Astro 404

Lecture 18

Oct. 4, 2021

Announcements:

- **Problem Set 6 due Friday**
- Distinguished Lecture Bonus on Canvas until Wed Oct 7
  can view video if you missed the talk

Last time:

hydrogen burning and the evolution of main sequence stars

Q: how does the Sun's core change during the main seq?

Q: Sun evolutions on the HR diagram during the MS?

main sequence = hydrogen burning

net effect $4p + 2e^- \to {}^4\text{He}$: less H, more He in solar core

- fewer particles
- higher average gas particle mass $m_{\rm g}$

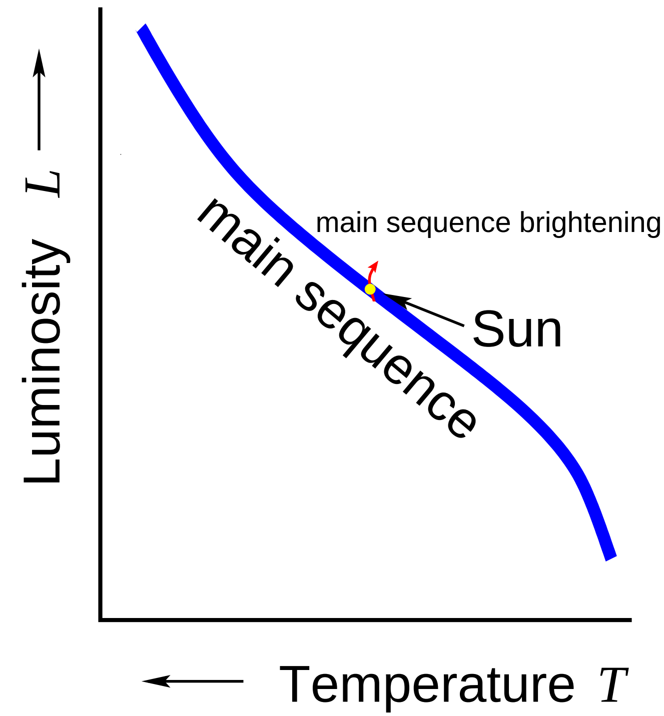
to maintain pressure & hydrostatic equilibrium

central density and temperature rise with time

increases nuclear energy generation rate

net effect:

luminosity increases during main sequence



Sun: "main sequence brightening"

- faint young sun paradox
- future Sun: the ultimate global warning

The Future Sun

main sequence brightening will continue in the future
unmeasurably small changes on human timescales
but eventually will profoundly affect the Earth

1 Gyr from now: Sun 10% more luminous
heating → evaporation of water vapor → adds to greenhouse
in upper atmosphere, UV from Sun breaks up H_2O molecules
and H lost to space:

- Earth hot and dry
- and losing water

3.5 Gyr from now: Sun 40% more luminous
oceans evaporated, hydrogen lost to space
runaway greenhouse effect
Uh oh. probably no life unless mitigation. *Q: suggestions?*

The Ultimate Global Warming

What is to be done–mitigation?

move Earth's orbit outward?
- perhaps by using asteroids to exchange energy with Jupiter
- a huge task, but we have lots of time

move the people: perhaps terraform Mars?
- a huge task, enormous energy cost to leave Earth
- "Mars ain't the kind of place to raise your kids" (E. John 1972)
  unclear how much water is available in permafrost
  Martian soil is poisonous–sorry Matt Damon!
  toxic concentrations of perchlorates (Cl-bearing compounds)

Reaction Rate Recap

nuclear reaction rates: there was a lot of formalism!
easy to get lost! but basic ideas are simple

key point:
almost all important nuclear reactions in stars are
the result of *two particles colliding: 2-body reactions*
that is: *reactions occur between pairs of particles*

a general example is $a + b \to c + d + \cdots$, with a

$$\text{reaction rate} \propto \text{number of pairs} \propto n_a n_b$$

and the number densities and mass densities are related

$$\text{number density } n_a \propto (\text{abundance of } a) \times \text{mass density } \rho \quad (1)$$

Q: so how does the rate of reactions depend on density?

rate of reactions: focusing on density, we have

$$\text{reaction rate} \propto \text{number of pairs} \propto \rho^2 \tag{2}$$

energy generation: each $ab \to cd \cdots$ reaction releases a fixed amount of nuclear energy, so

$$\mathcal{L} = \frac{dE}{dt\ dV} = \text{energy generation rate per volume} \propto \rho^2 \tag{3}$$

lesson: *2-body reaction* gives $\mathcal{L} \propto \rho^2$ density dependence

Poll: Three-Body Reaction Rates

Consider a three-body reaction rate $a + b + c \to \cdots$

How does the reaction rate per volume depend on density ρ?

A $\mathcal{L}_{abc} \propto \rho$

B $\mathcal{L}_{abc} \propto \rho^2$

C $\mathcal{L}_{abc} \propto \rho^3$

D $\mathcal{L}_{abc} \propto \rho^6$

Q: what about a 1-body reaction, e.g., a decay $a \to cd \cdots$*?*

Energy Generation: Velocity Dependence

we see: number of particles in initial state
sets exponent of density dependence

reaction rates also depend on *cross section and velocity*
through the average value of $\langle \sigma \, v \rangle$

Q: what condition(s) in a star determine particle speeds?

Energy Generation Rates

reaction cross sections and speeds depend on *temperature* T
recall: typical thermal speed $v_T = \sqrt{3kT/m_{\rm g}}$

so **nuclear energy generation rate per volume** has

$$\mathcal{L} \ \propto \ \rho^2 \ \langle \sigma \, v \rangle \tag{4}$$

- in nuclear reactions $\langle \sigma v \rangle$ *grows strongly with* T
- can approximate as $\mathcal{L} \propto \rho^2 \ T^s$
  with value of s specific to each nuclear reaction
  but generally $s \geq 4$, that is, $\mathcal{L} \propto \rho^2 T^4$ at least

energy generation rate is not uniform inside star
Q: where the highest? lowest?

we have seen: nuclear reactions determine

energy production rate per volume $\mathcal{L} = dE/dt\,dV$

Q: so what is physical significance of this?

$$l(r) = \int_0^r \mathcal{L}\ dV = 4\pi \int_0^r r^2\ \mathcal{L}(r)\ dr \tag{5}$$

Q: and of this?

$$l(R) = 4\pi \int_0^R r^2\ \mathcal{L}(r)\ dr \tag{6}$$

Summing Up: Power Generation

energy production rate per volume $\mathcal{L} = dE/dt\,dV$
is *power per volume* produced by nuclear reactions

so a volume integral out to radius r

$$l(r) = \int_0^r \mathcal{L}\ dV = 4\pi \int_0^r r^2\ \mathcal{L}(r)\ dr \qquad (7)$$

is the total power generated within radius r
i.e., the *enclosed power* generated!

and $l(R)$ is *total nuclear power made in the star!*

but if the star is in hydrostatic equilibrium
it has fixed potential and internal energy *Q: and so?*

Nuclear Power and Stellar Luminosity

in *hydrostatic equilibrium*, potential and internal energy constant
and so *total energy in star constant*

but then energy conservation demands:
all nuclear energy created inside the star
must be lost from its surface at exactly the same rate!

so *the total nuclear power generated* must be precisely
the total power radiated away
which is exactly the star's *luminosity*: $l(R) = L$

this also means that $l(r)$ is the **enclosed luminosity**

note also that $\mathcal{L}$ is luminosity per unit volume!
also known as luminosity density

note the close similarity with *mass density and mass*

$$\rho(r) = \frac{dM}{dV} = \text{mass per volume} \tag{8}$$

and so the *enclosed mass* within radius r is

$$m(r) = \int_0^r \rho \; dV = 4\pi \int_0^r r^2 \rho(r) \; dr \tag{9}$$

in fact we can write *enclosed luminosity* as

$$l(r) = \int \mathcal{L} \; dV = \int \frac{\mathcal{L}}{\rho} \; \rho \; dV = \int_0^{m(r)} q \; dm \tag{10}$$

where $q = \mathcal{L}/\rho \propto \rho T^s$ is the *power per unit mass*
and where total luminosity $L = \int_0^M q \; dm$

our enclosed luminosity satisfies

$$\frac{dl}{dr} = 4\pi \ r^2 \ \mathcal{L}(r) = 4\pi \ r^2 \ q(r) \ \rho(r) \tag{11}$$

this is the *second equation of stellar structure!*

- *relates nuclear energy generation to luminosity*
- expresses energy conservation

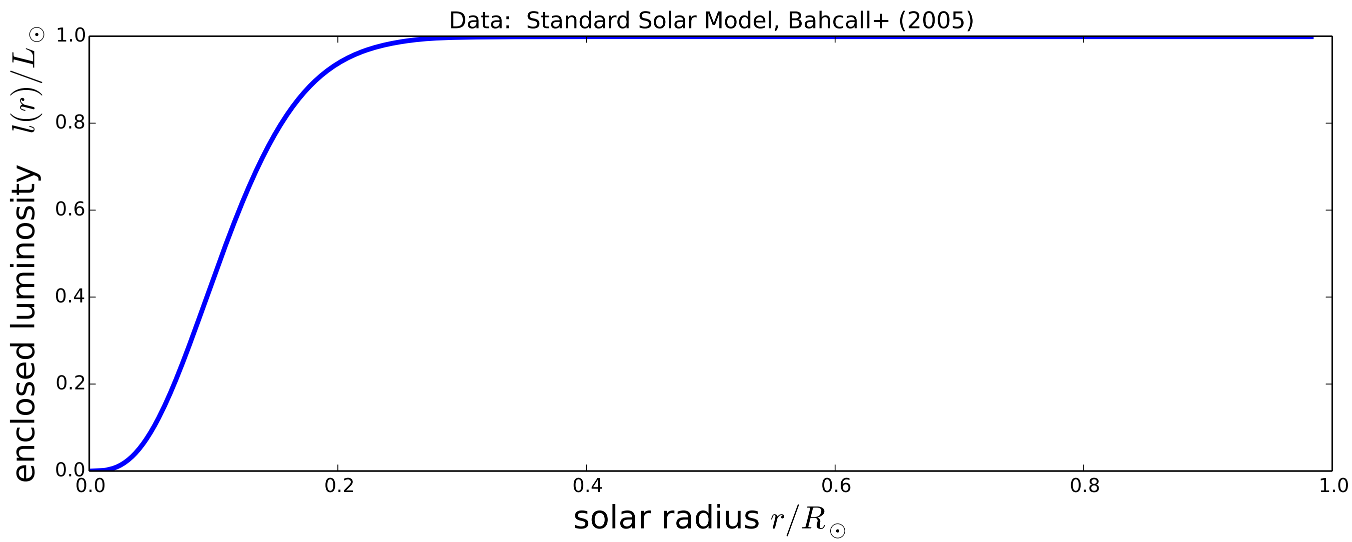
note that if we know the "local luminosity" $l(r)$
we can also find the "local energy flux"

$$F(r) = \frac{l(r)}{4\pi r^2} \tag{12}$$

which correctly gives $F(R) = L/4\pi R^2$ as the surface flux

Energy Generation: The Sun

models of the Sun show that
enclosed luminosity $l(r)$ *grows rapidly with radius*
and with enclosed mass



in the *innermost 20% of the Sun's radius*, $r = 0.2\,R_\odot$:

- enclosed mass $m(r) = 0.34\,M_\odot$
- enclosed luminosity $l(r) = 0.94\,L_\odot$

Q: lessons?

Solar Energy Comes from the Inner Core

in the *innermost 20% of the Sun's radius*, $r = 0.2\, R_{\odot}$:

- enclosed mass $m(r) = 0.34\, M_{\odot}$
- enclosed luminosity $l(r) = 0.94\, L_{\odot}$

mass is concentrated at center (density highest)

energy generation is entirely from innermost region
this define the **inner core**

in this core region temperature is close to our Virial estimate:
$T(0.2R_{\odot}) = 9 \times 10^6$ K

the outer bulk of the Sun is cooler and generates little power
but acts to compress the core enough to sustain nuclear reactions

Stellar Structure: The Story Thus Far

thus far, great progress!

density determines enclosed mass (*mass conservation*)

$$\frac{dm}{dr} = 4\pi r^2 \rho(r) \quad (13)$$

$$m(r) = 4\pi \int_0^r r^2 \rho(r)\, dr \quad (14)$$

density and and enclosed mass determine pressure

due to hydrostatic equilibrium (force balance)

$$\frac{dP}{dr} = -\frac{Gm(r)\ \rho(r)}{r^2} \quad (15)$$

$$P(r) = P_c - \int_0^r \frac{Gm\ dm}{dr} \quad (16)$$

density and temperature determine nuclear reaction rates
and thus determine *luminosity* (energy conservation)

$$\frac{dl}{dr} = 4\pi r^2 \mathcal{L} = 4\pi r^2 \, q \, \rho \tag{17}$$

$$l(r) = 4\pi \int_0^r r^2 \, \mathcal{L} = \int_0^{m(r)} q \, dm \tag{18}$$

one thing remains: temperature
comes from considering how *energy is transported*
from the core to the surface of the star
that is, how heat flows outward

Radiative Energy Transport

temperature set by heat flow, i.e., energy transport

for now: assume all energy flow due to photons
"**radiative energy transport**"

star interior is *opaque:* photons scatter frequently
with **mean free path** $\ell_{\rm mfp}$ very short

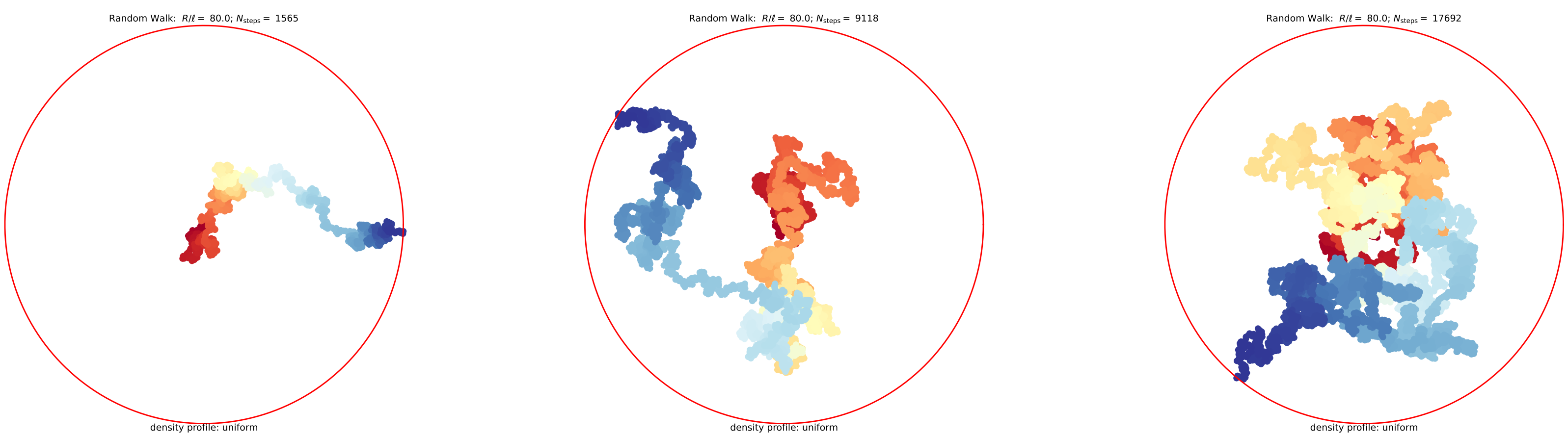
key idea: each scattering event randomizes photon direction
photon "forgets" previous history: "**random walk**"

Q: how can photons ever escape if each step randomized?

Photon Escape from Sun: Random Walk

solar photon scattering: motion is **random walk**:
each step in random direction

Simulations: illustrate photon escape, but not to scale!
color shows steps: red at photon birth, blue at escape
three examples shown below `www: run code for more`
Q: what strikes you? How does a photon escape?
How many steps needed?



Random Walk Warmup

Random walk: each step in random direction

- "progress" not organized, can go both inwards and outwards
- but after many steps, "stumble upon" the surface

PS6: shows how random walking photons escape

warmup: imagine *photon born at center* $\vec{r}_0 = 0$
first step has displacement $\vec{r}_1$
where $r_1^2 = |\vec{r}_1|^2 = \vec{r}_1 \cdot \vec{r}_1 = \ell_{\rm mfp}^2$

but if we *average over very many newborn photons*
going randomly with all directions chosen equally
then: $\langle \vec{r}_1 \rangle = 0$

Q: why?

Random Walk in 1D: Coin Flips

simplified random walk: 1-dimensional case
photon only moves on x-axis

then random walk is like *flipping coins:*
on average, each step has equal chance
of "heads" $+\ell_{\rm mfp}$ and "tails" $-\ell_{\rm mfp}$
so if *flip many coins for one step, averages to zero*

but if *flip one coin many times*, usually develop
random excess of heads over tails, or vice versa
which means *net progress away from origin!*

when net displacement gets to edge of star, escape!

Photon Mean Free Paths

photon mean free path $\ell_{\rm mfp} = 1/n_{\rm sc}\sigma_{\rm sc}$
where $n_{\rm sc}$ is the number density of scatters
and $\sigma_{\rm sc}$ is photon scattering cross section

recall that number and mass densities related by
$\rho_{\rm sc} = m_{\rm sc} n_{\rm sc}$, with scatterer mass $m_{\rm sc}$
so useful to define **opacity**

$$\kappa = \frac{\sigma_{\rm sc}}{m_{\rm sc}} \tag{19}$$

measures cross section per unit scatter mass, and

$$\ell_{\rm mfp} = \frac{1}{n_{\rm sc}\sigma_{\rm sc}} = \frac{1}{\kappa \rho_{\rm sc}} \tag{20}$$

Poll: Mean Free Paths in Stars

consider photons in the real Sun

How does photon mean free path change in Sun?

A ℓ_{mfp} longest in Sun's center, shortest at surface

B ℓ_{mfp} shortest in Sun's center, longest at surface

C ℓ_{mfp} is uniform in the Sun photon mean free path is

$$\ell_{\rm mfp} = \frac{1}{n_{\rm sc}\sigma_{\rm sc}} = \frac{1}{\kappa \rho_{\rm sc}} \tag{21}$$

in Sun:

- $\rho(r)$ decreases from center to surface
- and in addition sometimes κ also deceases towards surface

so: mean free path goes from short to long
solar "fog" thins as we go out

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our *Sun is a gas*, density smoothly drops with radius
it does not really have a surface!
yet it does show a *sharp edge* in images
Q: how's that? what is the surface really?

The Solar Photosphere

the surface of the Sun appears sharp
despite random scattering of photons and smooth density profile

photons we see are **not scattered** between Sun and us
and so originate from *final scattering* events in Sun

apparent edge of Sun is **surface of last scattering**
also know as the solar **photosphere**

sharpness of photosphere must mean:
- density drops very rapidly near apparent surface
- mean free path changes rapidly from short to long
  until $\ell_{\rm mfp} > R_\odot$: *escape!*