Astro 404
Lecture 14
Sept. 24, 2021

Announcements:

- **Problem Set 4 due 5pm today**
  last minute office hours after class
- **Problem Set 5 next Friday**

Last time: the Sun is a nuclear furnace
Q: how do we know this?
Q: how is the Sun unlike a cup of coffee?
atoms and nuclei
Q: what defines an element? an isotope?

How the Sun Shines: The Story Thus Far

the Sun is a $L_\odot = 3.85 \times 10^{26}$ Watt lightbulb
burning for at least Solar System present age $t_{\rm ss} = 4.55$ Gyr
needed energy per proton:

$$\epsilon_{\rm emit} = \frac{E_{\rm emit}}{N_{\rm p}} > 3 \times 10^5 \text{ eV/proton} = 0.3 \text{ MeV/proton}$$

atoms: sorted by number Z of protons
nuclei: ingredients: protons and neutrons – **nucleons**
  isotopes defined by Z and number N of neutrons

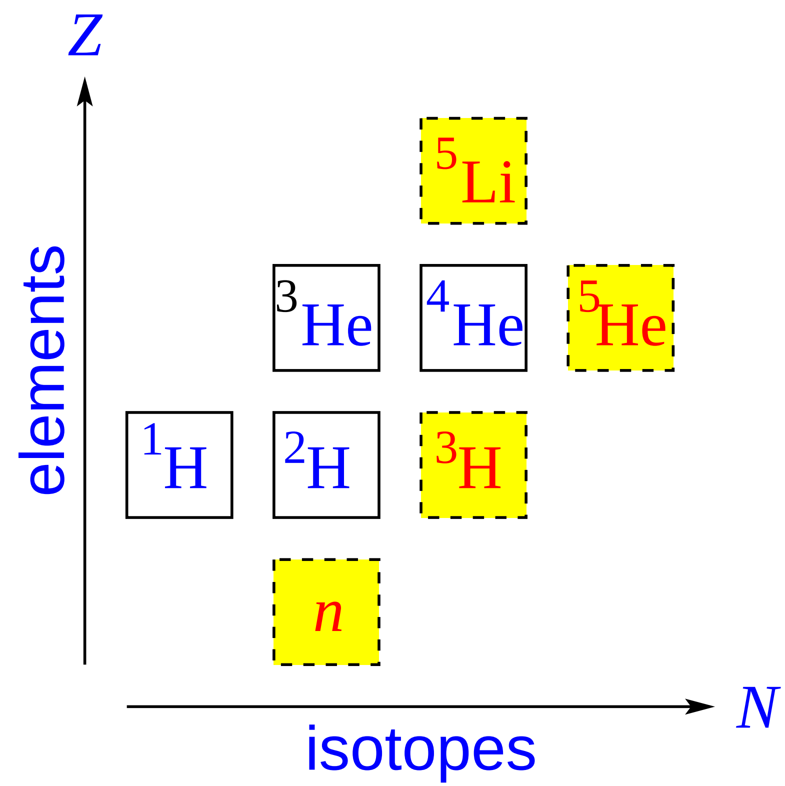
nuclear mass number total number of nucleons
$A = N + Z$

The Lightest Stable Isotopes

- **stable hydrogen isotopes**
  proton $p = {}^1\text{H}$,
  deuterium $D = {}^2\text{H}$
- **stable helium isotopes**
  helium-3 ${}^3\text{He}$
  helium-4 ${}^4\text{He}$

- these are the only stable nuclei with
  $A = N + Z = 1, 2, 3, 4$ nucleons

- there are *no stable nuclei*
  with $A = 5$ or 8 nucleons



`www: chart of nuclides`

Forces in Nuclei

nuclei made of *protons and neutrons:* **"nucleons"**
sizes similar

$$r_n \approx r_p \approx 1.4 \times 10^{-15}\ \text{m} = 1.4\ \text{femtometer} = 1.4\ \text{fermi}$$

in nucleus: nucleons are touching!
- nuclear size $\ll$ atom size (≈ 1 Å $= 0.1$ nm)
- protons very close $\rightarrow$ *huge electrostatic repulsion!*

electrostatic (Coulomb) energy between two protons in nucleus

$$E_C = \frac{e^2}{[4\pi\epsilon_0]r} \approx 1\ \text{MeV} \qquad (1)$$

if this is unopposed, nuclei would fly apart!
nuclear stability requires attractive force between nucleons

Q: how should nuclear forces behave at large distances?

Nuclear Forces

thus: existence of nuclei demands a stabilizing force
the **nuclear interaction / nuclear force**

properties of nuclear force:
- attractive at short distances
- stronger than Coulomb force at short distances
- with $\sim$ MeV scale strength
- weakens at long distances or all nuclei would merge to one!
  or react at room temperatures

good news:
nuclear forces lead to energies $\gtrsim 1$ MeV
very promising solar energy source!

Lessons: Nuclear Energy as Stellar Fuel

forces in nuclei vastly stronger than in atoms

- much harder to rip apart
- but much more energy at play when this happens

nuclear reactions: transformation of one set of nuclei to another

- can require net energy input (endothermic)
- or can release net energy (exothermic)

raises the question: how does the Sun–and all stars–do this?

Nuclear Fusion in the Sun

The Sun is a nuclear reactor
i.e., nuclear reactions occur inside the Sun
change *reactant nuclei* into different *product nuclei*
$\rightarrow$ changed nucleus often means element changes
$\rightarrow$ cosmic alchemy!

Mechanism: high-energy/high-speed collisions between nuclei

$$\text{nucleus}_1 + \text{nucleus}_2 \rightarrow \text{nucleus}_3 + \text{energy} \tag{2}$$

- nuke energy release $\rightarrow$ stellar power source
- lighter nuclei combine $\rightarrow$ heavier: **fusion**

Q: why are high energies, speeds needed?
Q: how do the nuclei get these energies & speeds?

Thermonuclear Reactions

recall forces at play in nuclei:

- *Coulomb repulsion* between nuclei with Z_1 and Z_2 protons:

$$E_{\rm C} = \frac{Z_1 Z_2 e^2}{[4\pi\epsilon_0] r} \tag{3}$$

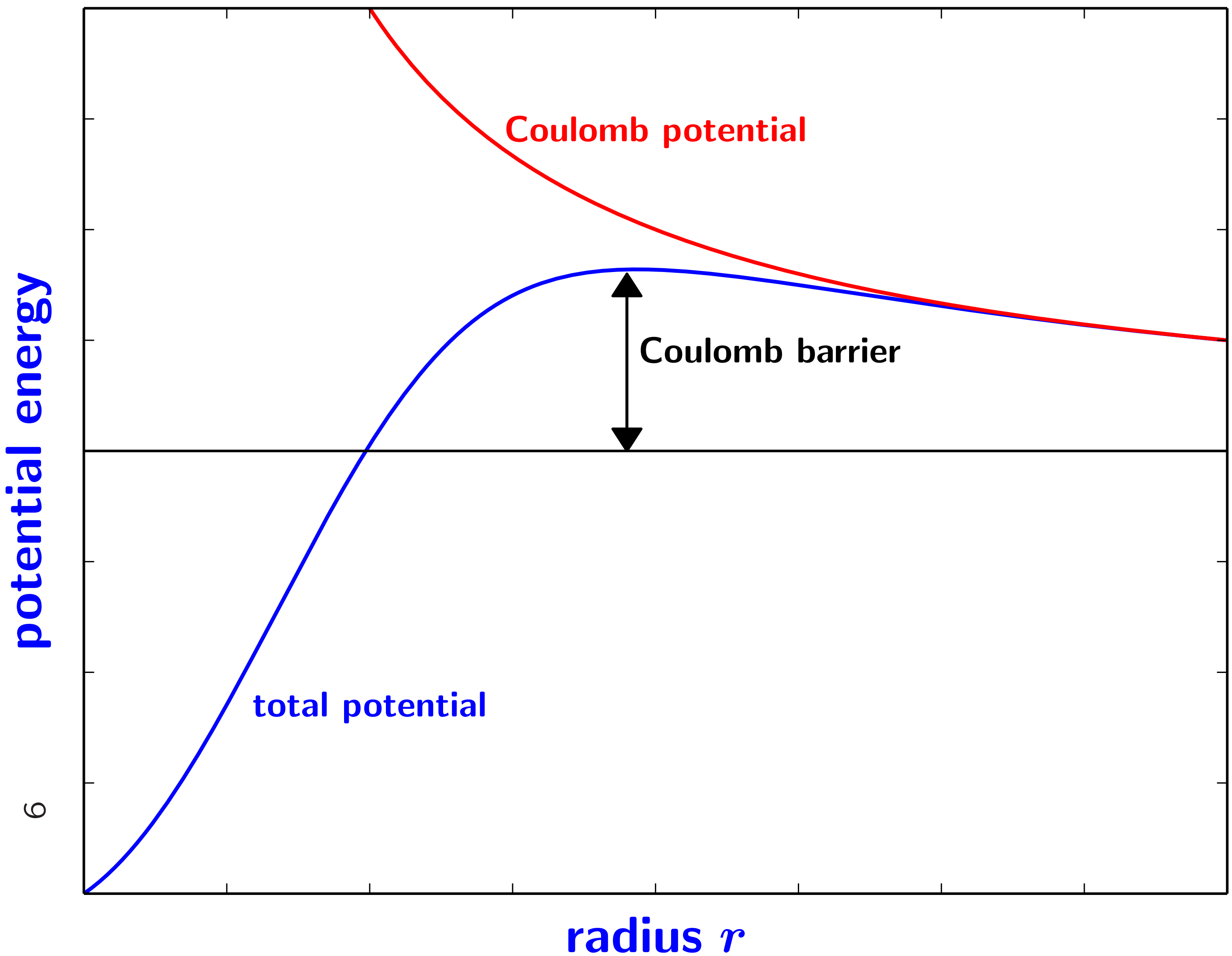
  long range, only goes to zero as $r \to \infty$

- *nuclear force attraction* is *short range*

nuclei must be nearly touching until nuclear force "wins" and reaction occurs

- thus nuclei must overcome "Coulomb barrier" to react
- this requires high energy particles
- and for thermal gas $\langle mv^2 \rangle \sim kT$: *need high temperature!*

Coulomb Barrier in Nuclei



Poll: Reaction Types

consider a gas of particles in random thermal motion

Which types of collisions will be more frequent?

A *collisions between two particles*
"two-body collisions" $a + b \to \cdots$

B *collisions of three particles*
"three-body collisions" $a + b + c \to \cdots$

C two and three body collisions should be *equally frequent*

Reaction Chains

In fact: many reactions can and do occur
but a small handful are the most important

Key reactions occur in "chains"

- first step involves pre-existing solar ingredients
- input for each new step is output from previous step
- important reactions involve collisions between two nuclei
- three-body reactions rare in main sequence stars
  to happen, need two particles to collide and a third to arrive
  before they scatter away: unlikely except at high density
  unimportant for main sequence, but important for later phases

Solar Composition

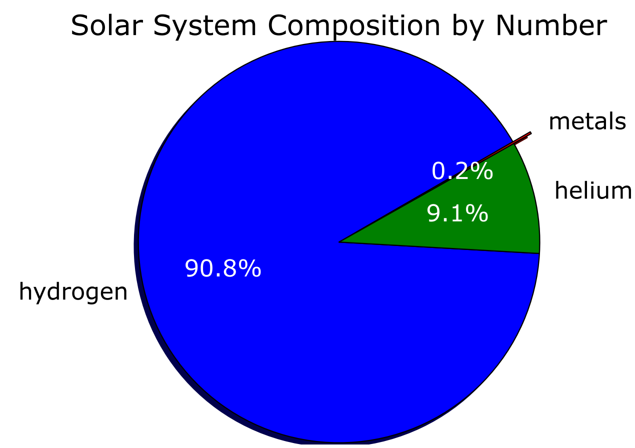
the Sun's ingredients are fuel for nuclear reactions

measured by observation of Sun's atmosphere
and by collecting solar wind (Apollo!)

spoiler: outer layers of Sun mostly preserve initial composition

Solar Ingredients

Element	Atom %	Mass %
hydrogen (protons)	91%	78%
helium (^{4}He)	9%	30%
other ("metals")	0.2%	2%



Q: so what are possible first steps in nuclear reactions?
Q: which are allowed?

Nuclear Burning in the Sun

Sun is mostly made of protons (hydrogen)
with small amounts of helium

possible first steps: pairs of ingredients

- $p + p \to p + p$ allowed but *no progress!* and no energy release ("elastic")
- $p + p \to {}^2\text{H}$ only possible $A = 2$ product but this reaction as written is incomplete/illegal! *Q: why? how to fix?*
- $p + {}^4\text{He} \to {}^5\text{Li}$ gives $A = 5$: unstable! instantly decays back ${}^5\text{Li} \to p + {}^4\text{He}$: *no progress!*
- ${}^4\text{He} + {}^4\text{He} \to {}^8\text{Be}$ gives $A = 8$: unstable! quickly decays back ${}^8\text{Be} \to {}^4\text{He} + {}^4\text{He}$: *no progress!*

first step: “p–p reaction”

$p + p \rightarrow$ ^{2}H $+ e^+ + \nu_e$

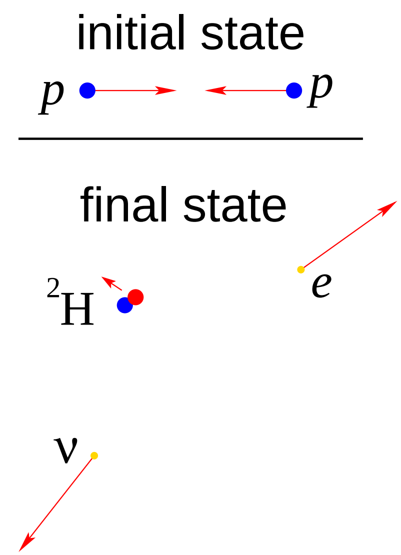
- ^{2}H$=$ $\boxed{np}$ **deuterium**
- e^+ **“positron”**

required by charge conservation

antimatter: anti-electron!

then $e^- + e^+ \rightarrow \gamma + \gamma$ energy!

annihilation



- ν_e **“electron-type neutrino”**

required by angular momentum conservation

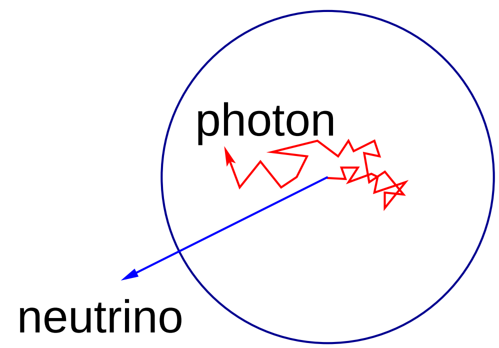
very low-mass ($m_\nu \ll m_e$) particle

only created in nuclear reactions (“weak” decays)

very weakly interacting particle

once born, go thru Sun, Earth, your body

but almost never interact



Q: next steps?

The pp Chain

Dominant nuclear reactions in the Sun: "pp" Chain

$$p + p \rightarrow {}^{2}\mathrm{H} + e^{+} + \nu$$

$$e^{-} + e^{+} \rightarrow \gamma + \gamma$$

$$^{2}\mathrm{H} + p \rightarrow {}^{3}\mathrm{He} + \gamma$$

$$^{3}\mathrm{He} + {}^{3}\mathrm{He} \rightarrow {}^{4}\mathrm{He} + 2p$$

Net effect:

$$4p + 2e^{-} \rightarrow \boxed{2n2p} = {}^{4}\mathrm{He} + \mathrm{energy} + \ldots$$

Fusion Energy

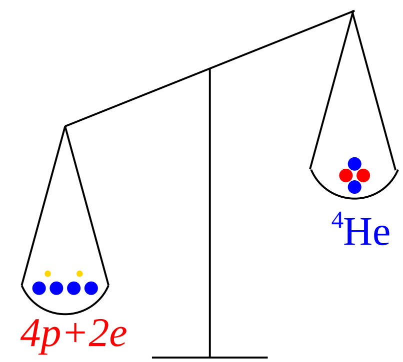
Where does the energy come from? **mass!**

Einstein: mass m at rest contains energy $\varepsilon = mc^2$

Observed fact:

$m(^4\text{He}_{\text{atom}}) < m(4p + 2e)$!

whole $<$ parts!



Do the math:

$$
\begin{array}{rcl}
 & m(4p + 2e) & = 6.694 \times 10^{-27} \text{ kg} \\
- & m(^4\text{He}) & = 6.644 \times 10^{-27} \text{ kg} \\
\hline
= & \Delta m & = 5 \times 10^{-29} \text{ kg}
\end{array}
$$

fusion $\rightarrow$ mass reduction!

$\rightarrow$ rest mass decrease $\rightarrow$ energy release!

Where Does the Energy Go?

energy "reservoir" is from changes in mass
but where does it go?

recall pp chain:

$$p + p \rightarrow {}^{2}\text{H} + e^{+} + \nu \quad (4)$$

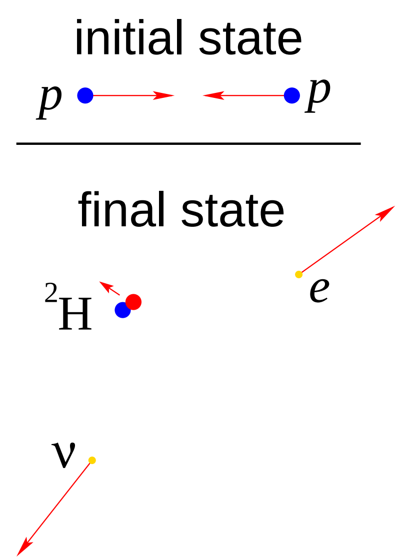
$$e^{+} + e^{-} \rightarrow \gamma\gamma \quad (5)$$

$${}^{2}\text{H} + p \rightarrow {}^{3}\text{He} + \gamma \quad (6)$$

$${}^{3}\text{He} + {}^{3}\text{He} \rightarrow {}^{4}\text{He} + p + p \quad (7)$$

in each reaction mass energy is released: $m_{\text{final}} < m_{\text{initial}}$
for each reaction: *Q: where does that energy go?*
Q: how does this ultimately lead to Sunlight?

$$
\begin{aligned}
p + p &\rightarrow {}^2\text{H} + e^+ + \nu \\
& \quad e^+ + e^- \rightarrow \gamma\gamma \\
{}^2\text{H} + p &\rightarrow {}^3\text{He} + \gamma \\
{}^3\text{He} + {}^3\text{He} &\rightarrow {}^4\text{He} + p + p
\end{aligned}
$$



★ for final state **nuclei**:
energy goes to *motion*: $v_{\text{nucleus}} \gg v_T$
$\Rightarrow$ large *kinetic energy*

then gradually slow, mostly via Coulomb scattering
$\rightarrow$ *heats* the plasma, also generates many photons

★ for final state **photons**:
carry momentum and very high energy: *gamma rays*!
then scatter violently, also *heat* the plasma

in each reaction mass $\rightarrow$ energy (kinetic, photons)
total for each $4p \rightarrow {}^4\text{He}$ fusion:
$Q = \Delta\varepsilon = \Delta mc^2 = 4.5 \times 10^{-12}$ Joules

Estimate Solar fusion energy supply:

$$E_{\text{fuse}} = \frac{\text{\# nuclei in Sun}}{\text{4nuclei/fusion}} \times Q \sim 1.3 \times 10^{45} \text{ Joules} \qquad (8)$$

if *all* Sun's hydrogen is fuel, can burn for
$\tau_{\text{fuse}} = E_{\text{fuse}}/L = 3 \times 10^{18}$ sec $=$ 100 billion years!

Poll: Solar Nuclear Lifetime

if all Sun's hydrogen is fuel, nuclear fusion can burn for
$\tau_{\text{fuse}} = E_{\text{fuse}}/L = 3 \times 10^{18}$ sec $= 100$ billion years!

Vote your conscience!

This is a crude estimate of the solar fusion lifespan–but how?

A this is an *over*estimate of the lifespan

B this is an *under*estimate of the lifespan

Solar Life Expectancy

We have overestimated fuel available for fusion:
assumed Sun can burn all if its hydrogen

$\rightarrow$ only fuse at high T, ρ
$\rightarrow$ core of Sun

true lifetime: $\tau \sim 1 \times 10^{10}$ yr $=$ 10 billion yrs
$\rightarrow$ Sun is middle aged
will last another ~ 5 billion yrs

Q: how test that sun is nuke powered?

How Do We Know?

By the 1930's we knew that the Sun is nuclear powered

`www: Nobel Prize: Hans Bethe`

> The Sun is a mass of incandescent gas
> a gigantic nuclear furnace
> Where hydrogen is burned into helium,
> at temperatures of millions of degrees
> – Lou Singer and Hy Zaret, 1959; cover: They Might Be Giants 1993

Q: how could we be so sure?

Can we get even more direct confirmation?

Q: is another way to confirms the Sun is a nuclear reactor? A "smoking gun" signature?

The Evidence: Solar Neutrinos

If the Sun takes $4p \rightarrow {}^4\text{He} = \boxed{\text{2p2n}}$

then it *must* convert $2p \rightarrow 2n$

$\rightarrow$ *must* produce neutrinos!

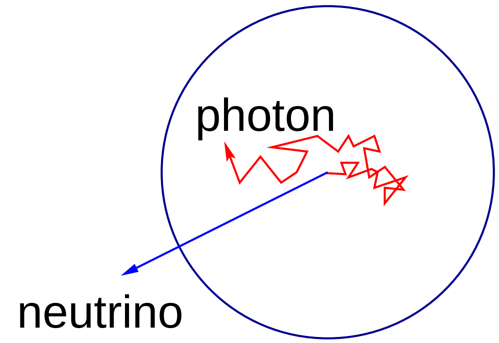
in fact: most made via $pp \rightarrow de^+\nu$

The Sun radiates neutrinos as well as photons!

...we are bathed in solar "neutrinoshine"

Moreover:

- since ν are weakly interacting
  they come directly from the solar core
  $\rightarrow$ messengers from the center of the Sun!
- but luckily, *weakly* interacting $\neq$ *non*-interacting
  $\Rightarrow$ solar neutrinos are potentially observable!
- clever experiments can try to "catch" them



In Search of Solar Neutrinos

experiments have been built to "see" solar neutrinos
by observing rare cases of ν interactions with atoms
all use huge underground detectors
Q: why huge? why underground?

Two types:
1. "radiochemical" – vats of fluid
see element change due to ν
ex: chlorine fluid $\nu + {}^{37}\mathrm{Cl} \rightarrow {}^{37}\mathrm{Ar} + e^-$
collect Ar atoms (radioactive!)
`www: Davis chlorine experiment`

2. "scattering" – vats of ultra-pure water
see light pulses from
high-energy e^- scattered by νs
`www: SNO, Borexino`
`www: Super-K Sun image`

Solar Neutrino Experiments: Results

- ⋆ All experiments detect solar νs!
- ⋆ Scattering experiments show neutrinos come from the Sun!
- ⋆ Amount (flux) is just as predicted!

Q: what fundamental fact(s) is/are confirmed?

Solar Neutrino Results

I. proof that Sun powered by nuke fusion

II. νs give direct view into solar core

III. these underground vats are ν telescopes!

A new window on the Universe:

Nobel Prize 2002!

Using the Sun to probe neutrino transformation and mass:

Nobel Prize 2015!

Solar Neutrino Experiments: A Deeper View

1960s: original chlorine radiochemical experiment (Ray Davis):
- sensitive only to a small component of very high-energy νs
- signal detected, but flux $\Phi_{\nu}^{\text{obs}} \approx \Phi_{\nu}^{\text{predicted}}/3$

  birth of **"solar neutrino problem"** – where did they go?

1990's: solar neutrino deficit confirmed

possible explanations:
- theory of solar nuclear reactions is wrong/incomplete
- neutrino theory incomplete

it was already known that: *neutrinos have 3 varieties ("flavors")*
ν_e, ν_μ, ν_τ: named for partner they appear with
solar neutrinos produced as ν_e: should remain so
$\rightarrow$ unless neutrinos can transform into different flavors!

Q: how to test for the latter possibility?

The Sun Reveals New Neutrino Physics

if neutrino flavor transformations exist

- some particles born in Sun as ν_e
- can arrive at Earth as ν_μ or ν_τ
- but radiochemical experiments only "see" ν_e

To test:

build detectors sensitive to *all flavors*

this was done: Sudbury Neutrino Observatory (SNO)

early 2000s: SNO results weigh in

- ν_μ and ν_τ *detected* from Sun!
- *total flux* for *all* ν *agrees* with Solar model!
- **confirms new neutrino physics**
- also *transformations require neutrinos have mass!*
  non-obvious property of the quantum flavor transformations

Director's Cut Extras

Nuclear Stability

stable atomic nuclei are bound states of nucleons

- that is: they can't "fall apart" on their own
- the same way bound atoms, planetary systems, binary stars don't fall apart

in other words:
to unbind a nucleus – to dismantle it to protons and neutrons requires an *input of energy*

Q: meaning for energies of the nucleus and its components?

Nuclear Binding Energy

so nucleus A, with Z protons and N neutrons
has **binding energy** B_A = energy required to rip apart
this means that

$$E_A + B_A = ZE_\mathrm{p} + NE_\mathrm{n} \tag{9}$$

that is

$$\begin{aligned} \text{binding} &= \text{parts} - \text{whole} & (10) \\ B_A &= ZE_\mathrm{p} + NE_\mathrm{n} - E_A & (11) \\ &= > 0 & (12) \end{aligned}$$

so energy of parts is more than whole!

but Einstein says $E = mc^2$
Q: what does this mean generally? implications for nuclei?

Nuclear Binding

Einstein $E = mc^2$ says: an object at rest, with mass m
contains energy $E = mc^2$ simply by having mass

- mass is a form of energy!
- not due to motion: “rest mass energy”

for nuclei (and similar any other bound system), binding energy

$$B_A = ZE_\mathrm{p} + NE_\mathrm{n} - E_A \ > \ 0 \tag{13}$$

implies a *mass difference*

$$B_A = Zm_\mathrm{p}c^2 + Nm_\mathrm{n}c^2 - m_Ac^2 = (Zm_p + Nm_n - m_A)c^2 \ > \ 0 \tag{14}$$

- mass of parts > mass of whole
- mass difference measures binding energy

bound system	binding energy B	binding energy per nucleon $B/(Z+N)$
hydrogen atom $\boxed{pe}$	13.6 eV	13.6 eV/nucleon
^{4}He nucleus $\boxed{2p, 2n}$	28.3 MeV	7.07 MeV/nucleon
^{56}Fe nucleus $\boxed{26p, 30n}$	492 MeV	8.79 MeV/nucleon
^{238}U nucleus $\boxed{92p, 146n}$	1801 MeV	7.57 MeV/nucleon

Q: atoms vs nuclei comparison?

Q: comparison among nuclei?

Q: lessons for Sun?