Astro 404

Lecture 13

Sept. 22, 2021

Announcements:

- **Problem Set 4 due Friday**
- instructor office hours: Today 11am-noon or by appt

  TA office hours: Thurs 2:30–3:30pm

Last time: internet fail–sorry! see recorded lecture

Energy generation in the Sun: How does the Sun shine?

- the Sun is not a cup of coffee *Q: the difference?*

The Sun is Not a Cup of Coffee

Coffee Thermodynamics

Demo: cup of coffee: cools

thermodynamic lesson:

- left alone, hot coffee cools (surprise!)
  $\rightarrow$ energy radiated, not replaced
- to keep your double-shot soy latte from cooling
  need Mr. Coffee™ machine–energy (heat) source

Contrast with the Sun

- surface $T_\odot$ constant over human lifetimes
  but energy *is* radiated, at enormous rate
- ergo: something must replace the lost energy

▷ What is solar heat source (fuel supply)?
  $\rightarrow$ a mystery in Astronomy until the 20th century

Energy Conservation and the Sun

recall: power is energy flow rate $L = dE/dt$

assume:

- Sun always emits energy at today's rate (L constant)
- radiation lasts for time $\tau_\odot$ = "lifetime" of Sun

  Q: what is a minimum value for $\tau_\odot$*?*

energy output over Sun's lifetime:

$$\boxed{E_{\text{out}} = L\tau}$$

Energy conservation:

solar energy supply = lifelong energy output

Solar Batteries: Required Lifetime

from radioactive dating of meteorites:
the solar system is very old: age $t_{\rm ss} = 4.55 \times 10^9$ yr
Sun's present age essentially the same:
$t_{\odot,\rm now} = t_{\rm ss} = 4.55$ billion years

total energy output over this time is huuuge!
$\rightarrow$ required huge energy reservoir

Breakout Discussion:
Q: possible sources–not just right answer, but any energy reservoirs?
https://docs.google.com/presentation/d/16HEoGG8x2vJsF6jTb_iuQln3mzExg

Poll: Rank the Energy Sources

Vote your conscience!

Of the proposed solar energy reservoirs

Which one is the largest, i.e., can power the Sun longest?

Which one is the smallest?

Q: how to sort the candidates? how to tell which is right?

Energy Sources in the Sun

to evaluate energy sources, need to study energy "budget"

- output: energy supply required to power Solar luminosity
- input: available energy sources that might act as fuel

PS4: sort sources by energy each particle must contribute
Here: look at the *time* an energy source can "burn"

Solar Energy: Required Supply

Sun must shine for at least the age of Solar System, emitting

$$E_{\rm emit} = L_\odot \, t_{\rm ss} \approx 6 \times 10^{50} \text{ erg} = 6 \times 10^{43} \text{ Joule} \qquad (1)$$

this a lot! but also huge mass $\rightarrow$ huge fuel supply

Worked Example: TNT Sun

imagine–completely unrealistically!–the Sun is made a TNT
i.e., high explosives: Sun as a (chemical) bomb!
How long could this power the Sun?

TNT energy release per unit mass: $u_{\rm TNT} = 4.7 \times 10^6$ Joule/kg
total energy content in TNT Sun:
$E_{\odot,{\rm TNT}} = u_{\rm TNT} \, M_\odot = 9.4 \times 10^{36}$ Joule
seems like a lot! but...

$$\frac{E_{\odot,{\rm TNT}}}{E_{\rm emit}} = 1.6 \times 10^{-7} \tag{2}$$

$$\frac{E_{\odot,{\rm TNT}}}{L_\odot} = 800 \text{ yr} \tag{3}$$

Q: and so?

Candidate Solar Energy Sources

• chemical energy: high-explosives?
energy supply > 1 million times too small
"TNT Sun" can only burn for ~ 800 yr! pitifully short!
...and the Sun is definitely not made of TNT!

• thermal energy: $E_{\odot,\mathrm{therm}} = U = N\langle kT \rangle \approx M_\odot \langle kT \rangle / m_p$
note: by virial theorem, related to grav pot energy Ω

- ▷ physical picture: energy supplied by cooling and contracting
- ▷ PS 4: will show–can power the Sun for millions of years
  ...which is way too short!

∞

Spoiler Alert!

there is **only one** viable candidate:

- Nuclear Energy

The Sun is a vast nuclear reactor
in hot core, hydrogen converted to helium
by nuclear reactions

Note: needed *quantitative* estimates of burn times
to answer *qualitative* question "What powers the Sun?"
$\rightarrow$ the power of (and necessity of) number crunching!

A Nuclear Powered Sun?

mystery of Solar energy source was recognized in the 1800s before atomic nuclei were discovered

Geologists came to understand the Earth is billions of years old but astronomers believed the Sun could only live for millions of years the story changed with the discovery of atomic nuclei and nuclear energy

Lessons:

- discoveries in the microscopic (subatomic!) world–inner space have profound consequences for astro/cosmo–outer space
- to understand how stars shine, we need to understand nuclei
- note that energy argument shows nuclear power needed but not *how* it works – what is the mechanism?
  need to figure out how stars "know" to release nuclear energy

Overview: the Structure of Matter

Zooming into microscopic structure of matter:

matter
$\downarrow$
molecules
$\downarrow$
atoms
$\swarrow \quad \searrow$
nucleus   electrons
$\swarrow \quad \searrow$
protons   neutrons

atoms come in **elements** `www: periodic table`

92 natural, 23+ artificial

element determined by *nuclear charge* Z = # protons

e.g., hydrogen H: $Z = 1$, uranium U: $Z = 92$

same element (same # p) can have different # neutrons

$\rightarrow$ "isotopes"

examples: most hydrogen is ^{1}H $= \boxed{1p,\ 0n}$

but $\sim 10^{-4}$ of hydrogen is deuterium ^{2}H $= \boxed{1p,\ 1n}$

most U is ^{238}U $= \boxed{92p,\ 146n}$; about $\sim 1\%$ is ^{235}U $= \boxed{92p,\ 143n}$

atom net charge fixed by # electrons

e = # $p \rightarrow$ neutral

e = # $p - 1 \rightarrow$ singly ionized

Note: all p, n, e are absolutely *identical* and *indistinguishable*

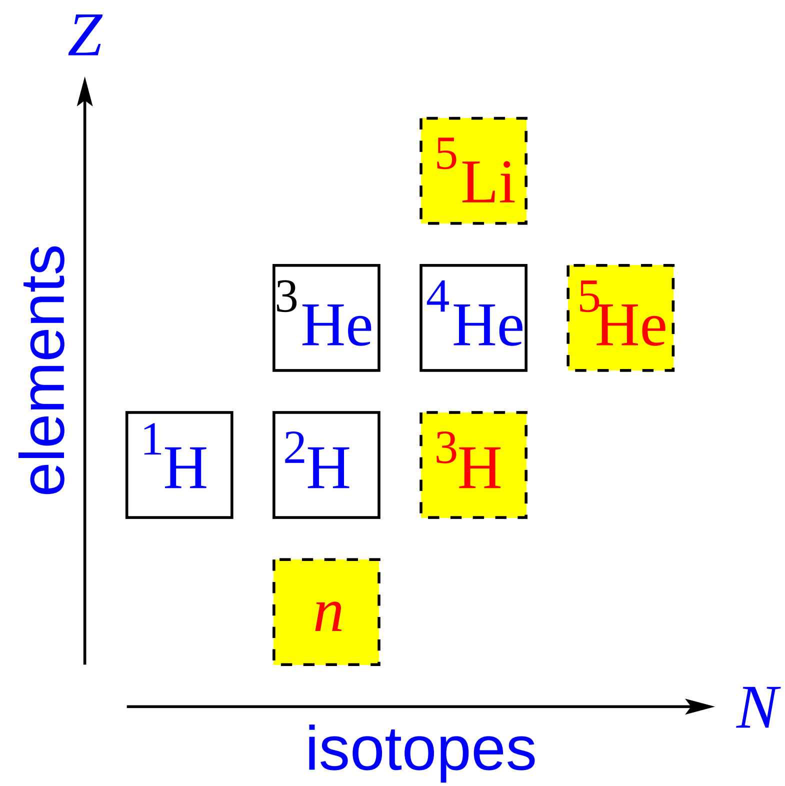
this turns out to be crucial for the understanding of matter

in a quantum mechanical way

The Lightest Stable Isotopes

- **stable hydrogen isotopes**
  proton $p = {}^1$H,
  deuterium $D = {}^2$H
- **stable helium isotopes**
  helium-3 ^{3}He

helium-4 ^{4}He



- these are the only stable nuclei with
  $A = N + Z = 1, 2, 3, 4$ nucleons

- there are *no stable nuclei*
  with $A = 5$ or 8 nucleons

`www: chart of nuclides`

Poll: Forces in Nuclei

Consider a nucleus, say $^{4}He = \boxed{2p+2n}$
maintains same size: not imploding, exploding

How many forces act on each proton?

A exactly 1

B more than 1

C fewer than 1

Forces in Nuclei

nuclei made of *protons and neutrons:* **"nucleons"**

sizes similar:

$r \approx 1.4 \times 10^{-15}$ m $= 1.4$ femtometer $= 1.4$ fermi

in nucleus: nucleons are touching!

- nuclear size $\ll$ atom size
- protons very close $\rightarrow$ *huge electrostatic repulsion!*

electrostatic (Coulomb) energy between two protons in nucleus

$$E_C = \frac{e^2}{[4\pi\epsilon_0]r} \approx 1 \text{ MeV} \tag{4}$$

if this is unopposed, nuclei would fly apart!

nuclear stability requires attractive force between nucleons

Nuclear Forces

thus: existence of nuclei demands a stabilizing force

the **nuclear interaction / nuclear force**

- attractive at short distances
- stronger than Coulomb force at short distances
- with $\sim$ MeV scale strength
- weakens at long distances or all nuclei would merge to one!

Nuclear Binding

stable atomic nuclei are bound states of nucleons

- that is: they can't "fall apart" on their own
- the same way bound atoms, planetary systems, binary stars don't fall apart

so nucleus A, with Z protons and N neutrons
has **binding energy** B_A = energy required to rip apart
this means that

$$E_A + B_A = ZE_{\rm p} + NE_{\rm n} \quad (5)$$

that is

$$\begin{aligned} \text{binding} &= \text{parts} - \text{whole} && (6) \\ B_A &= ZE_{\rm p} + NE_{\rm n} - E_A && (7) \\ &= > 0 && (8) \end{aligned}$$

so energy of parts is more than whole!

but Einstein says $E = mc^2$

Q: what does this mean generally? implications for nuclei?

Nuclear Binding

Einstein $E = mc^2$ says: an object at rest, with mass m contains energy $E = mc^2$ simply by having mass

- mass is a form of energy!
- not due to motion: "rest mass energy"

for nuclei (and similar any other bound system), binding energy

$$B_A = ZE_\mathrm{p} + NE_\mathrm{n} - E_A \ > \ 0 \tag{9}$$

implies a *mass difference*

$$B_A = Zm_\mathrm{p}c^2 + Nm_\mathrm{n}c^2 - m_Ac^2 = (Zm_p + Nm_n - m_A)c^2 \ > \ 0 \tag{10}$$

- mass of parts $>$ mass of whole
- mass difference measures binding energy

bound system	binding energy B	binding energy per nucleon $B/(Z+N)$
hydrogen atom $\boxed{pe}$	13.6 eV	13.6 eV/nucleon
^{4}He nucleus $\boxed{2p, 2n}$	28.3 MeV	7.07 MeV/nucleon
^{56}Fe nucleus $\boxed{26p, 30n}$	492 MeV	8.79 MeV/nucleon
^{238}U nucleus $\boxed{92p, 146n}$	1801 MeV	7.57 MeV/nucleon

Q: atoms vs nuclei comparison?

Q: comparison among nuclei?

Q: lessons for Sun?

Lessons: Nuclear Energy as Stellar Fuel

nuclei vastly more tightly bound than atoms
- much harder to rip apart
- but much more energy at play when this happens

nuclear reactions: transformation of one set of nuclei to another
- leads to changes in sum of binding energies
- can require net energy input (endothermic)
- or can release net energy (exothermic)

typical reaction energy per nuclear particle (nucleon $= n, p$):
$B/(Z+N) \sim few$ Mev/nucleon
this is more than enough to power the Sun!

raises the question: how does the Sun–and all stars–do this?

Nuclear Fusion in the Sun

The Sun is a nuclear reactor
i.e., nuclear reactions occur inside the Sun
change reactant nuclei into different product nuclei
$\rightarrow$ elements transformed into other elements
$\rightarrow$ cosmic alchemy!

Mechanism: high-energy/high-speed collisions between nuclei

$$\text{nucleus}_1 + \text{nucleus}_2 \rightarrow \text{nucleus}_3 + \text{energy} \tag{11}$$

- nuke energy release $\rightarrow$ stellar power source
- lighter nuclei combine $\rightarrow$ heavier: **fusion**

Q: why are high energies, speeds needed?
Q: how do the nuclei get these energies & speeds?

Thermonuclear Reactions

recall forces at play in nuclei:

- *Coulomb repulsion* between nuclei with Z_1 and Z_2 protons:

$$E_{\mathrm{C}} = \frac{Z_1 Z_2 e^2}{[4\pi\epsilon_0] r} \tag{12}$$

  long range, only goes to zero as $r \to \infty$

- *nuclear force attraction* is *short range*

nuclei must be nearly touching until nuclear force "wins" and reaction occurs

- thus nuclei must overcome "Coulomb barrier" to react
- this requires high energy particles
- and for thermal gas $\langle mv^2 \rangle \sim kT$: *need high temperature!*

Coulomb Barrier in Nuclei