Astro 404
Lecture 16
Sept. 29, 2021

Announcements:

- **Problem Set 5 due Friday**
  Office Hours: instructor after class or by appt
  TA: 2:30-3:30 tomomrrow
- Distinguished lecture **tonight!**
  Illinois alumna Vicky Kalogera
  "Einstein's Waves: Cosmic Sounds from Black Holes
  and Neutron Stars"
  Wed Sept 29, 7–8pm, Lincoln Hall Theater and online
  Canvas has registration link and *Bonus Points assignment*

Last time: solar neutrinos
Q: what's a neutrino?
Q: how do we detect neutrinos from the Sun?
Q: what is the main result of solar neutrino experiments?

Solar Neutrino Experiments: Results

- ⋆ All experiments detect solar νs!
- ⋆ Scattering experiments show neutrinos come from the Sun!
- ⋆ Amount (flux) is just as predicted! (PS5)

  `www: Gran Sasso Laboratory and Borexino Experiment`

Q: what fundamental fact(s) is/are confirmed?

Solar Neutrino Results

I. proof that *the Sun is powered by nuke fusion*

II. νs give a direct view into *solar core*
“a solar thermometer”

III. these underground vats are ν telescopes!

A new window on the Universe:
Nobel Prize 2002!
Raymond Davis Jr. and Masatoshi Koshiba

neutrinos forevermore part of **Multimessenger Astronomy**
probing the cosmos is not just for photons anymore!

Solar Neutrino Experiments: A Deeper View

1960s: original chlorine radiochemical experiment (Ray Davis):
- sensitive only to a small component of very high-energy νs
- signal detected, but flux $\Phi_\nu^{\text{obs}} \approx \Phi_\nu^{\text{predicted}}/3$

birth of **"solar neutrino problem"** – where did they go?

1990's: solar neutrino deficit confirmed

possible explanations:
- theory of solar nuclear reactions is wrong/incomplete
- neutrino theory incomplete

it was already known that: *neutrinos have 3 varieties ("flavors")*

ν_e, ν_μ, ν_τ: named for partner they appear with

solar neutrinos produced as ν_e: should remain so

$\rightarrow$ unless neutrinos can transform into different flavors!

Q: how to test for the latter possibility?

The Sun Reveals New Neutrino Physics

if neutrino flavor transformations exist

- some particles born in Sun as ν_e
- can arrive at Earth as ν_μ or ν_τ
- but radiochemical experiments only "see" ν_e

To test:
build detectors sensitive to *all flavors*
this was done: Sudbury Neutrino Observatory (SNO)

early 2000s: SNO results weigh in

- ν_μ and ν_τ *detected* from Sun!
- *total flux* for *all* ν *agrees* with Solar model!
- **confirms new neutrino physics**
- also *transformations require neutrinos have mass!*
  non-obvious property of the quantum flavor transformations

Neutrinos and Mass

neutrino flavor transformations confirmed in lab experiments:
use nuclear reactors as ν_e sources
detect neutrino disappearance with distance
characteristic of quantum "oscillation" into other flavors
`www: oscillation data`

confirms *neutrinos have mass*,
but only measures mass differences!

Using the Sun to probe neutrino transformation and mass:
Nobel Prize 2015!
Arthur MacDonald and Taakaki Kajita

Cosmic Gall

by John Updike

Telephone Poles and Other Poems

1963

Neutrinos, they are very small.
They have no charge and have no mass
And do not interact at all.

The earth is just a silly ball
To them, through which they simply pass,
Like dustmaids down a drafty hall
Or photons through a sheet of glass.

They snub the most exquisite gas,
Ignore the most substantial wall,
Cold-shoulder steel and sounding brass,
Insult the stallion in his stall.

And, scorning barriers of class,
Infiltrate you and me! Like tall
And painless guillotines, they fall
Down through our heads into the grass.

At night, they enter at Nepal
And pierce the lover and his lass
From underneath the bed–you call
It wonderful; I call it crass.

Cosmic Gall

by John Updike

Telephone Poles and Other Poems

1963 + 2019 Update!

Neutrinos, they are very small.
They have no charge and ~~have no~~ tiny mass
And ~~do not~~ hardly interact at all.

The earth is just a silly ball
To them, through which they simply pass,
Like dustmaids down a drafty hall
Or photons through a sheet of glass.

They snub the most exquisite gas,
Ignore the most substantial wall,
Cold-shoulder steel and sounding brass,
Insult the stallion in his stall.

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Neutrinos versus Dark Matter

dark matter: unseen mass that dominates cosmic matter
dark matter form unknown but:

- must have mass and gravitate–this is how we infer it to exist!
- DM particles must interact weakly – else already discovered

DM similarity to neutrinos is obvious! And connections are deep:

- neutrinos *are* a component of dark matter
  produced in the early universe, remain today
  but masses too small – can't be most of DM
- dark matter experiments very similar to neutrino telescopes
  underground, look for scattering events
  soon will reach sensitiviy to solar neutrinos
  then will have diffiulty searching beneath this "neutrino floor"
- dark matter theories inspired by neutrino success story
  where new invisible particles found to exist & play critical role

Poll: How do Stars Shine?

We have proven the Sun is nuclear powered in core: energy generated by $4p \rightarrow {}^4$He

Vote your conscience!

What can we infer about other stars?

A *all other stars* burn hydrogen $\rightarrow$ helium

B *only* $1M_\odot$ *stars* burn hydrogen $\rightarrow$ helium

C *all main sequence* stars burn hydrogen $\rightarrow$ helium

D none of the above

Main Sequence: Hydrogen Burning Phase

HR diagram teaches:

- the Sun is a typical main sequence star
- main sequence is the longest phase in a star's life

energy conservation teaches:
main sequence luminosity and lifetime demand large energy source
only nuclear energy can sustain

so we infer:
all main sequence stars are nuclear reactors
converting hydrogen to helium
nuclear power makes stars shine!

Another Way to Burn Hydrogen

the Sun and other stars are mostly made of hydrogen
with about 28% helium by mass (less by number–*Q: why?*)
and about 2% by mass of heavier elements

some of most abundant heavy elements ("metals")
are carbon, oxygen, nitrogen (CNO)
these allow for another set of reactions

$$\begin{aligned}
{}^{12}\mathrm{C} + p &\rightarrow {}^{13}\mathrm{N} + \gamma \\
{}^{13}\mathrm{N} &\rightarrow {}^{13}\mathrm{C} + e^{+} + \nu_e \quad \text{radioactive decay} \\
{}^{13}\mathrm{C} + p &\rightarrow {}^{14}\mathrm{N} + \gamma \\
{}^{14}\mathrm{N} + p &\rightarrow {}^{15}\mathrm{O} + \gamma \\
{}^{15}\mathrm{O} &\rightarrow {}^{15}\mathrm{N} + e^{+} + \nu_e \quad \text{radioactive decay} \\
{}^{15}\mathrm{N} + p &\rightarrow {}^{12}\mathrm{C} + {}^{4}\mathrm{He}
\end{aligned}$$

Q: what is total net input? total net output?
Q: what is the role of CNO?

The CNO Cycle

$$
\begin{aligned}
{}^{12}\mathrm{C} + p &\rightarrow {}^{13}\mathrm{N} + \gamma \\
{}^{13}\mathrm{N} &\rightarrow {}^{13}\mathrm{C} + e^{+} + \nu_e \quad \text{radioactive decay} \\
{}^{13}\mathrm{C} + p &\rightarrow {}^{14}\mathrm{N} + \gamma \\
{}^{14}\mathrm{N} + p &\rightarrow {}^{15}\mathrm{O} + \gamma \\
{}^{15}\mathrm{O} &\rightarrow {}^{15}\mathrm{N} + e^{+} + \nu_e \quad \text{radioactive decay} \\
{}^{15}\mathrm{N} + p &\rightarrow {}^{12}\mathrm{C} + {}^{4}\mathrm{He}
\end{aligned}
$$

then repeat–recycle the ^{12}C!

⋆ *same net effect as pp chain: another way to burn hydrogen!*

⋆ *total CNO unchanged:* acts as a *catalyst!*

⋆ CNO morphs to different forms but comes back: cyclic!
can start anywhere in the cycle!

this chain: the **CNO cycle**

Poll: Hydrogen Burning in Stars

Vote your conscience!

Q: Which chain dominates hydrogen production in stars?

A pp dominates for all stars

B CNO cycle dominates for all stars

C pp dominates for stars with cooler cores, CNO for hotter

D pp dominates for stars with hotter cores, CNO for cooler

Hydrogen Burning: pp versus CNO

reaction chain speed/importance set by slowest link
the most difficult and thus "*rate limiting step*"

pp *chain:* rate limited by $pp \to de^+\nu_e$
- weak reaction required
- three body final state disfavored

CNO cycle: rate limited by $p + {}^{14}\text{N} \to {}^{15}\text{O} + \gamma$
- large Coulomb repulsion due to ^{14}N charge $Z = 7$
- but CNO has no weak reactions, only weak decays

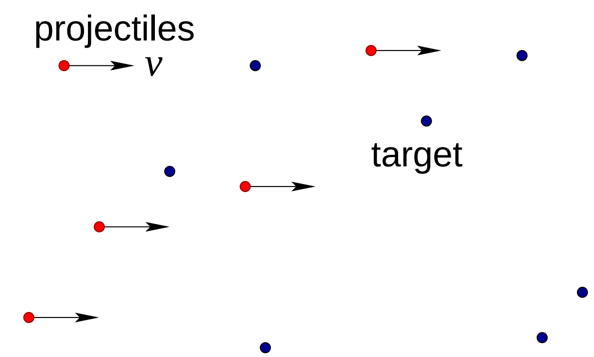
which is dominant depends on star core temperature!
cooler stars can't overcome large CNO Coulomb barrier
but hot stars can, then can burn fast

for main sequence stars:
- pp dominates of mass $M \lesssim 1.3 M_\odot$
- CNO dominates for higher masses

Collision Technology: Reaction Rates and Cross Sections

We need to connect *particle collisions and reactions (micro scale)* to *energy generation in stars (macro scale)*

Imagine some general reaction: $a + b \rightarrow c + d$



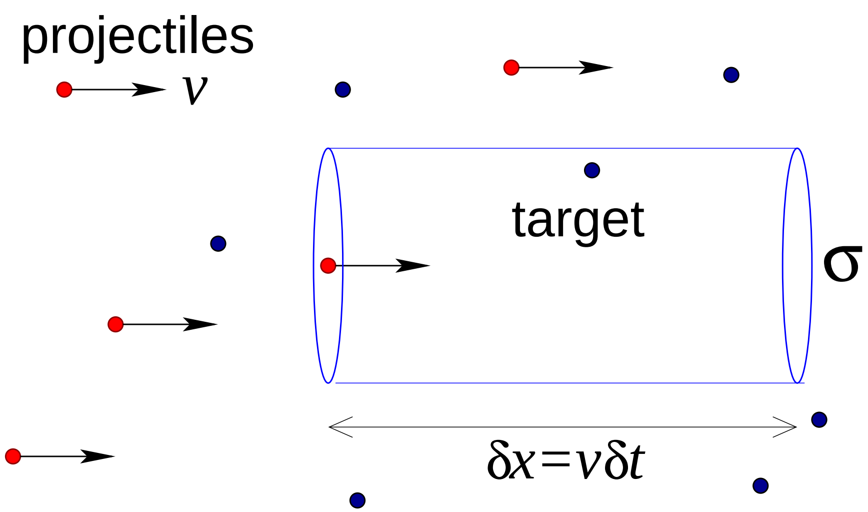
Consider particle beam:
"projectiles," number density n_a
incident w/ velocity v
on targets of number density n_b

Due to interactions, targets and projectiles "see" each other as spheres of projected area $\sigma(v)$: the **cross section**

- ⋆ fundamental measure interaction strength/probability
- ⋆ *microphysics meets astrophysics via* σ

interaction zone: as seen by each projectile
a targe sweeps out "*scattering tube*"

- tube area σ
- length $\delta x = v\delta t$



scattering tube volume around target:
$\delta V = \sigma \delta x = \sigma v \delta t$

collide if a target is in the volume

Cross Section, Flux, and Collision Rate

in scattering tube volume $\delta V = \sigma v \, \delta t$,
average number of targets in tube $= \mathcal{N}_{\text{targ,tube}} = n_b \delta V$
so: *average number of collisions in δt:*

$$\delta \mathcal{N}_{\text{coll}} = \mathcal{N}_{\text{targ,tube}} = n_{\text{b}} \sigma v \delta t \quad (1)$$

so $\delta \mathcal{N}_{\text{coll}} / \delta t$ gives

$$\text{avg collision rate } \textit{per projectile } a \quad \boxed{\Gamma_{\text{per}\, a} = n_b \sigma_{ab} v} \quad (2)$$

Q: Γ units? sensible scalings n_b, σ, v? why no n_a?

Q: average collision time interval for a target?

Q: average projectile distance traveled in this time?

Reactions: Characteristic Length and Time Scales

estimate *average time between collisions for projectile* a:
mean free time τ
collision rate: $\Gamma = d\mathcal{N}_{\text{coll}}/dt$
so wait time until next collision set by $\delta N_{\text{coll}} = \Gamma_{\text{per}\, a}\tau = 1$:

$$\tau = \frac{1}{\Gamma_{\text{per}\, a}} = \frac{1}{n_b \sigma v} \tag{3}$$

in this time, projectile a moves distance: **mean free path**

$$\ell_{\text{mpf}} = v\tau = \frac{1}{n_b \sigma} \tag{4}$$

no explicit v dep, but still $\ell(E) \propto 1/\sigma(E)$
Q: physically, why the scalings with n, σ*?*

PS5: alternative derivation of mean free path

Q: what sets σ *for billiard balls?*
Q: what set σ *for* $e^- + e^-$ *scattering?*

Cross Section vs Particle "Size"

if particles interact only by "touching"
(e.g., billiard balls)
then $\sigma \leftrightarrow$ particle radii: $\sigma = \pi(r_a + r_b)^2$

but: if interact by force field
(e.g., gravity, EM, nuclear, weak)
cross section σ *unrelated* to physical size!

For example: e^- has $r_e = 0$ (as far as we know!)
but electrons scatter via Coulomb (and weak) interaction
"touch-free scattering"

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Q: what is collision or reaction rate per volume?

Reaction Rate Per Volume

recall: collision rate *per target* b is $\Gamma_{\text{per}\,a} = n_b \sigma_{ab} v$
total collision rate *per unit volume* is

$$r_{ab} = \frac{\text{collision rate}}{\text{volume}} = \frac{\text{collision rate}}{\text{projectile}} \times \frac{\text{projectiles}}{\text{volume}} \quad (5)$$

$$= \Gamma_{\text{per}\,a} n_a = n_a n_b \sigma v \quad (6)$$

Note: *symmetric*–can choose either particle type as projectile

also note: $n_a n_b \propto \mathcal{N}_a \mathcal{N}_b =$ *number of* ab *pairs*
reflects the fact that $ab \to cd$ reactions
are initiated by ab pairs!

Q: What if particles have more than one relative velocity?
What is energy generation rate per volume?

Reaction and Energy Generation Rates

If $v \in$ distribution, rates is average over velocities:

$$\langle r_{ab} \rangle = n_a n_b \langle \sigma v \rangle \tag{7}$$

energy generation rate per volume:
depends on reaction rate r_{ab}
and energy release per reaction Q_{ab} :

$$\dot{\epsilon}_{ab} = \frac{dE_{ab}}{dV\ dt} = Q_{ab} \frac{dN}{dV\ dt} = Q_{ab}\ r_{ab} = Q_{ab}\ n_a n_b \langle \sigma v \rangle \tag{8}$$

Finally, number densities proportional to mass density $n_a \propto \rho$:
$n_a = \rho_a / m_a = X_a \rho / m_a$
where m_a is mass of particle a
and $X_a = \rho_a / \rho$ is fraction of mass density in a, so

$$\dot{\epsilon}_{ab} = Q_{ab}\ n_a n_b \langle \sigma v \rangle = \frac{Q_{ab}}{m_a m_b} X_a X_b \rho^2 \langle \sigma v \rangle \tag{9}$$

Hydrogen Burning Rates

nuclear energy generation rate per volume:

$$q_{ab} = \frac{\dot{\epsilon}_{ab}}{\rho} = X_a X_b \frac{Q_{ab}}{m_a m_b} \rho \langle \sigma v \rangle \tag{10}$$

- proportional to *density*: $q \propto \rho$
- depends on *temperature* via particle speeds: $\langle \sigma(v) \; v \rangle$

for hydrogen burning, roughly have:

$$q_{pp} \propto X_p^2 \rho \; T^4 \tag{11}$$

$$q_{\mathrm{CNO}} \propto X_p X_{\mathrm{CNO}} \rho \; T^{16} \tag{12}$$

note strong CNO temperature dependence:
important for stars with high T_{c}

$\Rightarrow$ huge luminosity for massive main sequence stars

Director's Cut Extras

Time Reversal and Particle Interactions

the claimed if ν were truly *non-interacting*
and can't collide and react with ordinary particles
then they can't be made in the first place!

Why? *time reversal invariance*
almost without exception: if a microscopic process can occur
then the "time reversed" process is also physically possible
$\rightarrow$ run the movie backwards, and this must be allowed

so consider the observed reaction: $\nu_e p \rightarrow n e^+$
• *neutrino absorbed* by proton, creates neutron and positron
this requires time-reversed $n e^+ \rightarrow \nu_e p$ is possible
• *neutrino emitted*

Lesson: time reversal invariance implies that
absorbers most also be emitters
both must occur if an interaction exists

Energy Generation Per Unit Mass

recall that we can describe a star's properites vs radius or mass

using radius, energy generation rate per volume is most useful
total energy generation rate:

$$\ell(r) = \int \dot{\epsilon}_{ab} \, dV = \int_0^r 4\pi r^2 \, \dot{\epsilon}_{ab} \, dr \tag{13}$$

using enclosed mass m, need energy generation rate *per mass*

$$\ell(m) = \int q_{ab} \, dm \tag{14}$$

and since $dm = \rho dV$, we have $\int q_{ab}\rho \, dV = \int \dot{\epsilon}_{ab} \, dV$
and thus energy generation per unit mass:

$$q_{ab} = \frac{\dot{\epsilon}_{ab}}{\rho} = X_a X_b \frac{Q_{ab}}{m_a m_b} \rho \langle \sigma v \rangle \tag{15}$$