Astro 404
Lecture 17
Oct. 1, 2021

Announcements:
- **Problem Set 5 due today**
- **Problem Set 6 due next Friday**
- Distinguished Lecture Bonus on Canvas until Wed Oct 7
  can view video if you missed the talk

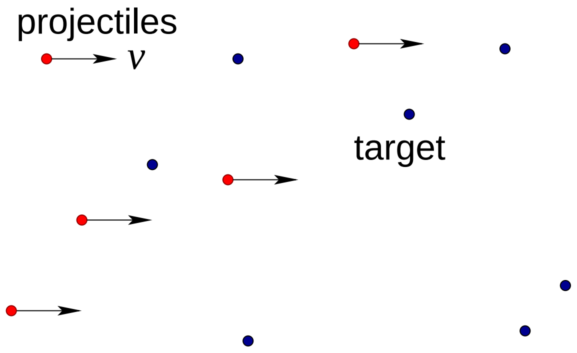
Last time: finished solar neutrinos
Q: main lessons from detections of solar neutrinos

Today: energy generation in stars

Collision Technology: Reaction Rates and Cross Sections

We need to connect *particle collisions and reactions (micro scale)* to *energy generation in stars (macro scale)*

Imagine some general reaction: $a + b \rightarrow c + d$



Consider particle beam:

"projectiles," number density n_a

incident w/ velocity v

on targets of number density n_b

Goal: understand reaction/collisions

- rate of collisions
- particle distances traveled between collisions
- rate of energy generation from reactions

Cross Sections

If particles don't interact at all: no scattering!
pass through each other

But when interactions can occur:
targets and projectiles "see" each other
as spheres of projected area $\boxed{\sigma(v)}$: the **cross section**

⋆ fundamental measure interaction strength/probability
⋆ generally depends on particle velocity/energy–often strongly!
⋆ *microphysics meets astrophysics via* σ

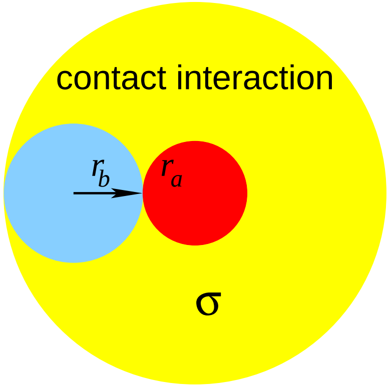
How do we know cross sections?

- calculate σ given particle and interaction properties
- measure σ in collision experiments

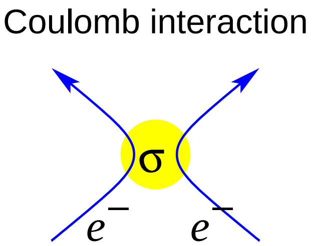
Q: what sets σ *for billiard balls?*
Q: what set σ *for* $e^- + e^-$ *scattering?*

Cross Section vs Particle “Size”

if particles interact only by “touching”
that is, direct contact (e.g., billiard balls)
then $\sigma \leftrightarrow$ particle radii: $\sigma_{\text{contact}} = \pi(r_a + r_b)^2$



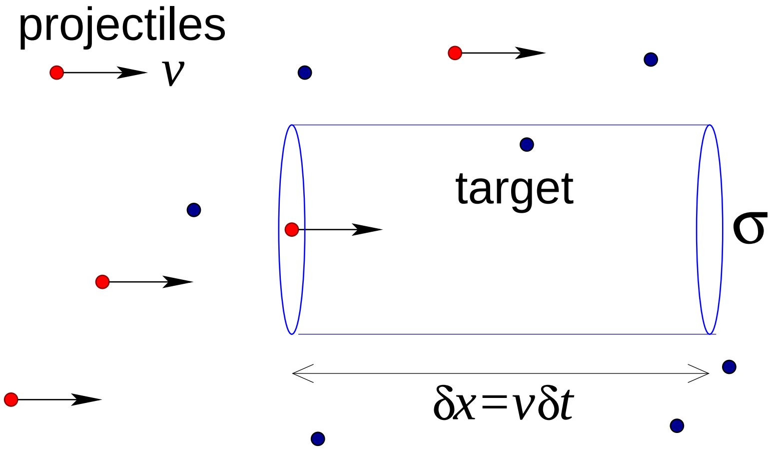
but: if interact by force field
(e.g., gravity, EM, nuclear, weak)
cross section σ *unrelated* to physical size!
this is the case for all collisions we will study



For example: e^- has $r_e = 0$ (as far as we know!)
but electrons scatter via Coulomb (and weak) interaction
“touch-free scattering”

Cross Sections and Collisions

in *time interval* δt:
each projectile sweeps out cylindrical "*scattering tube*"



scattering tube acts as "interaction zone"

- tube area σ
- tube length $\delta x = v\ \delta t$
- *scattering tube volume*: $\delta V = \sigma\ \delta x = \sigma\ v\ \delta t$

Collision happens: if a target is in the scattering tube
Q: how many targets b *in scattering tube, given number dens.* n_b*?*

Cross Section, Flux, and Collision Rate

in scattering tube volume $\delta V = \sigma v \, \delta t$,
average number of targets in tube $= \mathcal{N}_{\text{targ,tube}} = n_b \; \delta V$
so: *average number of collisions in δt:*

$$\delta \mathcal{N}_{\text{coll}} = \mathcal{N}_{\text{targ,tube}} = n_{\text{b}} \;\; \sigma \;\; v \;\; \delta t \tag{1}$$

so $\delta \mathcal{N}_{\text{coll}} / \delta t$ gives

$$\text{avg collision rate } \textit{per projectile } a \;\; \boxed{\Gamma_{\text{per}\, a} = n_b \;\; \sigma_{ab} \;\; v} \tag{2}$$

Q: Γ units? sensible scalings n_b, σ, v? why no n_a?

Q: average collision time interval for a projectile?
Q: average projectile distance traveled in this time?

Reactions: Characteristic Length and Time Scales

estimate *average time between collisions for projectile a*:

mean free time τ

collision rate: $\Gamma = d\mathcal{N}_{\rm coll}/dt$

so wait time until next collision set by $\delta\mathcal{N}_{\rm coll} = \Gamma_{{\rm per}\, a}\tau = 1$:

$$\tau = \frac{1}{\Gamma_{{\rm per}\, a}} = \frac{1}{n_b \sigma v} \tag{3}$$

in this time, projectile a moves distance: **mean free path**

$$\ell_{\rm mpf} = v\tau = \frac{1}{n_b \sigma} \tag{4}$$

no explicit v dep, but still $\ell(E) \propto 1/\sigma(E)$

Q: physically, why the scalings with n, σ?

PS5: alternative derivation of mean free path

Q: what is collision or reaction rate per volume?

Reaction Rate Per Volume

recall: collision rate *per target b* is $\Gamma_{\text{per}\,a} = n_b \sigma_{ab} v$
total collision rate *per unit volume* is

$$r_{ab} = \frac{\text{collision rate}}{\text{volume}} = \frac{\text{collision rate}}{\text{projectile}} \times \frac{\text{projectiles}}{\text{volume}} \tag{5}$$

$$= \Gamma_{\text{per}\,a} n_a = n_a n_b \sigma v \tag{6}$$

Note: *symmetric*—can choose either particle type as projectile

also note: $n_a n_b \propto \mathcal{N}_a \mathcal{N}_b =$ *number of ab pairs*
reflects the fact that $ab \to cd$ reactions
are initiated by ab pairs!

∞ *Q: What if particles have more than one relative velocity?*
What is energy generation rate per volume?

Reaction and Energy Generation Rates

If $v \in$ distribution, rates is average over velocities:

$$\langle r_{ab} \rangle = n_a n_b \langle \sigma v \rangle \tag{7}$$

energy generation rate per volume:
depends on reaction rate r_{ab}
and energy release per reaction Q_{ab} :

$$\dot{\epsilon}_{ab} = \frac{dE_{ab}}{dV\ dt} = Q_{ab} \frac{dN}{dV\ dt} = Q_{ab}\ r_{ab} = Q_{ab}\ n_a n_b \langle \sigma v \rangle \tag{8}$$

Finally, number densities proportional to mass density $n_a \propto \rho$:
$n_a = \rho_a / m_a = X_a \rho / m_a$
where m_a is mass of particle a
and $X_a = \rho_a / \rho$ is fraction of mass density in a, so

$$\dot{\epsilon}_{ab} = Q_{ab}\ n_a n_b \langle \sigma v \rangle = \frac{Q_{ab}}{m_a m_b} X_a X_b \rho^2 \langle \sigma v \rangle \tag{9}$$

Hydrogen Burning Rates

nuclear energy generation rate per volume:

$$\dot{\epsilon}_{ab} = X_a X_b \frac{Q_{ab}}{m_a m_b} \rho^2 \langle \sigma v \rangle \tag{10}$$

- proportional to *density*: $\dot{\epsilon} \propto \rho^2$
- depends on *temperature* via particle speeds: $\langle \sigma(v)\ v \rangle$

for hydrogen burning, roughly have:

$$\dot{\epsilon}_{pp} \propto X_p^2 \rho^2\ T^4 \tag{11}$$

$$\dot{\epsilon}_{\mathrm{CNO}} \propto X_p X_{\mathrm{CNO}} \rho^2\ T^{16} \tag{12}$$

note strong CNO temperature dependence:
important for stars with high T_{c}
⇒ huge luminosity for massive main sequence stars

Main Sequence: Core Composition Over Time

in the core of a main sequence star

hydrogen fusion ("nuclear burning") reactions: $4p \rightarrow {}^4\text{He} + 2e^+ + 2\nu_e$

- positrons annihilate $e^+ + e^- \rightarrow \gamma + \gamma$
- neutrinos ν_e escape

so in core: net change in matter is $4p + 2e \rightarrow {}^4\text{He}$

so hydrogen burning in core:

- *reduces* the number of gas particles (electrons and nuclei)
- *increases* average mass m_g of a gas particle

Poll: Core Pressure

compare the Sun's core at start of H burning vs present day and *imagine core temperature and volume held fixed*

What effect does H burning have on core pressure?

A core pressure *reduced*

B core pressure *increased*

C core pressure *constant*

Main Sequence Evolution

hydrogen burning $4p + 2e \rightarrow {}^4\mathrm{He}$
reduces number of gas particles in Sun's core
and increases average gas particle mass m_{g}

core pressure: ideal gas law

$$P = \frac{N\ kT}{V} = n\ kT \tag{13}$$

if V fixed and T fixed, fewer particles → lower N
core pressure P decreases!

but pressure supports the core against gravity
reduced pressure → can't maintain hydrostatic equilibrium!

Q: how would the star respond?

Main Sequence Evolution

Virial theorem, ideal gas

$$U = \frac{3}{2}\frac{M}{m_{\rm g}}\langle kT\rangle \quad = \quad -\frac{1}{2}\Omega \sim \frac{GM^2}{R} \tag{14}$$

$$\langle kT\rangle \quad \sim \quad \frac{GMm_{\rm g}}{R} \tag{15}$$

main sequence: H→^{4}He burning gives $m_{\rm g}$ increase
→ *contraction*: core density increase
→ $\langle kT\rangle$ increase, also density ρ increase

recall pp chain energy release per mass: $\dot{\epsilon}_{pp} \propto \rho^2 T^4$
core increase in ρ, T → *higher energy production!*

star luminosity increases – “main sequence brightening”

Main Sequence Evolution: H–R Diagram

main sequence core H burning: luminosity increases with time

in detailed models of the Sun

- initial *zero age main sequence* luminosity

$$L_{\odot,\mathrm{init}} = 0.7 L_{\odot,\mathrm{today}} \tag{16}$$

- turns out: star radius R increases too
  small change in T_{eff} – in Sun, slight increase

Q: consequences for HR diagram? how to test?

Main Sequence Evolution on the H–R Diagram

Sun in H–R diagram over time:

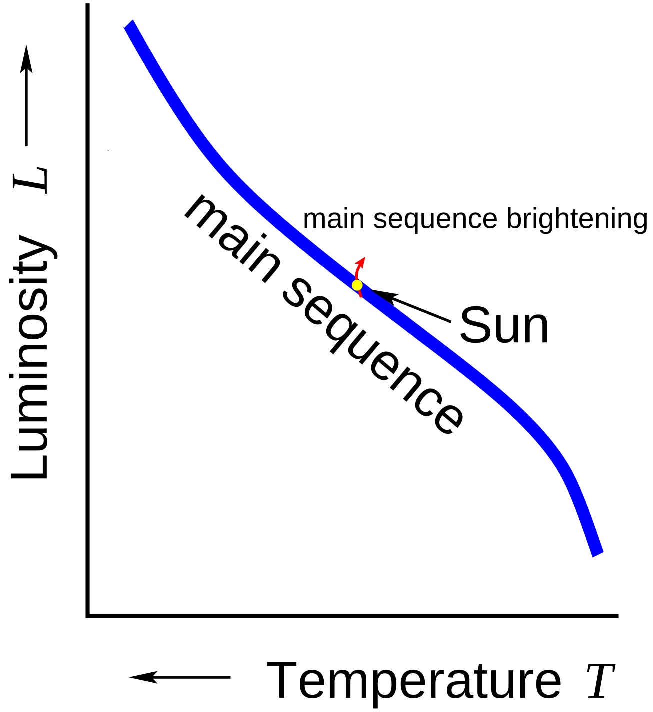
Sun point moves upward on main sequence

other stars evolve similarly

but sometimes change in $T_{\rm eff}$

for a group of stars with mixed ages

“smears out” the main sequence width



to test: find $1M_\odot$ *“solar twins” in young star clusters*

these indeed show lower L!

Q: implications of 30% *less luminous young Sun for Earthlings?*

The Faint Young Sun

consequences of Sun's main sequence brightening

in the past the Sun was less luminous
at zero age on main sequence $L_{\odot,\mathrm{ZAMS}} = 0.7 L_{\odot,\mathrm{today}}$
so lower flux $F_{\odot}(1\ \mathrm{au})$: "faint young Sun"

but this sets Earth's temperature, so: cooler early Earth!
if Earth absorbs same sunlight as now (same albedo)

$$T_{\mathrm{Earth,init}} = \left(\frac{L_{\odot,\mathrm{init}}}{L_{\odot,\mathrm{today}}}\right)^{1/4} T_{\mathrm{Earth,today}} \approx 263\ \mathrm{K} = -12^{\circ}\ \mathrm{C} \quad (17)$$

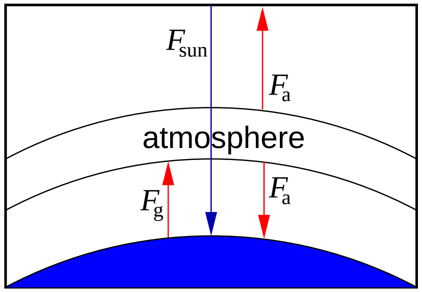
Cold enough to freeze seawater!
Earth frozen for first 2 Gyr (2.5 Gyr ago)! Yikes.

but: evidence for liquid water, and even life, up to 3.8 Gyr ago
Q: possible explanations?

Fain Young Sun Problem: Possible Solutions

liquid water on Earth back to Archean era 2.5 – 3.8 Gyr ago
how to reconcile with Solar evolution?

Greenhouse Effect – add blanket to Earth
atmospheric greenhouse gases trap Earth's thermal IR radiation
warm Earth's surface above airless temperature



today: greenhouse effect warms Earth by $\sim 30^\circ$
in past: *if thicker greenhouse gases, Earth warmer*
e.g., Sagan & Mullen (1972) proposed ammonia in early Earth
later shown unlikely, but basic idea remains

Solar Mass Loss – move the Earth
if early Sun had stronger mass loss than today
then *initial solar mass larger*

but Earth's angular momentum $J_{\text{Earth}} = M_{\text{Earth}} v a$ conserved
this and Kepler's laws say
higher $M_{\odot} \rightarrow$ smaller semimajor axis a: Earth closer!
and sees *higher solar flux* $F = L_{\odot}/4\pi a^2$: hotter!

this would require a higher solar mass in the past
would also affect orbits of other planets
good: lots of evidence early Mars had liquid water

Q: what about the future Sun? mitigation?

The Future Sun

main sequence brightening will continue in the future
unmeasurably small changes on human timescales
but eventually will profoundly affect the Earth

1 Gyr from now: Sun 10% more luminous
heating → evaporation of water vapor → adds to greenhouse
in upper atmosphere, UV from Sun breaks up H_2O molecules
and H lost to space:

- Earth hot and dry
- and losing water

3.5 Gyr from now: Sun 40% more luminous
oceans evaporated, hydrogen lost to space
runaway greenhouse effect
Uh oh. probably no life unless mitigation. *Q: suggestions?*

The Ultimate Global Warming

What is to be done–mitigation?

move Earth's orbit outward?

- perhaps by using asteroids to exchange energy with Jupiter
- a huge task, but we have lots of time

move the people: perhaps terraform Mars?

- a huge task, enormous energy cost to leave Earth
- "Mars ain't the kind of place to raise your kids" (E. John 1972)
  unclear how much water is available in permafrost
  Martian soil is poisonous–sorry Matt Damon!
  toxic concentrations of perchlorates (Cl-bearing compounds)