Astro 404

Lecture 9

Sept. 13, 2021

Announcements:

- **Problem Set 3** posted, due Friday 5pm
  Q1: build a model of a star!
  longest question of the semester – take it one step at a time
  leave time to plot results

Last time:

lifespans of main sequence stars: *Q: what determines? trends?*

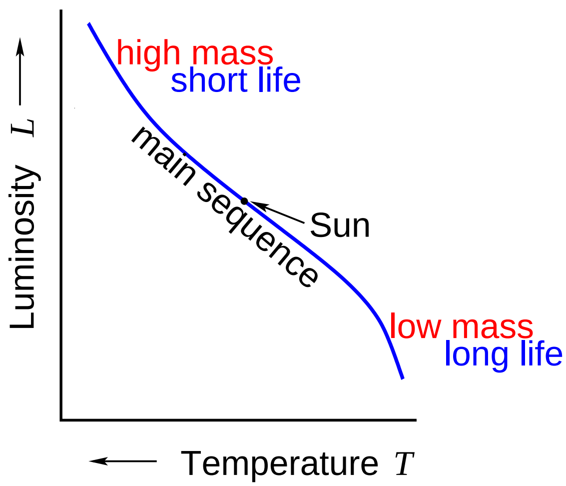
if star cluster born with range of masses *Q: MS over time?*

Q: how do test?

energy conservation: MS lifespan $\tau \approx E_{\rm fuel}/L$

for $E_{\rm fuel} \propto M$, we infer

$$\tau(M) \approx \frac{E_{\rm fuel}}{L} \sim \frac{1}{M^{\nu-1}} \longrightarrow \frac{1}{M^{2.5}} \quad (1)$$



star cluster evolution: **`www: animated HR diagram`**

- highest mass stars → huge L → short lifespan → die first
- then next most massive stars
- process continues for all stars gone with $\tau(M) \leq$ cluster age

A Theory of Stars

Towards a Theory of Stars

our goal: understand the nature and evolution of stars

we want to
- peer into the hearts of stars
- see how they change with time
- determine their fate

`www: schematic of solar interior`

what in detail do we want to know
Q: at a given moment of a star's life?
Q: over a star's life?

Q: how can we hope to do this?

at any moment on the star's life, want to account for
★ interior structure, composition, and motions
★ light emitted from surface (flux, luminosity, spectrum)
★ effects of binary partner if there is one
over time account for how these change

how to do this: **use physics!**

a big job! a *full model* should include
- gravitation
- gas dynamics: compressible fluid
- thermodynamics and statistical mechanics
- electromagnetic radiation and interaction with atoms
- spoiler alert: nuclear and neutrino physics too
- rotation `www: Sun movie`
- magnetic fields `www: solar magnetogram`
- mass loss `www: solar wind`

Yikes! This is a lot! Q: simplifying assumptions?

Basic Stellar Models

"Everything should be made as simple as possible, but no simpler."
–Albert Einstein/Roger Sessions

begin stellar model building by making
simplest realistic model possible:

- *a single star in isolation*: no binary partner
- *non-rotating*: angular frequency $\Omega_{\rm rot} = 0$
- *non-magnetic*: $\vec{B} = 0$ throughout

so in equilibrium, star has **spherical symmetry**

after building these basic star models
we will see effect of relaxing these assumptions

Describing the Physical State of the Sun

note: stars are not point masses, but extended objects

bad news: Sun contains $\sim 10^{57}$ particles!
good news: we dont have to –or want to!–describe them all

Q: how to physically describe the star's interior?
Hint: think of schematic cutaway diagrams of Sun

Building Models of Stars

a star is an extended object, so must physically characterize its properties at every radius r

- **mass density** ρ
- **composition** (which elements)
- **temperature** T
- **pressure** P

dependence of each on r: *radial profile*

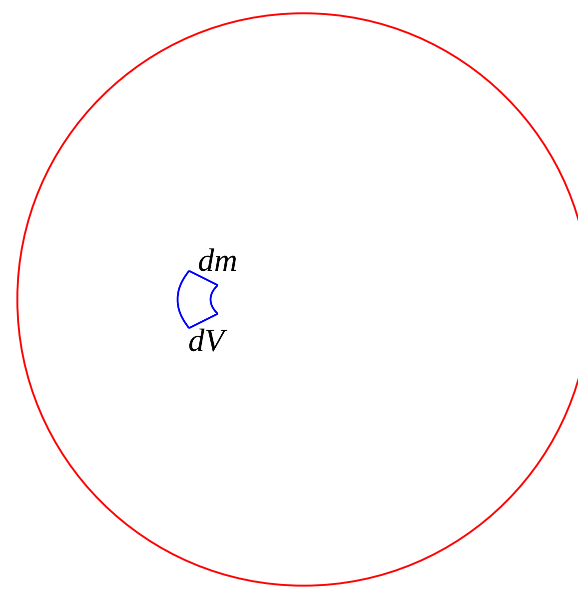
star composed of **gas:** a compressible fluid

often useful to consider small parcel of gas

"fluid element"

∞ with mass dm and volume dV

Q: how are these related?



Mass and Density

a fluid element with

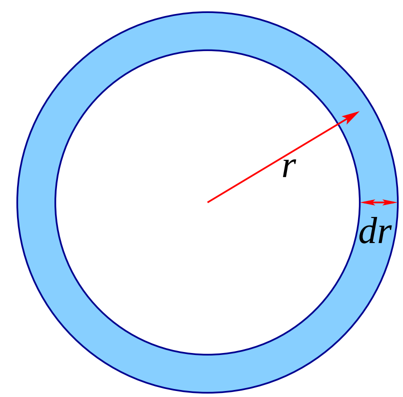
- *mass* dm and
- *volume* dV

has **mass density**

$$\rho = \frac{dm}{dV}$$

need not be uniform throughout an object! in general $\rho \neq m/V$!

in spherical symmetry: volume element in thin shell of radius r and thickness dr is



$$dV \stackrel{\text{sph}}{=} 4\pi r^2 dr = A_{\text{shell}}\ dr$$

and thus (useful for PS3!)

$$\rho \stackrel{\text{sph}}{=} \frac{1}{4\pi r^2}\frac{dm}{dr}$$

Enclosed Mass

for spherical mass distribution:

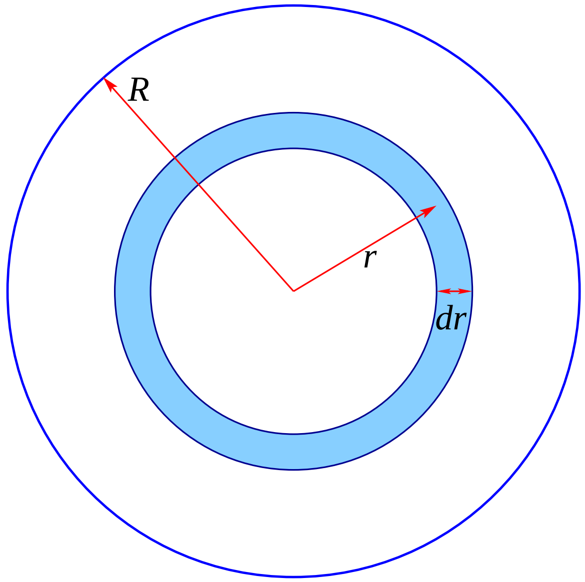
enclosed mass defined as

mass inside radius r

$$m(r) = \int_0^r \rho \; dV = 4\pi \int_0^r \rho(r) \; r^2 \; dr$$

Q: what is $m(0)$?

Q: as r increases, $m(r)$ behavior?



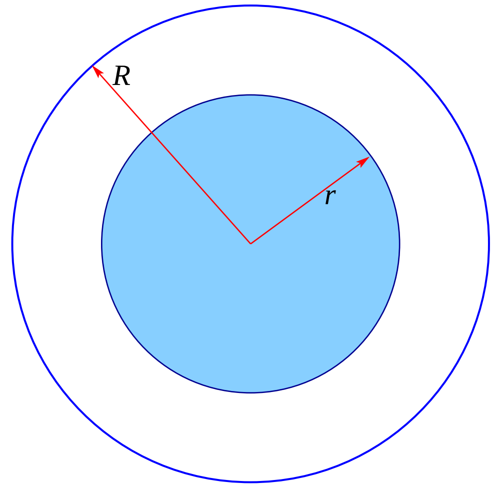
for star of radius R:

Q: what is $m(R)$? what is $m(r)$ for $r > R$?

Q: what changes if star expands or contracts?

Q: what doesn't change?

Mass Coordinate



enclosed mass for *star of radius* R:

$m(r) = 4\pi \int_0^r \rho(r)\ r^2\ dr$

$\rho > 0$, so $m(r)$ grows monotonically with r

- $m(0) = 0$: nothing to enclose at center
- $m(R) = M$: *total mass of star*
- for $r > R$, still $m(r) = M$

for stars: **total mass** M fixed (when mass loss negligible)
but expansion/contraction changes density profile $\rho(r)$

lesson: can label star interior regions with r
but also useful to label star interior via $m(r)$

- $m(r)$ sometimes called **"mass coordinate"**
- tracks stellar matter if expansion or contraction
- sometimes called a *Lagrangian* coordinate ("follows" fluid)

Newtonian Gravitational Field

Galileo: for point "test" mass m_{test}:

- acceleration independent of test mass
- thus only depends on "source" M
- "equivalence principle" (Einstein; more later on this)

formally: can write test mass force $\vec{F}_m = m_{\text{test}}\vec{g}$
and thus in the presence of a gravity source M
  i.e., given the existence and amount *mass*
any and all test particles at point $\vec{r}$ feel acceleration

$$\vec{a} = \vec{g}(\vec{r}) \tag{2}$$

$\Rightarrow$ physical interpretation: each mass M sets up
  its own gravitational field $\vec{g}$ throughout space

Poll: Gravitational Strength Inside a Massive Sphere

Consider a sphere with mass density $\rho(r)$, *possibly non-uniform*

Vote your conscience!
Where is the gravitational acceleration $g(r)$ the largest?

A at the center

B between the center and surface

C at the surface

D above the surface

E it depends on $\rho(r)$

Bonus! Where is $g(r)$ **smallest**?

Gravity from many sources: Superposition

Thus far: only considered single point masses
what if we add more gravity sources–i.e., more masses?

If one point particle of mass m at $\vec{r}$
gravity is

$$\vec{g} = -\frac{Gm}{r^2}\hat{r} \quad (3)$$

for many particles: use principle of superposition
$\Rightarrow$ take vector sum of gravitational acceleration

bad news: this can be complicated!
good news: spherical symmetry drastically simplifies
best news: you already have the technology in hand
Q: what's that? hint–it was in PHYS 212

Gravitation and Electrostatics: Family Resemblance

how sum up? how do the integral?

You already have the technology! Notice similarity:

	Electrostatics	*Gravity*
"charge"	q	m
force	$qQ/4\pi\epsilon_0 r^2\ \hat{r}$	$-GmM/r^2\ \hat{r}$
field	$\vec{F}_q = q\vec{E}$	$\vec{F}_m = m\vec{g}$

formally identical inverse square law forces!
(except sign, and $\pm q$ allowed, $m \geq 0$)

So: can import electrostatics technology
Memory lane: Gauss' Law from EM
`www: PHYS 212`

Gauss' Law in E&M

consider a point charge Q

enclose in sphere: $\vec{E}$ normal to surface $\vec{S}$

$$\int_S \vec{E} \cdot d\vec{S} = E \int_S dS = \frac{Q}{4\pi\epsilon_0 r^2} 4\pi r^2 = \frac{q}{\epsilon_0} \tag{4}$$

miracle: holds for all $\vec{E}$ and surfaces $\vec{S}$

$$\text{electric flux} = \int_S \vec{E} \cdot d\vec{S} = \frac{q_{\text{enc}}}{\epsilon_0} \tag{5}$$

where q_{enc} is total charge enclosed in surface S

Gauss' Law for gravity: for point mass M

$$\int_S \vec{g} \cdot d\vec{S} = -\frac{GM}{r^2} 4\pi r^2 = -4\pi GM \tag{6}$$

and in general:

$\int_S \vec{g} \cdot d\vec{S} = -4\pi G M_{\text{enc}}$

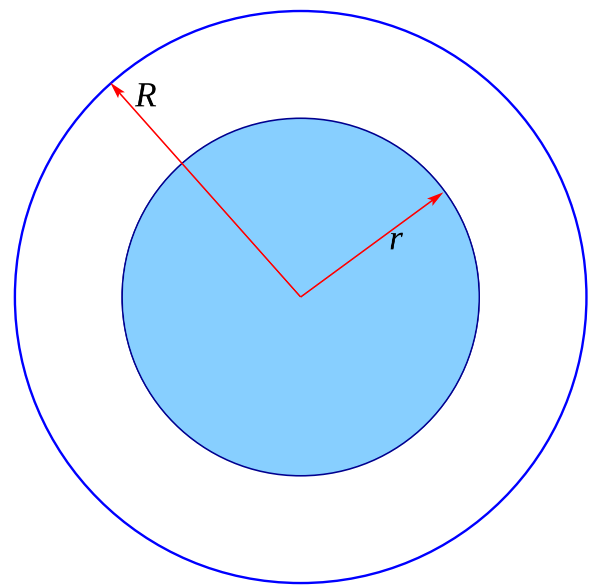
A Gravitating Sphere

spherical mass distribution $\rho(r)$
and $\vec{g}(r, \theta, \phi) = \vec{g}(r)$

Gauss' Law: choose spherical surface

$$\int_S \vec{g} \cdot d\vec{S} = 4\pi r^2 g(r) = -4\pi G m(r) \quad (7)$$

where $m(r) = 4\pi \int dr r^2 \rho(r)$
is the *enclosed mass!*



solve:

$$\vec{g}(r) = -\frac{Gm(r)}{r^2}\hat{r} \tag{8}$$

note similarity to point-source formula

but this works for *any* spherical mass distribution

and works inside, outside mass distribution!

Q: field at center?

Q: field if hollow out inside and you're there?

$\Rightarrow$ field is same as if interior mass concentrated at center!

iClicker Poll: Maximal Gravity

imagine the Earth's density were uniform (constant)

Where would the gravitational acceleration be the strongest?

A at the center

B at the surface

C at the Moon's distance

D none of the above