: expect inequalities to reverse...and they do

obs=Jodie: $r_{\text{obs}} > R_{\text{Sch}}$; em=us: $r_{\text{em}} \to \infty$:

- our watches: $\Delta t_{\text{obs}}/\Delta t_{\text{em}} = \sqrt{1 - R_{\text{Sch}}/r_{\text{em}}} < 1$
  $\to \Delta t_{\text{obs}} < \Delta t_{\text{em}}$! appears to tick fast!
- wavelengths: $\lambda_{\text{obs}} < \lambda_{\text{em}}$! blueshift!

When Jodie returns:

then $r_{\text{em}} = r_{\text{obs}}$

- $\Delta t_{\text{obs}} = \Delta t_{\text{em}}$: her watch ticks at same rate as ours!
- but the *elapsed time* is shorter on her watch
  and so she is younger than her twin!

# Poll: Black Holes

From a safe distance, you drop an object (nuclear waste? Voldemort?) on an isolated black hole.

Will you see it fall in?

A yes, no matter your distance from the hole

B maybe, depends on how far you are from the hole

C no, because it never actually falls in

∞ D no, although it does actually fall in

# Falling Into a Black Hole

No barrier, bells, or whistles at horizon
infalling objects go right through

seen from afar, time dilation and redshift progressively severe as object approaches horizon

progressively strong relativistic flux reduction

so as seen from afar:

- time elapse slows until appears “frozen”
- signal redshifts
- image fades until last photon emitted before horizon crossing and then object gone–and black hole mass higher

## Life Inside a Black Hole

once inside $R_{\text{Sch}}$, no getting out
all matter $\rightarrow$ center $\rightarrow$ point (?): "singularity"
i.e., finite mass $M$ in volume $V = 0 \rightarrow$ density $\rho = M/V \rightarrow \infty$!
D'oh! known laws of physics break down

A few remarks:

- we know that all observers travel to center
- don't know what happens once there
- regardless, certain that you die if you go in
- in a way, it's not a relevant question, since can't get info out even if went in (no Nobel Prize!)
- once crushed to $< 10^{-33}$ cm, quantum mechanics important
  i.e., need quantum theory of relativistic gravity!
  ... but there isn't one...yet
- if you have quantum gravity theory, please tell instructor and we'll publish it (your name may even go first!)

## Poll: You Thought the BP Spill Was Bad

Experiment:
Industrial accident causes Sun to be crushed to black hole
Spokesdroid from Interplanetary BP: “Mistakes were made.”

Vote your conscience!

What happens to Earth’s orbit?

A nothing: same orbit!

B spirals in: aaargh!

C stronger gravity, but does not fall in

# Life Far From a Black Hole

No change in orbit!

Newtonian explanation: wrong in detail, but correct spirit:
when **outside** of Sun, gravity acceleration is
$a = GM_\odot/r^2$: only $M$ matters
gravity same as if Sun were $1M_\odot$ BH

gravity outside star **not** increased by becoming BH
no more pull than before!
$\rightarrow$ "black hole threat" not any more dangerous than
"nearby star gravity" threat

So sleep well tonight!

# Black Holes: From Theory to Observations??

So far: discussed *predicted* black hole properties
that is: General Relativity says
black holes *can* exist in nature
but question remains: is there *evidence*
that black holes *do* exist in nature?

recall: in death of some massive stars
- core collapse
- crush to high density: proto neutron star

we observe neutron stars and pulsars
thus: some proto neutron stars stable against collapse
...but not necessarily all!

*Q: how could we detect black holes? No light escapes!*

# Evidence for Black Holes

how detect? no light emitted from BH, but:
*can observe matter near a BH*, interacting with it

**X-ray binaries: stellar-mass black holes (few $M_\odot$)**
massive star born in bound system with less massive star
larger star $\rightarrow$ SN $\rightarrow$ BH left behind
if supergiant companion, close orbit:
some gas falls onto BH $\rightarrow$ compressed, heated $\rightarrow$ X-rays

what you see: giant star orbiting unseen massive companion,
and emitting X-rays
`www: Cygnus X-1`

# Our Own Galactic Center

central $\sim 30$ pc of Galaxy:
can't see optically (*Q: why?*), but can in other wavelengths:
extended (non-point) radio emission (Sagittarius A)
from high-energy electrons

radio source at center: $\boxed{\text{Sgr A}^*}$
size 2.4 AU(!), variable emission in radio, X-ray
`www: X-ray Sgr A*`

in infrared wavelengths: can see stars near Sgr $A^*$
and they move! `www: Sgr A* movie`
elliptical paths! closest: period $P = 15.2$ yr
semi-major axis: $a = 4.64 \times 10^{-3}$ pc
$\rightarrow$ enclosed mass $(3.7 \pm 1.5) \times 10^6 M_\odot$
*Q: and so?*

the center of our Galaxy contains a black hole!

Sgr A* Schwarzschild radius

$$r_{\text{Sch}} = 1.1 \times 10^7 \text{ km} = 0.74 \text{ AU} = 3.6 \times 10^{-7} \text{ pc} \quad (2)$$

→ not resolved (yet) but: *Event Horizon Telescope*
has data and right now is processing possible first images!

Galactic black hole is a triumph: **Nobel Prize 2020**!
But also raises many questions:

- how did it get there?
- Sgr A* low luminosity, "quiet"
  compared to more "active" galactic nuclei `www: AGN: M87`
  why? open question....
- in last few months: discovery of high-energy "bubbles"
  above & below Galactic center `www: gamma-ray images`
  → remains of the most recent Sgr A* belch?