Astro 350
Lecture 29
April 4, 2022

Announcements:

- **Discussion due Wednesday**
- **Homework due Friday**

Last time: comic dynamics – what controls cosmic evolution?

Q: time changes of galaxy speeds in empty U?

Q: time changes of galaxy speed in U with matter?

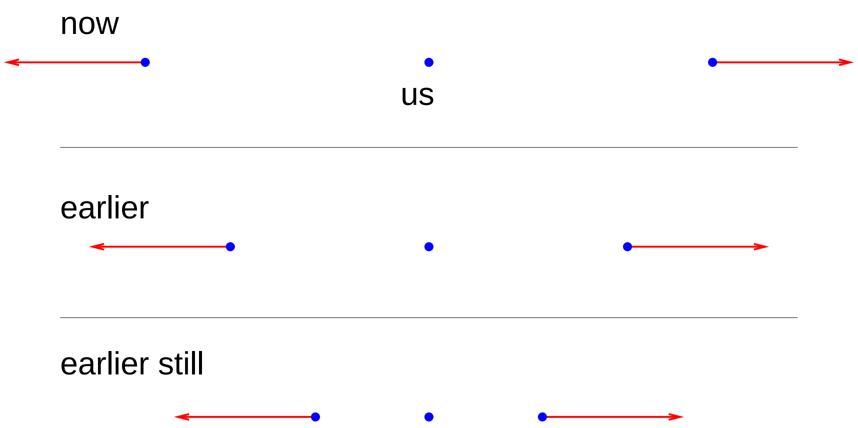
Q: fate of U with low matter density ρ? high ρ?

Matter-Free “Empty” Universe No matter → no attraction between galaxies

→ nothing to change galaxy speeds

→ galaxies “coast” keeping constant velocity

Empty Universe: "Coasting"



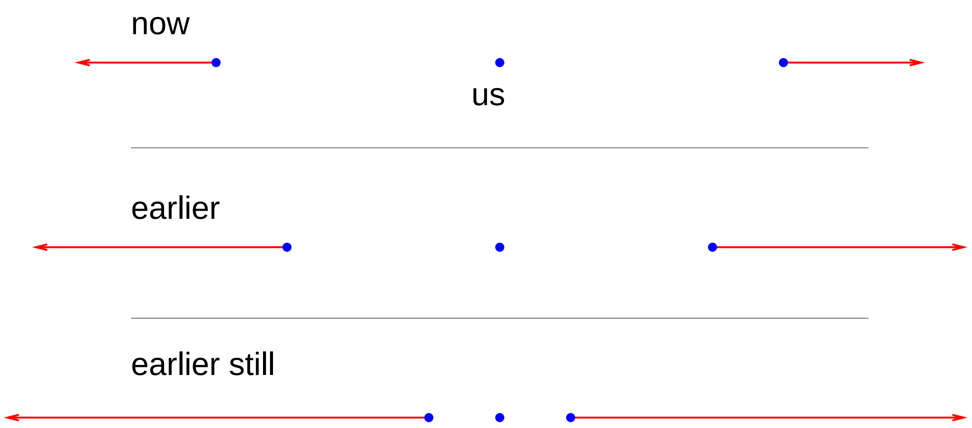
expansion rate:

neither accelerated **nor** decelerated fate: the **universe expands forever**!

everything ever more spread out
Universe becomes ever more empty and cold
→ known as “**the Big Chill**”

Matter-full Universe

The real universe has galaxies with mass → attract each other
→ inward gravity slows expansion
→ speeds constantly *decreasing*, galaxies *decelerating*
⇒ to achieve observed speed today, had to be *faster* in past!



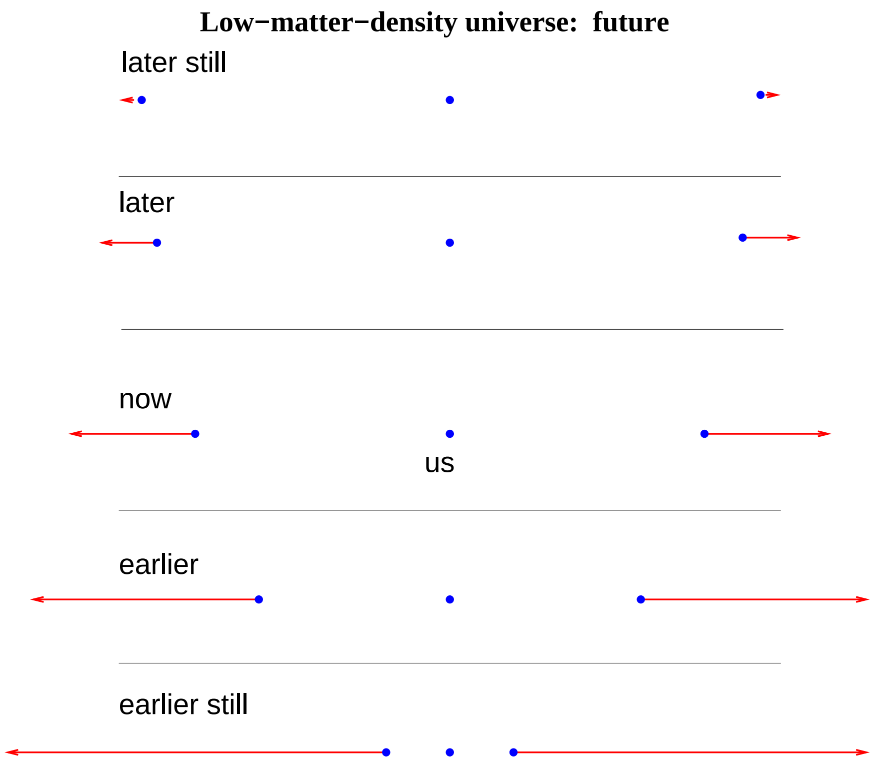
Normal Gravity and Matter: Decelerating Universe

expansion rate: **decelerated**
Q: plot of a vs t for this universe?
Q: fate of such a universe? why?

Low-Density Universe: Fate

in a *low*-density universe: weak gravity

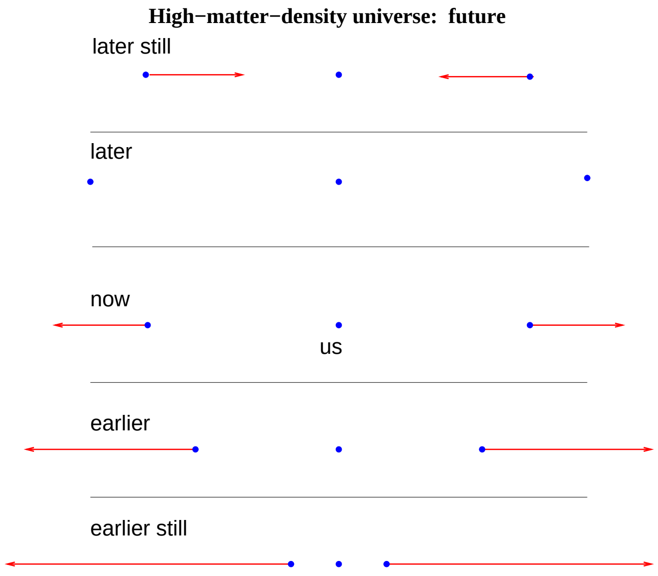
- expansion slows, but never stops
- fate: expand forever, but speeds slower than now



High-Density Universe: Fate

in a *high*-density universe:

- high density → strong gravity
  strong enough to overcome inertia!
- expansion slows until *stopping* momentarily
- but gravity will not stop!
  galaxies still attract each other!
- galaxies now move *toward* each other
- Universe begins to *contract*
  as they get closer, gravity stronger → galaxies faster
- continues until Universe *collapses* on itself!
  fate known as the **Big Crunch**



Q: plot of a vs t for this universe?

Q: what lessons do we draw about cosmic history and evolution?

Density and Destiny

We have seen:

- a high-density universe has a different expansion history than a low-density universe
- namely: a normal-matter high-ρ universe decelerates & slows more rapidly than a low-ρ universe and expanded *even faster* in the past
- the future fate of the cosmos is very different depending on the cosmic density

Lessons:

- different cosmic fates are possible!
- the evolution and fate of the Universe depends on what's in the Universe
- namely: cosmic fate depends on cosmic density
- weight is fate! density is destiny!

Cosmic Dynamics: Friedmann Equations

Newton: gravity acceleration due to gravitating mass M

$$\text{acceleration} = \frac{d^2r}{dt^2} = -\frac{GM}{r^2}$$

Einstein: acceleration on universe with scale factor a density ρ, and pressure P

$$\begin{aligned} \text{acceleration} &= \frac{d^2a}{dt^2} = \ddot{a} = -\frac{G(\rho + 3P)\ 4\pi a^3/3}{a^2} \\ \frac{\ddot{a}}{a} &= -\frac{4\pi}{3} G\,(\rho + 3P) \end{aligned}$$

Newton: energy of "test particle" of mass m
object moving in gravity of mass M

$$\begin{aligned} E_{\text{tot}} &= -\frac{GMm}{r_{\text{average}}} = \frac{1}{2}mv^2 - \frac{GMm}{r} \\ v^2 &= \frac{2GM}{r} - \frac{2GMm}{r_{\text{average}}} \end{aligned}$$

Einstein: expansion rate in universe with a and ρ

$$H^2 a^2 = \frac{8\pi}{3} G\rho a^2 - K$$

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G\rho - \frac{K}{a^2} \tag{1}$$

with K a constant

The Friedmann Equations

Friedmann Acceleration Equation

$$\text{cosmic acceleration} = \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3P}{c^2}\right) \tag{2}$$

important features:

- *Q: significance of − sign?*

Friedmann Equation ("Energy Eq.")

$$(\text{cosmic expansion rate})^2 = H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\rho - \frac{K}{a^2} \tag{3}$$

where K is a constant

- *Q: how does expansion rate depend on contents of U?*

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3P}{c^2}\right) \tag{4}$$

note $-$ sign:

- due to attractive nature of gravity
- galaxy gravity on each other slows expansion

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\rho - \frac{K}{a^2} \tag{5}$$

- for any time t, relates expansion rate $H(t) =$ change in a to constant K and values of $\rho(t), a(t)$ at t
- cosmic contents (density) influences expansion
- K term can be important – or zero!
  value, sign of constant K has to be measured

Q: so what's the big picture–what just happened?

Post-Math Aftermath

What just happened?

Inputs:

- Newton's laws
- homogeneous, isotropic Universe, that is
- expanding

Outputs: Friedmann equations

expressions for how scale factor a changes with time

- expansion rate: time change of a
- acceleration rate: time change of expansion

These give a precise mathematical statement
of what we already found:
in the expanding universe: galaxies have inertia
resist change in speed gravity acts to change speeds

Studying the History of Cosmic Expansion

at any cosmic "moment"
(“epoch”− measured by time t and/or redshift z)
the cosmic expansion H directly related to
cosmic contents ρ: gravity sources that change expansion

So: imagine we could measure expansion rate H
at many different epochs, finding $H(z)$ at many z

this determines the cosmic expansion history
then Friedmann → density history, e.g, $\rho(z)$

★ cosmic motion ↔ cosmic contents
★ expansion measures cosmic density!
★ can compare with expectations if $\rho = \rho_{\text{matter}} + \rho_{\text{radiation}}$

iClicker Poll: Effect of Dark Matter

We have seen that galaxies are *mostly* made of *dark matter*, which holds galaxies together

compared to a universe with only the visible galaxies
the effect of dark matter should be to?

A add gravity, slowing expansion over time

B add gravity, speeding up expansion over time

C reduce gravity, slowing expansion over time

D reduce gravity, speeding up expansion over time

Expansion Archaeology

Goal: measure expansion rate at past times

Strategy:

recall Hubble's law $v(t) = H(t)\ r(t)$: always true
but as you look at distant objects
light travel time becomes large ("time machine effect")
$\rightarrow$ no longer probing expansion today,
  when rate is $H(t = t_0) = H(z = 0) = H_0$
  but rather expansion in past $H(\text{high} - z)$
$\Rightarrow$ Can use this to get expansion history
  key requirement: need distance r to high-z objects

Q: what techniques are available?

Supernovae and Cosmodynamics

goal: measure expansion out to high z

key tool: **standard candle** $\equiv$ object of known L
measure flux F, then $d_{\text{lum}} = \sqrt{F/4\pi L}$

need objects which:

- have fixed L indep of z, environment
- can see at high z → high L → supernova explosions

- Massive stars → SN: Type II
  bright, but: L varies w/ mass, metallicity ✗ bad idea!
- SN Type Ia: exploding white dwarf
  WD → fixed mass of ^{56}Ni (radioactive) → ^{56}Fe
  decay sets $L(t)$ → std candle!
  `www: SN 1994D`

iClicker Poll: Past Expansion Rate

in the real universe, with gravity

How should the expansion rate change with time?

A expansion slows with time → faster in past

B expansion constant with time → same in past

C expansion speeds up with time → slower in past

SN Ia and Expansion History

SN Data: distance indicator ($d_{\text{lum}}(z)$ "luminosity distance") traces expansion history $H(z) = \dot{a}/a$

- but expansion $H = \dfrac{\text{rate of change in } a}{a}$: a kind of "velocity"
  so variation in $H \to$ change in "velocity"
- H history measures cosmic acceleration

expectations: acceleration $= \ddot{a}/a = -\dfrac{4\pi G}{3}\left(\rho + 3\dfrac{P}{c^2}\right)$

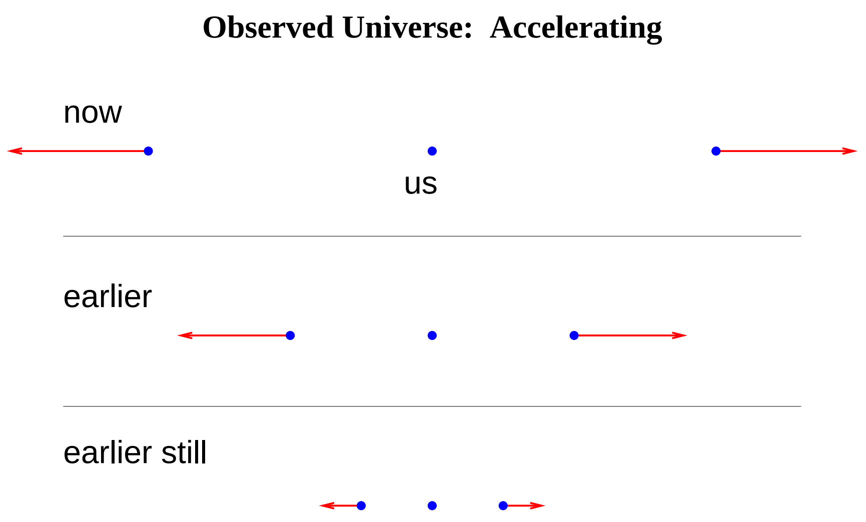
$\Rightarrow \ddot{a} < 0$ gravity slows expansion

Pop fly Analogy

galaxies moving apart today ↔ upgoing ball
would coast forever, but gravity attractive
slowing galaxies ↔ slowing ball
→ faster in past

Distant Supernovae: The Verdict

Our actual observed universe:
galaxies *slower* in the past!



SN data: $H(z)$ **smaller** in the past (high z and small t)
$\Rightarrow H(z)$ **increases** with time!
$\Rightarrow \ddot{a} > 0$!
expansion rate: **accelerated!**
Q: what would this mean in the pop fly analogy?

Accelerating Universe: Pop Fly Analogy

Pop fly: ball thrown up in the air
ordinary baseballs: made of matter, feel Earth's gravity
→ moves ever slower on the way up
→ decelerated

but the *Universe* does the opposite!
a pop fly acting like the Universe
would get *faster* as it gets higher!
and so would launch itself to space!?!

2011 Nobel Prize in Physics

given to Saul Perlmutter, Brian Schmidt, and Adam Riess

`www: 2011 Nobel Prize`

> for the discovery of the accelerating expansion of the Universe through observations of distant supernovae

Q: why is this Nobel-worthy?

Q: notice what the prize does not *mention?*
that is, what does their work not *tell us?*

An Accelerating Universe: Implications

Recall: expected deceleration because ordinary matter
(even dark matter!) has gravitational attraction
But: found acceleration → something present which
has gravitational repulsion! "antigravity"!?!
...and enough of it to overwhelm the attraction
of ordinary matter!

Pop Fly Analogy

Universe is accelerating
→ galaxies faster as they move apart
so upgoing ball would speed up, zoom away from view!!

In more detail: supernova data says $\ddot{a} > 0$

but the Friedmann equation for acceleration says

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3\frac{P}{c^2}\right)$$

$\Rightarrow$ positive acceleration means $\rho + 3P/c^2 < 0$

$\Rightarrow$ which also means $P < -3\rho c^2$: negative pressure!?!

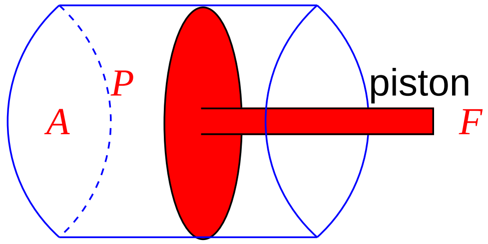
Physical "interpretation":

Pressure is force per area

so $F =$ pressure × area

$P > 0$: outward force (e.g., ideal gas)

$P < 0$: inward force (e.g., elastic)



Cosmic Expansion History: the Full Story

in fact, observations show that cosmic expansion
had *two* phases

⋆ *today and in the recent past*
expansion is **accelerating**
opposite of prediction from matter + General Relativity

⋆ *in the more distant past*
at redshifts $z > 0.3$, times before $t < 10$ billion years
that is, more than about 3 billion years ago
expansion was **decelerating**
agrees with prediction from matter + General Relativity!

Cosmic Acceleration: Who Ordered That?!

ordinary matter and ordinary gravity: **attractive**
gravity acts to draw galaxies together,
$\Rightarrow$ slows outward expansion
should give cosmic **deceleration**

but we observe cosmic **acceleration**!
two known options:

1. Universe contains something bizarre that pushes objects apart!
in fact, this repulsion has to be so strong that it overcomes gravity attraction from matter!

2. *Q: what's the other option?*