Astro 404

Lecture 36

Nov. 19, 2021

Announcements:

- **PS11 due today**
- **PS12–last one!–due Friday Dec 3**

  Note: lowest HW score is dropped

  but all HW are fair game on exam

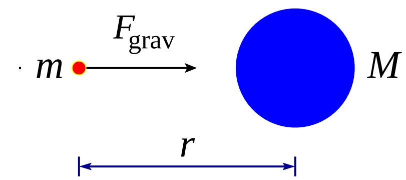
Last time: path to relativistic gravity

Q: why does Einstein object to Newtonian gravity?

Q: equivalence principle–what's equivalent? what's the principle?

Newton gravity force law

$$F_{\text{grav}} = \frac{GMm}{r^2}$$



implies that *if M moves* and thus *r changes:*

$\rightarrow$ *gravity force changes instantaneously* over all space!

Einstein sez: *this is totally illegal! an unmitigated disaster!*

no signal–including gravity–can move faster than c!

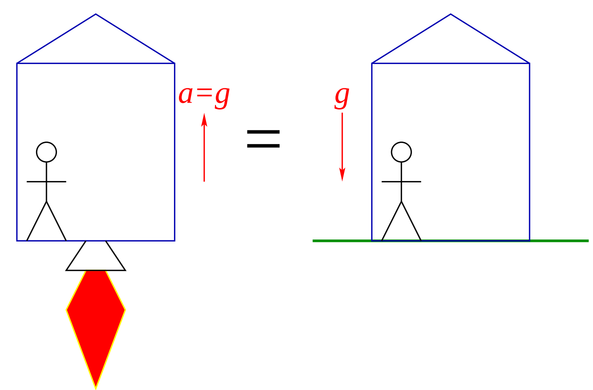
Einstein's Equivalence Principle:

in a closed room

no experiment can distinguish

gravity-free acceleration vs

gravity and no acceleration



Q: what does the equivalence principle imply when comparing an observer on planet with g, and accelerating rocket $a = g$?

Gravity Bends Light

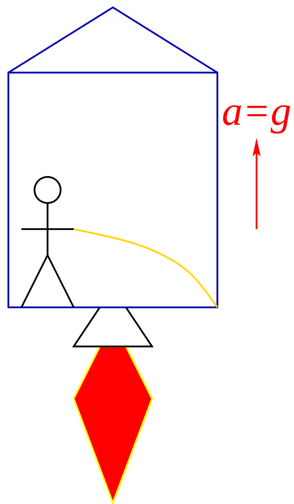
Rocket Experiment: `www: illuminating animation`

in accelerating rocket, shoot a horizontal beam

⋆ entire light path bent (in fact, a parabola!)

shooting a vertical beam

⋆ upstairs sees redshift, downstairs sees blueshift



Poll: Rocket Lasers

Install lasers and detectors in rocket basement and attic
measure λ_{obs} during acceleration

Resulting effect on photons?

A no effect: λ unchanged if emitter and detector both accelerate with rocket

B attic detectors see *blueshift*
basement detectors see *redshift*

C attic detectors see *redshift*
basement detectors see *blueshift*

D both detectors see *redshift*

E both detectors see *blueshift*

Gravitational Redshifting

Rocket experiment: as photon travels,
acceleration changes detector v relative to emitter

- light travels height Δz in time $\Delta t \approx \Delta z/c$
- detector velocity change $\Delta v = a\,\Delta t \approx a\Delta z/c$
- redshift $\Delta\lambda/\lambda = \Delta v/c = a\Delta z/c^2$

for **upgoing** photon: $\Delta z > 0$ and $\Delta v > 0$
during travel time, *detector gains speed away*
$\Delta\lambda > 0$: photon observed as *redshifted*

for **downgoing** photon: $\Delta v < 0$
during travel time, *detector gains speed toward*
$\Delta\lambda < 0$: photon observed as *blueshifted*

Q: implications for gravity via equivalence principle?

Gravitational Redshifting

In accelerating rocket:
upgoing photon seen as *redshifted*
downgoing photon seen as *blueshifted*

But by equivalence principle:
must find *same result due to gravity*, so:
⋆ gravity bends light rays
gravitational lensing
⋆ observers in basement see blueshift of attic photons!
and observers in attic see redshift of basement photons!
gravitational redshift/blueshift

Note: gravitational red/blueshift confirmed in lab!
Q: how would you do the experiment? what are you looking for?
`www: Pound-Rebka expt`

Rocket Experiment: Redshifting

rocket redshifting: $\Delta\lambda/\lambda \approx a\,\Delta z/c^2$
equivalence principle: gravitational redshift

$$\frac{\Delta\lambda}{\lambda} \approx \frac{g\,\Delta z}{c^2} = \frac{\Delta\phi}{c^2} \tag{1}$$

with $\Delta\phi$ the change in gravitational potential

And there's more:
redshift = decrease in light frequency f: $\Delta f/f = -\Delta\lambda/\lambda$

but $f = 1/P$, light wave oscillation period

so redshift $\rightarrow$ P increases
but *light oscillations are like clock ticking*
Q: and so?

Gravitational Time Dilation

⋆ clocks in basement appear to run slow
when viewed from attic!
and attic clocks appear fast when viewed from basement!

viewed from attic, basement clocks appear slower by

$$\Delta t = t_{\rm basement} - t_{\rm attic} = \frac{gh}{c^2} = \frac{\phi}{c^2} \qquad (2)$$

where $\phi = gh$ is the change in gravitational potential

⋆ time "warping" due to gravity:
"**gravitational time dilation**"
⋆ gravity influences "flow" of time!
∞ deeper potential → slower apparent "time flow"
Q: so which clock is really right?

Twofer Poll: Comparing Clocks With Gravity

experiment: Adele and Beyoncé start at same place

- Adele remains still
- Beyoncé goes deeper in potential, hangs out, then returns

They meet again at starting point and compare clocks:

How do the tick rates compare upon reunion?

- A Beyoncé's clock ticks faster
- B Beyoncé's clock ticks slower
- C tick rates are the same

How do the elapsed times compare upon reunion?

- A Beyoncé's clock shows more elapsed time
- B Beyoncé's clock shows less elapsed time
- C both clocks show the same elapsed time

Gravity and Time

when they meet again:

- *both* clocks tick at same rate
- but total elapsed time is larger for Adele!

Who's right–attic or basement observers?

⋆ *both* are reporting accurately

⋆ *both* see their own clocks tick normally

Q: how to test these effects in real world?

General Relativity

Einstein's gravity: General Relativity
relativity generalized to include fast motion *and* gravity

Newton: matter causes force (gravity)
→ particles follow curved lines in
"flat" (Euclidean geometry) space

Einstein: bold leap, rejected Newton–twice

- special relativity: space & time linked as *spacetime*
- general relativity: matter causes spacetime to be "curved"
  → particles follow straight lines ("geodesics")
  in curved space

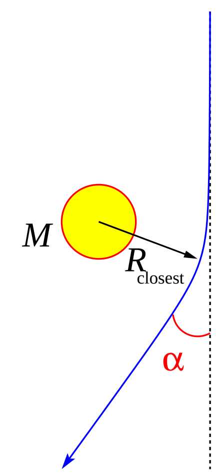
in general relativity: space and time dynamic
respond to gravitating sources (matter and energy): not fixed
once and for all

Light Bending: The Sun

In principle: *all* gravitating objects bend light
including you, me, the earth...
In practice: need strong gravity source to create
effect large enough to observe

Einstein (1915) devised first test: *the Sun*

- Sun's gravity deflects starlight rays
- the stronger the gravity along the path
  the bigger the deflection
  bending angle $\alpha = 4GM_\odot/R_{\rm closest}c^2$

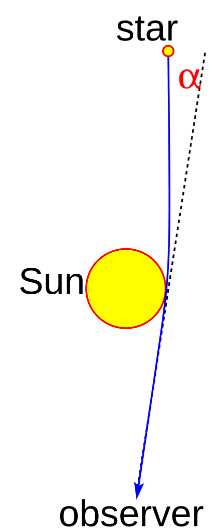


biggest effect for starlight "grazing" Sun edge: $R_{\rm closest} = R_\odot$
Q: why is this technically challenging to see?
Q: how to get around the problem?

1919 Eclipse: Give it up for Big Al!

Problem: Sun's glare obscures surrounding starlight
Solution: block glare with eclipse!

1919: total solar eclipse in Southern hemisphere
expedition led by Sir Arthur Eddington
`www: expedition results paper to Royal Society`
★ starlight bent! Woo hoo!
★ relativistic gravity confirmed!
★ Einstein an instant celebrity
`www: NY Times announcement`



Now tested many times, and very accurately
seen in clusters of Galaxies
`www: HST gravitational lens Abell 2218`

- all starlight bending experiments confirm Einstein!

Black Holes

Laplace (1790's)
recall: escape velocity $v_{\text{esc}} = \sqrt{2GM/R}$
What if star has M, R with $2GM/c^2R > 1$?
then $v_{\text{esc}} > c$!
light cannot escape! $\rightarrow$ black hole

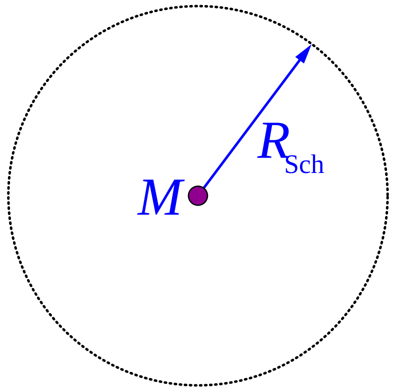
Wrong argument (Newtonian gravitation)
...but right answer!

in death of $M > 30M_\odot$*: gravity wins
collapse unstoppable
black hole formed
$\Rightarrow$ inevitable part of star formation

* "threshold mass" for SN $\rightarrow$ BH uncertain!

Black Hole Properties

any object of *any* mass M can (in principle)
become a black hole!
here: non-spinning case



size: Schwarzschild radius

$$R_{\mathrm{Sch}} = \frac{2GM}{c^2}$$

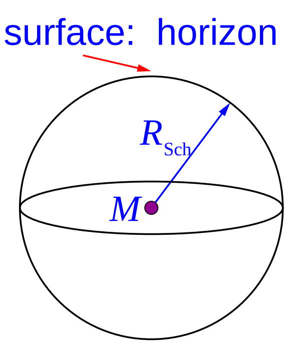
radius also provides BH "recipe":

- *crush* object M *smaller than* R_{Sch} $\rightarrow$ *get BH!*
- example: for mass of Sun $R_{\mathrm{Sch}} = 2GM_\odot/c^2 = 3.0$ km
  but actual $R_\odot = 7 \times 10^6$ km
  $\rightarrow$ the Sun is not a black hole! (whew!)
- for mass of Earth: $R_{\mathrm{Sch}} \approx 1$ cm!

The Black Hole Horizon

Why call R_{Sch} the BH radius? nothing is there!

True, but: R_{Sch} marks "point of no return"
horizon: surface enclosing the BH
i.e., horizon is surface of sphere w/ radius R_{Sch}



horizon is one-way "membrane"
once inside $r \le R_{\text{Sch}}$ nothing can escape...even light!
cosmic roach motel!

Hence:
no light escapes $\rightarrow$ black
but nothing else moves as fast $\rightarrow$ nothing else escapes $\rightarrow$ hole

Life Near a Black Hole

Experiment: lower astronaut (Jodie) near $R_{\rm Sch}$
we are at mission control, far away ($r_{\rm us} \gg R_{\rm Sch}$)
communicate w/ light signals

when viewing photons (or clock ticks)
emitted at $r_{\rm em}$, observed at $r_{\rm obs}$
general rule, handy for PS12:

$$\frac{\Delta t_{\rm obs}}{\Delta t_{\rm em}} = \frac{\lambda_{\rm obs}}{\lambda_{\rm em}} = \sqrt{\frac{1 - R_{\rm Sch}/r_{\rm obs}}{1 - R_{\rm Sch}/r_{\rm em}}} \tag{3}$$

What do we see?
obs=us: $r_{\rm obs} \rightarrow \infty$; em=Jodie: $r_{\rm em} > R_{\rm Sch}$
- Jodie's watch: $\Delta t_{\rm obs}/\Delta t_{\rm em} = 1/\sqrt{1 - R_{\rm Sch}/r_{\rm em}} > 1$
  $\rightarrow \Delta t_{\rm obs} > \Delta t_{\rm em}$! appears to tick slow! time dilation!
- wavelengths: $\lambda_{\rm obs} > \lambda_{\rm em}$! redshift!

Q: and Jodie?

$$\frac{\Delta t_{\text{obs}}}{\Delta t_{\text{em}}} = \frac{\lambda_{\text{obs}}}{\lambda_{\text{em}}} = \sqrt{\frac{1 - R_{\text{Sch}}/r_{\text{obs}}}{1 - R_{\text{Sch}}/r_{\text{em}}}} \tag{4}$$

What does Jodie see?

intuitively: expect inequalities to reverse...and they do

obs=Jodie: $r_{\text{obs}} > R_{\text{Sch}}$; em=us: $r_{\text{em}} \to \infty$:

- *our watches*: $\Delta t_{\text{obs}}/\Delta t_{\text{em}} = \sqrt{1 - R_{\text{Sch}}/r_{\text{em}}} < 1$
  $\to \Delta t_{\text{obs}} < \Delta t_{\text{em}}$! *appear to tick fast!*
- wavelengths: $\lambda_{\text{obs}} < \lambda_{\text{em}}$! *blueshift!*

When Jodie returns:

then $r_{\text{em}} = r_{\text{obs}}$

- $\Delta t_{\text{obs}} = \Delta t_{\text{em}}$: her watch ticks at same rate as ours!
- but the *elapsed time* is shorter on her watch
  and so she is younger than her twin!

Poll: Black Holes

From a safe distance, you drop an object (nuclear waste? Voldemort?) on an isolated black hole.

Will you see it fall in?

A yes, no matter your distance from the hole

B maybe, depends on how far you are from the hole

C no, because it never actually falls in

D no, although it does actually fall in

Falling Into a Black Hole

No barrier, bells, or whistles at horizon
infalling objects go right through

seen from afar, time dilation and redshift progressively severe as object approaches horizon

progressively strong relativistic flux reduction

so as seen from afar:

- time elapse slows until appears "frozen"
- signal redshifts
- image fades until last photon emitted before horizon crossing and then object gone—and black hole mass higher

Poll: Thanksgiving Travel

How far from Urbana will you be next week?

A < 10 miles

B $10 - 100$ miles

C $100 - 1000$ miles

D $1000 - 10,000$ miles

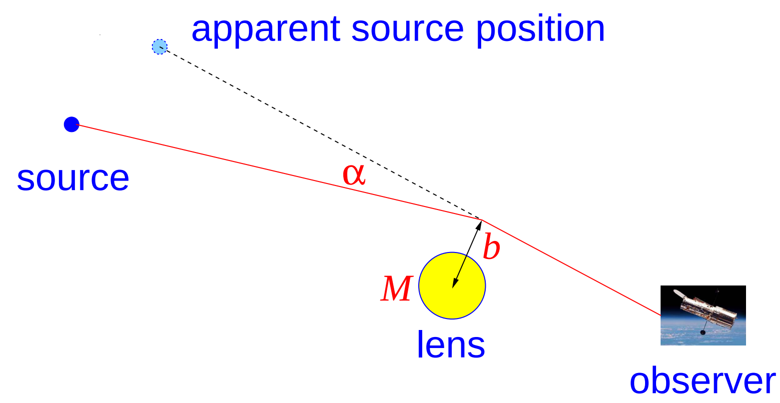
Have a great break!

Director's Cut Extras

Light Bending Quantified: Point Mass

the setup:

- light ray incident on
- a point mass M
- with distance of closest approach b



Einstein result: light deflected by angle

$$\alpha = \frac{4GM}{c^2 b} \tag{5}$$

Q: how could this be useful as a tool?

Supermassive Black Holes

MH has supermassive BH: quiet

QSO have supermassive BH: active

recent result:

all galaxies have supermassive BH! ...but most quiet

$\rightarrow$ maybe active galaxies are phase in evolution?

BH mass **correlated** with host gal stellar (spheroid) mass

$\rightarrow$ $M_{\rm BH}/M_{\rm sph} \sim const \sim 0.006$

constant "BH fraction"

$\rightarrow$ supermassive BH formation is part of gal formation!

Open Questions:

- how does a $10^{7-8} M_\odot$ BH ($R_{\text{Sch}} \sim$ AU) know about the $10^{11-12} M_\odot$ galaxy it lives in (and vice versa)?
- how does a SMBH "grow" – what are the "seeds," and how are they "fed"?
- Are there any galaxies without SMBH? Are there any SMBH without galaxies? Either way, what does this mean?