Astro 404
Lecture 6
Sept. 3, 2021

Announcements:

- **Problem Set 1 due today at 5pm**
  upload pdf file on Canvas, check that it is legible
- **Problem Set 2 will be posted today, due next Friday**

Note on lecture notes:

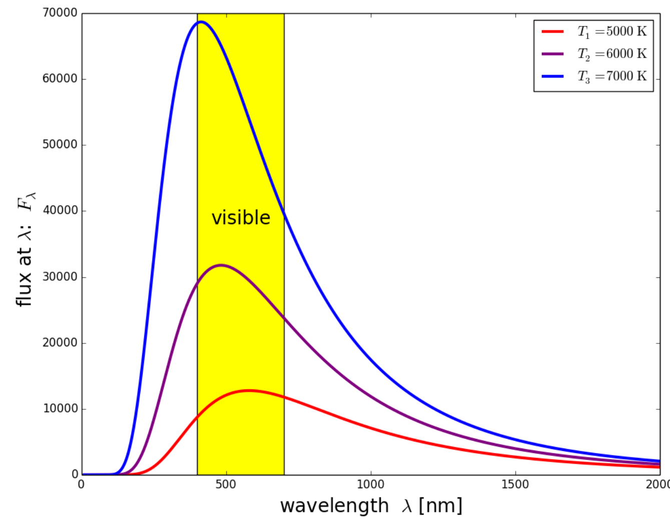
- sometimes updated after class (errors fixes)
- sometimes include "Director's Cut Extras" – like today

Last time:
thermal/blackbody radiation: laws and stellar thermometry

- *Q: Wien's law and color temperature?*
- *Q: Stefan-Boltzmann law, luminosity, and $T_{\rm eff}$?*

Blackbody Radiation Reminder

blackbody = perfect absorber of radiation
re-emits according to temperature T
always measured in absolute units (Kelvin)



spectrum: nonzero emission at all λ
peak position depends on T:
hotter $\rightarrow$ shorter $\lambda_{\rm max}$ $\rightarrow$ more blue

$$\lambda_{\rm peak} = \frac{0.29 \text{ cm K}}{T} \propto \frac{1}{T} \quad \text{Wien's law}$$

total flux (integral of spectrum):

$$F(T) = \sigma T^4 \quad \text{Stefan-Boltzmann law}$$

with Stefan-Boltzmann constant
$\sigma = \pi^2 k^4 / 60 \hbar^3 c^2 = 5.67 \times 10^{-8}$ Watt m^{-2} K^{-4}

A Census of Stars: the Hertzsprung-Russell Diagram

- the “Rosetta Stone” of stellar astrophysics
- *the central plot of this course–memorize, it’s on exams!*

plots star L vs T (theorist-friendly)
or *absolute magnitude vs color* (observer-friendly)
hence also known as *color-magnitude diagram = CMD*

unfortunate convention: **color/temperature axis backwards**
left $\rightarrow$ right goes from blue $\rightarrow$ red, hot $\rightarrow$ cold

for a “fair sample” of stars (i.e., not a specially picked cluster)
trends emerge

`www: Gaia HR diagram for 4+ million stars`
note: on *Gaia* plots, colormap gives number of stars
Q: features?

Poll: Main Sequence Trend

For **Main Sequence stars**:

A hotter $\leftrightarrow$ more luminous

B hotter $\leftrightarrow$ less luminous

C hotter $\leftrightarrow$ redder

D none of the above

H-R Diagram: Main Features

⋆ *most stars (∼ 90%) fall on curve:* **main sequence**
(including the Sun!); "dwarfs"
MS trend/correlation: hotter ↔ more luminous

⋆ *most of the rest: cooler but more luminous*–**giants**
Q: how do we know they are giant?

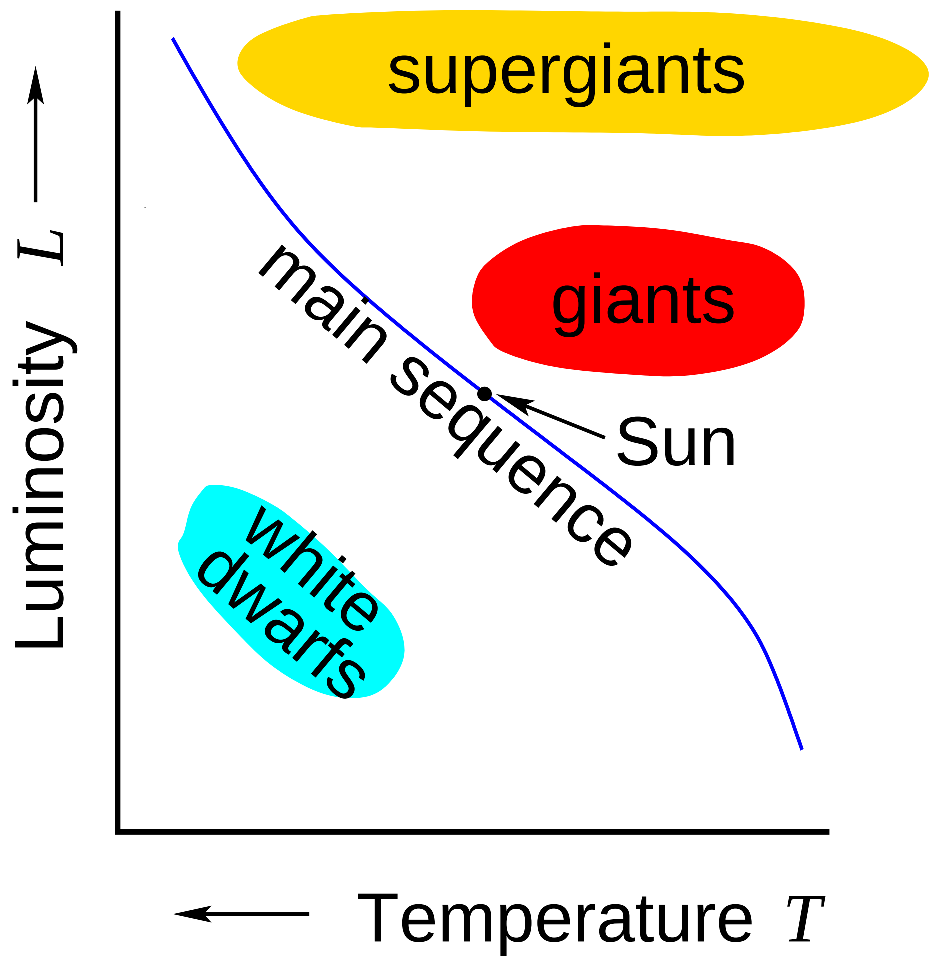
⋆ *a rare few: hot but luminous*–**supergiants**

not rare but dim and hard to find:

⋆ *very hot but very low-L objects:* **white dwarfs**
Q: how do we know they are teeny?
WD trend/correlation: hotter ↔ more luminous

note huge range in luminosity – more than $10^{-4} L_{\odot} < L < 10^{4} L_{\odot}$
and in temperature: $3000 \text{ K} < T < 30,000 \text{ K}$

HR Diagram Sketch for All Stars



Q: what does the HR diagram tell us about the Sun?

H-R and the Sun

The Sun on H-R diagram:

- found on the main sequence
- position is in the middle of the curve

but the main sequence is where most stars are found!

thus: *the Sun is a typical star!*

- lies in heart of main sequence L vs T trend
- neither most nor least luminous, not hottest or coolest

---

most stars are on Main Sequence
Q: what is this trying to tell us?

The H-R Diagram and Stellar Evolution

counting stars on the HR diagram:
⋆ 90% of stars are on the Main Sequence
⋆ and most of the rest are giants

these population statistics are due to *stellar evolution!*

recall: stars have life cycles–birth, midlife, death
these are ongoing–we see stars in all stages of life

analogy: hyperintelligent mosquito trying to understand humans
but mosquito lifetime $\ll$ human lifetime

- measures height h, weight w for people of all ages at Sox/Cubs/Cards/Bears game
- plots (w, h) for fair sample of people *Q: trends? why?*
  Q: which regions most populated? why?

Q: lessons for HR diagram detectives?

The H-R Diagram Encodes Stellar Evolution!

stellar life stages that last the *longest time*
are where *most stars* on HR will accumulate

lesson:
main sequence is the longest phase in a star's life
good news, and Copernican, that Sun is in this phase

Other questions arise:
- *why* do stars lie on the main sequence?
- what controls their position on the diagram?
- what's up with the giants, supergiants, and white dwarfs?

...most of the course is detective work to find answers

Weighing Stars

We saw that clever measurements give a stars

- luminosity
- surface temperature
- radius

What about **mass?**

For single stars:
mass determination difficult, very indirect
but we *can* find masses for stars in **binary** systems

Q: how to measure dynamics if both star orbits resolved?
Q: how to measure dynamics if only one orbit resolved? neither?

Binary Star Systems

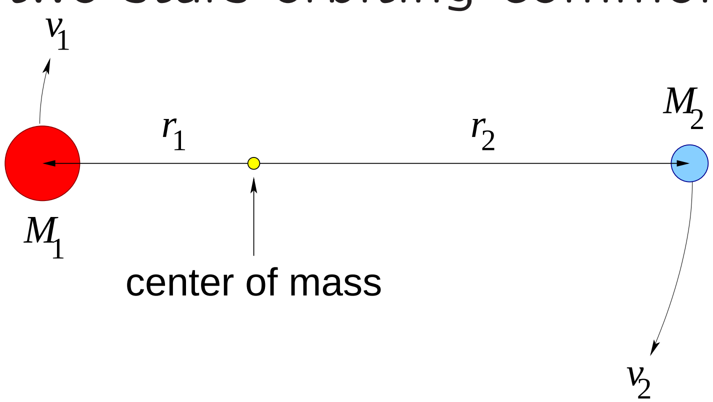
almost half of all stellar objects are multiple systems
  gravitationally bound sets of 2 or more stars
binary pairs are most common (1/3 of all stars)
but $\gtrsim$ 10% of stars are in triple systems or higher order!

observational classes:
visual binaries both stars resolved
  can track orbit around each other
astrometric binaries only the brighter star resolved
  moves in orbit around unseen partner
spectroscopic binaries appear as single point in scope
  but spectrum shows lines that split into pairs
  due to different Doppler shifts along sightline
eclipsing binary stars orbit plane seen edge-on
  when aligned one blocks the other

Measuring Star Masses: Binary Systems

for single stars without companions: can't accurately find mass

But can find masses for **binary** systems:
two stars orbiting common center of mass



binary orbit info + gravity physics $\rightarrow$ star masses!

Q: what gravity force between point masses?

Universal Gravitation: Point Masses

consider point masses m_1 and m_2 at separation $\vec{r}$
gravitational force of 2 on 1:

$$\vec{F}_1 = -\frac{G m_1 m_2}{r^2} \hat{r} \tag{1}$$

- *inverse square* *Q: why minus sign?*
- $\hat{r} = \vec{r}/r$: unit vector along $\vec{r}$
  force is along line between particle centers: *central force*

Q: motion in center of mass system?
Q: equation of motion?

Motion in Center of Mass System

PS2: for two interacting particles with no external forces

- center of mass feels no net force
- particles stay on opposite sides of center of mass
- relative motion: particle separation $\vec{r}$ set by

$$\mu \frac{d^2\vec{r}}{dt^2} = \vec{F} \tag{2}$$

where

- $\mu = m_1 m_2/(m_1 + m_2)$ is *reduced mass*
- $\vec{F}$ is force between particles

so for 2-body gravitational interaction

$$\mu \frac{d^2\vec{r}}{dt^2} = -\frac{G m_1 m_2}{r^2}\hat{r} \tag{3}$$

$$\frac{d^2\vec{r}}{dt^2} = -\frac{G(m_1 + m_2)}{r^2}\hat{r} \tag{4}$$

Q: conditions for circular motion?

Kepler Motion: Circular Case

for circular motion

- particle separation unchanged: constant radius $r = a$
- all *acceleration* is radial and thus centripetal v^2/r
- circular speed $v = v_{\text{circ}} = 2\pi a/P = a\omega$
  with orbit period P

$$\frac{d^2\vec{r}}{dt^2} = -\frac{G(m_1 + m_2)}{r^2}\hat{r} \tag{5}$$

$$\frac{v_{\text{circ}}^2}{a} = \frac{G(m_1 + m_2)}{a^2} \tag{6}$$

and motion has

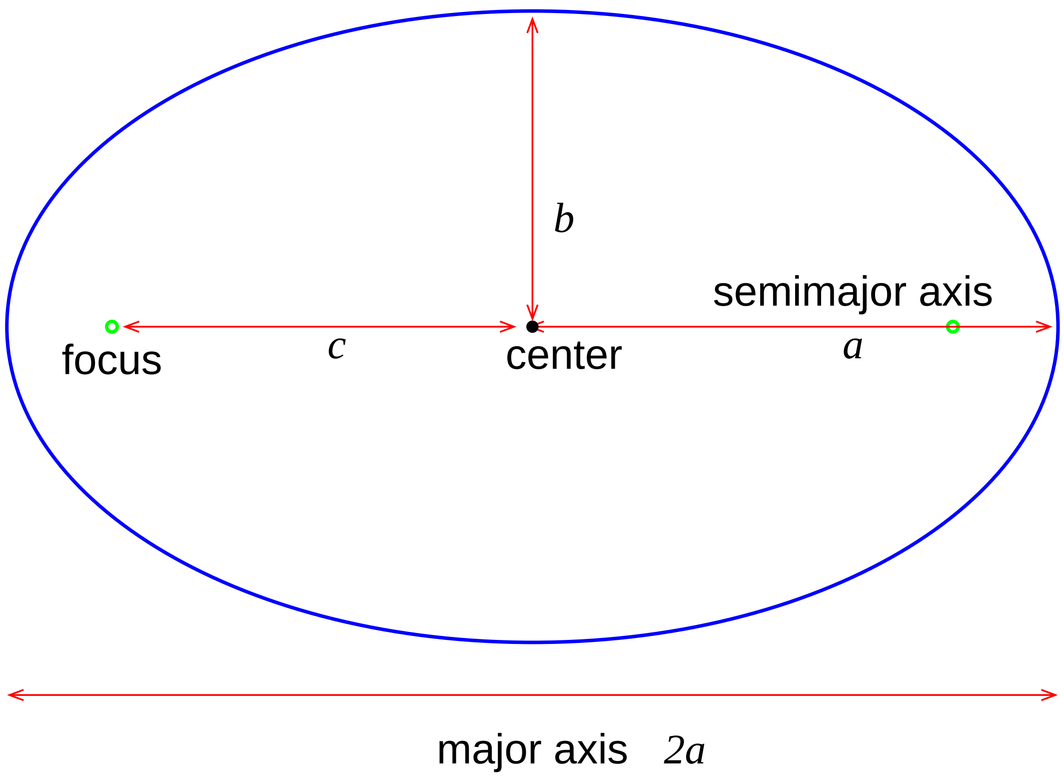
- constant circular speed v_{circ} and angular speed ω
- $4\pi^2 a^3 = G(m_1 + m_2)P^2$

Q: generalize to non-circular bound orbits?

General Bound Orbit

in general,for two gravitatational bound bodies

- orbits are **ellipses**
- with center of mass at one focus



Newtonian Orbits: Kepler's Laws

in general, orbits of gravitationally bound point masses

I. *in space:*
orbits are ellipses, with *center of mass at one focus*

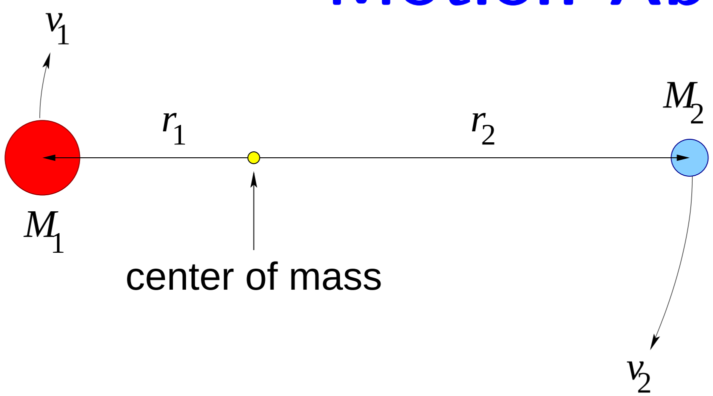
II. *speed:* **orbits sweep equal areas in equal time**

III. *orbit period and orbit size related:*

$$4\pi^2 a^3 = G(m_1 + m_2)P^2 \tag{7}$$

Q: how does the circular case fit in?
Q: equal mass particle motions about COM? unequal masses?

Motion About Center of Mass



COM positions: $r_1/r_2 = m_2/m_1$ (PS2)
star separation: $r = r_1 + r_2$

measure P, and r_1, r_2
$\rightarrow$ find mass ratio

problem: must measure r's
Q: how to do this for visual binaries? spectroscopic?

Types of Binary Stars Revisited

visual binary

can see both stars! can measure each distance from COM

`www: visual binary orbit`

eclipsing binary

stars pass in front of each other

can see this in flux vs time: *light curve* `www: examples`

$\rightarrow$ get orbit radius r from width of eclipse features *Q: how?*

spectroscopic binary

periodic Doppler shifts in spectrum

see $\Delta\lambda_1$, $\Delta\lambda_2$

$\rightarrow$ radial velocity $v_r/c = \Delta\lambda/\lambda_0$

then $v_1 = r_1\omega = 2\pi r_1/P$

can solve for r!

Director's Cut Extras

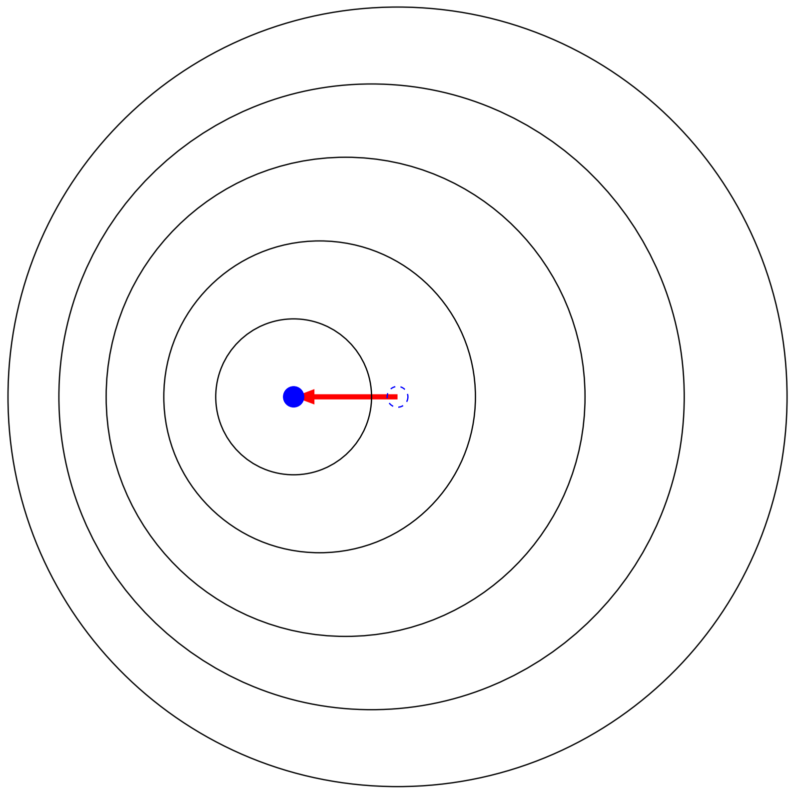
Doppler Effect

consider a moving light source

- moves at constant speed v
- emits light of wavelength λ_{em}
  as measured in emitter's rest frame

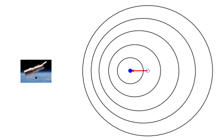
Each wave crest propagates spherically from emission point
but emission points move, so...
Q: how does this affect observed wavelength λ_{obs}*?*
Q: does the effect depend on viewing angle? how or why not?

Wave Crests from Moving Emitter



in front of emitter: wave crests "bunch up"

$\rightarrow$ approaching objects observed at smaller wavelength



$\rightarrow$ shorter λ: "blue shift"

behind emitter: wave crests "stretched out"

$\rightarrow$ receding objects observed at longer wavelength

$\rightarrow$ longer λ: "red shift"

shift depends only on

relative motion in **radial** direction ("line of sight")

$$\frac{\lambda_{\text{obs}} - \lambda_{\text{em}}}{\lambda_{\text{em}}} = \frac{\Delta\lambda}{\lambda} = \frac{v_r}{c} \tag{8}$$

where $v_r > 0$ means moving away

Observer's Scorecard

Doppler effect: speed $\leftrightarrow$ λ shift

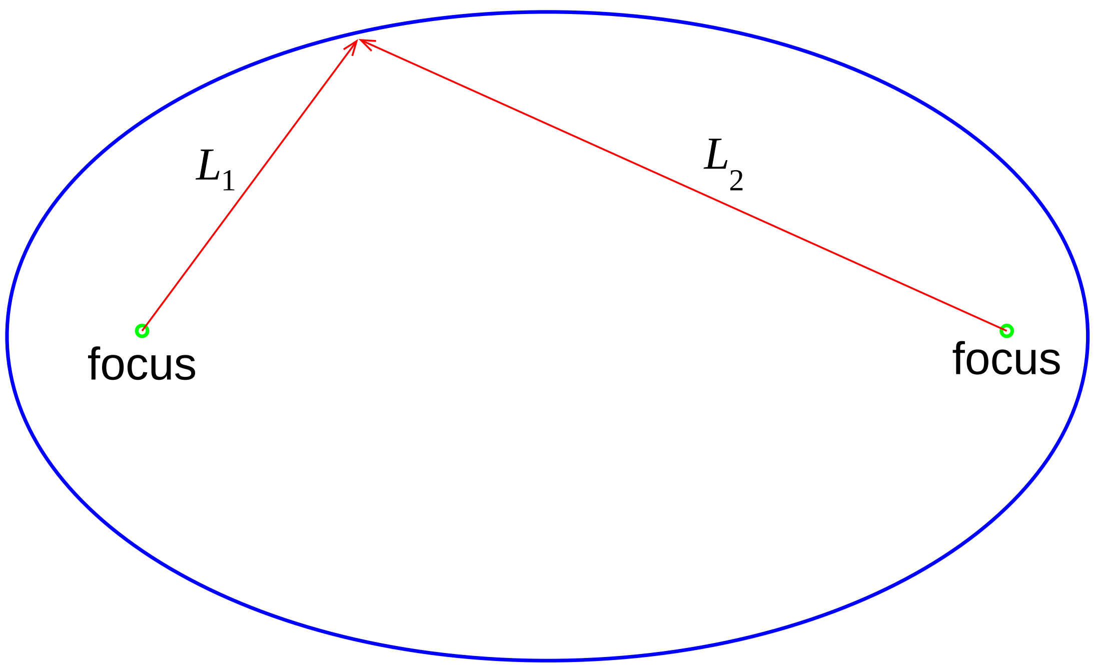
redshifts/blueshifts $\rightarrow$ speedometer

namely: measure $\lambda_{\rm obs}$, know $\lambda_{\rm em}$ $\rightarrow$ find $v_r = \dfrac{\Delta\lambda}{\lambda}\, c$

Q: but how does it work in practice?
how do you know a line is shifted?

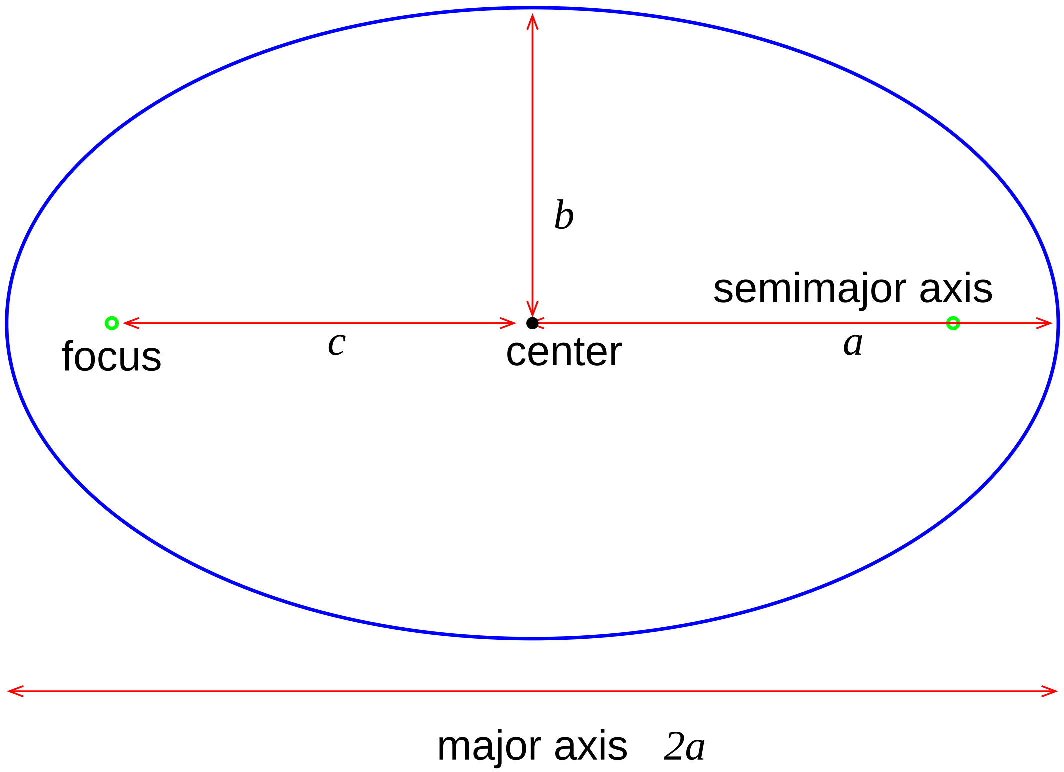
Kepler I Generalized: Law of Ellipses

for a two-body gravitating system
each body's orbit is an **ellipse**
with the **center of mass at one focus**



$$L_1 + L_2 = constant$$

Ellipse Anatomy



- two foci
- semi-major axis a
- focal length c
- semi-minor axis $b = \sqrt{a^2 - c^2}$

any ellipse fully characterized by:

a and eccentricity $e = c/a$

Q: what do we get for $e = 0$? $e = 1$?