Astro 350

Lecture 28

April 1, 2022

Announcements:

- **Homework 7 due today** – no April Fools
- **Discussion 7 due Wednesday**

Last time: explaining Hubble–an expanding universe

Q: what is the cosmic scale factor?

Q: what does it measure?

Q: what is its value today? in the past? the future?

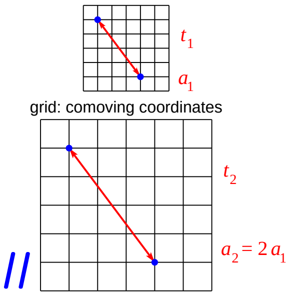
Q: what is scale factor in static, non-expanding universe?

at *any time* t, distance ℓ between A and B is

$$\ell(t) = \quad a(t) \quad \times \quad \vec{\ell}_0$$

AB distance at t — scale factor — present AB distance

time varying — *time varying* — *fixed once and for all*



cosmic time *today* $t = t_0$: scale factor $a(t_0) = 1$

in cosmic *past* $t < t_0$: scale factor $a(t) < 1$

in cosmic *future* $t > t_0$: scale factor $a(t) > 1$

in a static, non-expanding universe: $a(t) = 1$ always!

Expansion: Einstein $\rightarrow$ Hubble

for two arbitrary observers (e.g., "galaxies")
scale factor gives distances $\vec{r}(t) = \vec{r}_0 a(t)$
so velocity is

$$\vec{v}(t) = \frac{\dot{a}}{a}\, a\vec{r}_0 = H(t)\vec{r} \tag{1}$$

with shorthand notation: time rate of change $\dot{a} = da/dt$

This means:
$\Rightarrow$ In expanding U, everyone observes Hubble law!

- now interpret "Hubble parameter" $H(t)$ as

$$\boxed{H(t) = \dot{a}/a} \tag{2}$$

  expansion rate at time t
- $H(t_0) = H_0 =$ expansion rate today
- but expansion rate need not be (and usually isn't) constant!
- In static universe: $\dot{a} = 0$ so $H = 0$, no expansion!

Redshifts

Recall: General Relativity predicts gravitational redshifts

in cosmology: wavelengths are *lengths*!
...it's right there in the name!
expansion stretches photon wavelength: $\lambda \propto a$

if emit photon at $t_{\rm em}$ with wavelength $\lambda_{\rm em}$, then
$\lambda(t) = \lambda_{\rm emit} a(t)/a(t_{\rm em})$

if observe later,
$\lambda_{\rm obs} = \lambda_{\rm em}\ a_{\rm obs}/a_{\rm em}$
measure redshift today:

$$z = \frac{\lambda_{\rm obs} - \lambda_{\rm em}}{\lambda_{\rm em}} = \frac{1 - a_{\rm em}}{a_{\rm em}} \quad \Rightarrow \quad a_{\rm em} = \frac{1}{1+z}$$

Scale factor $\leftrightarrow$ redshift corresondence

$a = 1/(1+z)$

$z = 1/a - 1$

`www: Sloan Digital Sky Survey spectra`

`www: redshift recordholder`

Example: most distant galaxy has $z = 11.09$
$\rightarrow$ scale factor $a = 1/(1+11.09) = 0.083$
interparticle (intergalactic) distances 8.3% of today!
$\rightarrow$ galaxies 1+11.09=12.09 times closer
squeezed into volumes $(12.09)^3 = 1800$ times smaller!

Recall from General Relativity, black hole discussions
gravitational redshifting often accompanied by...
Q: what? and how might you observe this?

Cosmic Time Dilation

GR: gravitational redshifting goes hand-in-hand with gravitational time dilation
$\rightarrow$ i.e., redshifted objects also appear to have slow clocks
and blueshifted objects appear to have fast clocks

Cosmic time dilation observed! And only recently!
Challenge: need "standard clock" in order to know
that it's running slow
Tool: exploding stars (supernovae) – know timing of brightness
observe high-z supernovae, see lengthening of
duration in explosion and aftermath!
Woo hoo!

Q: how does expansion affect photon energy?
Q: for blackbody, how does expansion affect T?
hint: $T \leftrightarrow \lambda$ connection?

Expansion and Radiation Energy & Temperature

since $E_\gamma = hc/\lambda \propto 1/\lambda$, then
$E_\gamma \propto 1/a$ $\rightarrow$ photon energy redshifts, i.e., decreases with time

for thermal radiation, Wien's law: $T \propto 1/\lambda_{\mathrm{max}}$ so $T \propto 1/a \Rightarrow T$ decreases $\rightarrow$ U cools!
the universe cools as it expands

today: cosmic thermal radiation peaks at $\lambda \sim 1$ mm
"cosmic microwave background" radiation (CMB)
CMB temperature today: $T_0 = 2.725 \pm 0.001$ K
≈ 3 degrees above absolute zero

in past $\rightarrow$ CMB, universe hotter:
distant but still "garden variety" quasar: $z = 3$
"feels" $T = 8$ K (effect observed!)

iClicker Poll: A Pop Fly

A ball is launched upwards from the Earth's surface

What will happen later?

A it will eventually fall back down

B it will leave earth and never return

C either (a) or (b), depending on launch speed

∞

Cosmodynamics II

$a(t)$ gives expansion history of the Universe
but: *How does scale factor $a(t)$ grow with time?*

Cosmic Evolution: Intuition

Ballpark analogy: a pop fly
Q: what are possible fates?
Q: what factors influence which occurs?
Q: how would we predict which will occur?

The Universe is a pop fly!
Given current expansion of the Universe
Q: what are possible future outcomes?
Q: what factors influence which occurs?
Q: how would we predict which will occur?

Gravity & Fate: Baseball vs Cosmology

Pop fly: upgoing ball in Earth's gravity field; possible fates:
(1) fall back
(2a) leave Earth; $v \neq 0$ at infinity
(2b) leave Earth; $v = 0 \rightarrow$ "barely escape"

Factors:
gravity (downward)
vs inertia (upward)

How predict? Pop Fly
gravity $\rightarrow$ escape speed $v_{\rm esc} = \sqrt{2GM/R}$
inertia $\rightarrow$ launch speed v_0
$\rightarrow$ fate set by ratio $v_{\rm esc}/v_0$

What about the Universe?
same ideas! Gravity vs inertia!
will find similar key: gravity/inertia ratio

Cosmic Gravity and Expansion

consider two objects today, say two galaxies
presently at some distance r_0, say 100 Mpc

right now moving apart due to cosmic expansion
at speed $v_0 = H_0 r_0$

imagine "turning off" gravity–then:
Q: what are speeds at earlier times? later times?

Q: what sort of Universe (not necessarily ours!)
would actually have no gravity? Hint–what's the source of gravity?

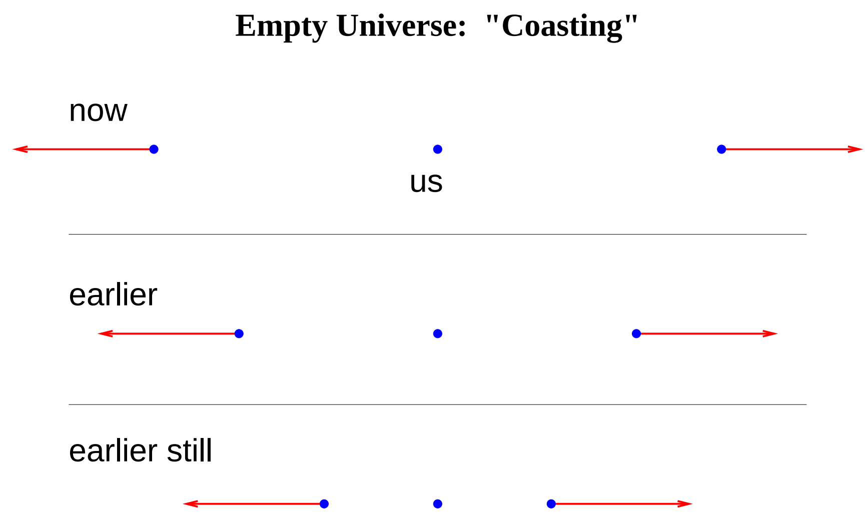
Matter-Free "Empty" Universe

Gravity source is *mass*
so a universe without mass has *no gravity*
$\Rightarrow$ this corresponds to an *empty universe*
with density $\rho = 0$

Obviously, this cannot be our Universe! *Q: why not?*

But: this case still useful:
- same as the "egoist" universe discussed earlier
- corresponds to a universe in which expansion (inertia) is much more important than gravity

No matter $\to$ no attraction between galaxies
$\to$ nothing to change galaxy speeds
$\to$ galaxies "coast" keeping constant velocity
$\Rightarrow$ same speeds in past



expansion rate:
neither accelerated **nor** decelerated

Q: what is the final fate of such a Universe?

Empty Universe: Final Fate

in "empty" universe, galaxies coast forever
i.e., expansion continues without slowing or stopping
$\Rightarrow$ the **universe expands forever**!

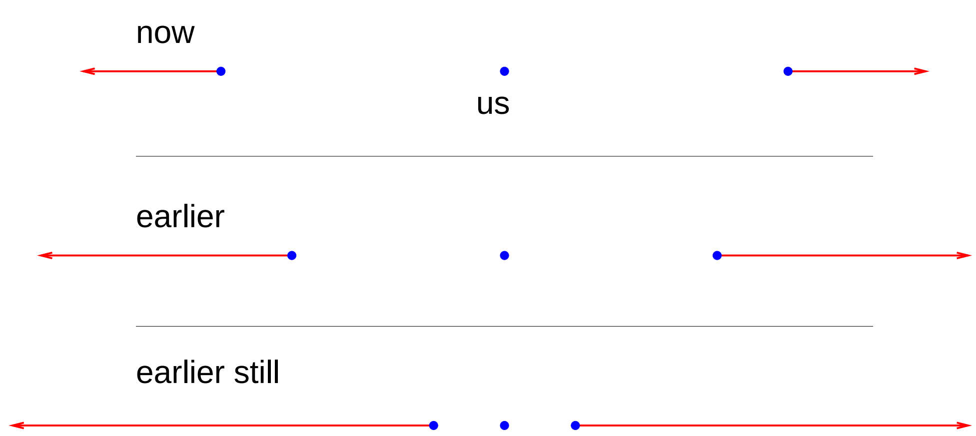
everything ever more spread out
Universe becomes ever more empty and cold
$\rightarrow$ known as "**the Big Chill**"

Q: so what do we expect in the real Universe?
Q: would galaxy speed be different in past?

Ordinary Gravity and Matter

The real universe has galaxies with mass → attract each other
→ inward gravity slows expansion
→ speeds constantly *decreasing*, galaxies *decelerating*
⇒ to achieve observed speed today, had to be *faster* in past!

Normal Gravity and Matter: Decelerating Universe



expansion rate: **decelerated**

A Denser Universe

now imagine we cram *more galaxies & mass*
in the every bit of cosmic volume today

this corresponds to a universe with *larger density* ρ

Q: will this change what we infer about the cosmic past? if so, how?

A High-Density Universe: the Past

higher density $\rightarrow$ more galaxies closer together
but every galaxy exerts gravity force on all others
so higher $\rho \rightarrow$ closer $\rightarrow$ stronger gravity

and for matter: stronger gravity = stronger attraction
and thus more drastic slowdown of expansion

so: for a *high-density* Universe, in the past
galaxies must have moved *even faster*
than in a low-density Universe

Q: what about the future in a low-ρ universe? what is it's fate?

A Low-Density Universe: the Future

We have seen: in a real universe with matter
(and thus nonzero density $\rho > 0$)
the attraction of gravity *slows* expansion

and thus:

- in the *past*, galaxies moved faster than now, and so
- in the *future*, galaxies will move *slower* than now

in a *low*-density universe:

- expansion slows, but never stops
- low-density $\rightarrow$ weak gravity
  too weak to overcome inertia!
- fate: expand forever, but speeds slower than now
  Big Chill strikes again

Q: how will the future and the fate be different in a very high-ρ universe?

A High-Density Universe: the Future

in a *high*-density universe:

- high density $\rightarrow$ strong gravity
  strong enough to overcome inertia!
- expansion slows until *stopping* momentarily
- but gravity will not stop! galaxies still attract each other!
- galaxies now move *toward* each other
- Universe begins to *contract*
  as they get closer, gravity stronger $\rightarrow$ galaxies faster
- continues until Universe *collapses* on itself!
  fate known as the **Big Crunch**

Q: what lessons do we draw about cosmic history and evolution?

Density and Destiny

We have see:

- a high-density universe has a different expansion history than a low-density universe
- namely: a normal-matter high-ρ universe decelerates & slows more rapidly than a low-ρ universe and expanded *even faster* in the past
- the future fate of the cosmos is very different depending on the cosmic density

Lessons:

- different cosmic fates are possible!
- the evolution and fate of the Universe depends on what's in the Universe
- namely: cosmic fate depends on cosmic density
- weight is fate! density is destiny!

Director's Cut Extras

Math Alert!

the next few slides are more math-y than usual
and are aimed students with more technical backgrounds

see how much of the math you can follow
but don't worry about parts you *don't* follow

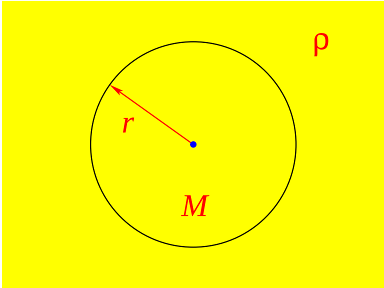
but *do* understand the basic ideas that
go into the analysis, and what we get out

strategy: **Newton says:** $F = ma$
apply this to a Universe that is

- homogeneous
- expanding

Cosmic Evolution: Quantitative Analysis

full description: comes from General Relativity
quick 'n dirty: Non-relativistic (Newtonian) cosmology



at time t, pick arbitrary point as origin $\vec{R} = 0$,
enclose in arbitrary sphere of radius $R(t)$:

enclosed mass $M(R) = 4\pi/3\ R^3\rho = const$
consider a small "test" mass m on edge of sphere
"feels" gravity due to sphere mass
Q: what is Newtonian acceleration of test mass?

Newtonian Cosmodynamics

a mass m accelerates due to force: $m \times \text{accel} = F$
if force due to gravity–free fall–then $F = GM(R)m/R^2$
and so acceleration is

$$m\ddot{R} = -\frac{G\ M(R)m}{R^2} \quad (3)$$

where $-$ sign reminds us gravity is *attractive Q: how*?

but note–"test" mass cancels (equivalence principle), so

$$\ddot{R} = -\frac{G\ M(R)m}{R^2} = -\frac{4\pi}{3}G\rho R \quad (4)$$

Newton sez:

$$\ddot{R} = -\frac{4\pi}{3} G \rho R \tag{5}$$

Hubble & Einstein say:

Universe is expanding, so sphere radius moves according to scale factor: $R(t) = a(t)\ R_0$

$$\ddot{a} R_0 = -\frac{4\pi}{3} G \left(\rho + 3\frac{P}{c^2}\right) a R_0 \tag{6}$$

$$\ddot{a} = -\frac{4\pi}{3} G \left(\rho + 3\frac{P}{c^2}\right) a \tag{7}$$

- R_0 cancels! scale factor accel indep of sphere size! had to be this way $\rightarrow$ cosmo principle
- Einstein adds term with pressure P

Q: what is Newtonian energy of test mass?

Newtonian Cosmodynamics II: Energy

test mass m at edge of gravitating sphere has energy

$$\text{kinetic} + \text{potential} = \text{total} \tag{8}$$

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = E \tag{9}$$

solve for speed v:

$$v^2 = 2\frac{GM}{R} + 2\frac{E}{m} \tag{10}$$

$$= \frac{8\pi}{3}G\rho R^2 + 2\frac{E}{m} \tag{11}$$

Q: what do Hubble and Einstein say about v?

Newton says:

$$v^2 = \frac{8\pi}{3} G\rho R^2 + \frac{2E}{m} \tag{12}$$

Hubble and Einstein say:
speed $v = HR = \frac{\dot{a}}{a} R$, so

$$H^2 R^2 = \dot{R}^2 \;=\; \frac{4\pi}{3} G\rho R^2 + \frac{2E}{m} \tag{13}$$

expansion technology: $R(t) = a(t) R_0$

$$H^2 a^2 = \dot{a}^2 = \frac{4\pi}{3} G\rho a^2 - K \tag{14}$$

The Friedmann Equations

Friedmann Acceleration Equation

$$\text{cosmic acceleration} = \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3P}{c^2}\right) \tag{15}$$

important features:

- *Q: significance of − sign?*

Friedmann Equation (“Energy Eq.”)

$$(\text{cosmic expansion rate})^2 = H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\rho - \frac{K}{a^2} \tag{16}$$

where K is a constant

- *Q: how does expansion rate depend on contents of U?*

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3P}{c^2}\right) \tag{17}$$

note $-$ sign:

- due to attractive nature of gravity
- galaxy gravity on each other slows expansion

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G\rho - \frac{K}{a^2} \tag{18}$$

- for any time t, relates expansion rate $H(t)$ = change in a to constant K and values of $\rho(t), a(t)$ at t
- cosmic contents (density) influences expansion
- K term can be important – or zero!
  value, sign of constant K has to be measured

Q: so what's the big picture–what just happened?

Post-Math Aftermath

What just happened?

Inputs: • Newton's laws
• homogeneous, isotropic Universe, that is
• expanding

Outputs: Friedmann equations
expressions for how scale factor a changes with time
• expansion rate: time change of a
• acceleration rate: time change of expansion

These give a precise mathematical statement
of what we already found: