Astronomy 501: Radiative Processes

Lecture 16

Sept 28, 2022

Announcements:

- **Problem Set 5 due Friday**
- Office hours today after class

Last time:

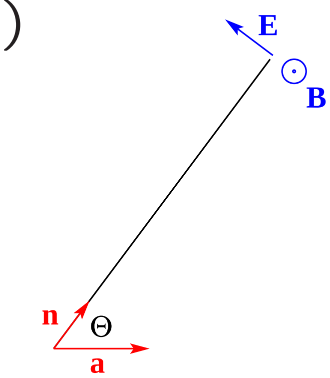
- intensity, polarization and the radiation $\vec{E}_{\rm rad}$ field

  Q: what determines intensity? polarization?

Non-Relativistic Acceleration: Polarization

field magnitude determines intensity
field direction determines polarization

$$\vec{E}_{\rm rad} \propto \hat{n} \times (\hat{n} \times \vec{a}) = -\vec{a}_\perp \quad (1)$$



where $\vec{a}_\perp = \vec{a} - (\hat{n} \cdot \vec{a})\hat{n}$

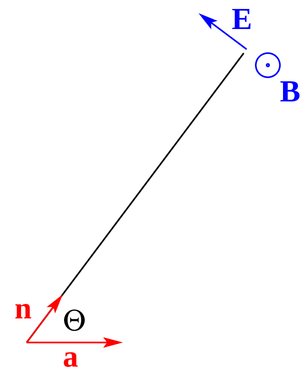
Lesson: $\vec{a}_\perp$ and hence *polarization direction* is

- along component of acceleration $\perp$ sightline
- the projection of $\vec{a}$ onto the observer's sky

also last time: dipole approximation

Q: when valid? Radiation from dipole?

Larmor formula: non-relativistic motion



power per unit solid angle is

$$\frac{dP}{d\Omega} = \frac{q^2 a^2}{4\pi c^3} \sin^2 \Theta$$

with angle Θ *between* $\vec{a}$ *and* $\hat{n}$: a $\sin^2\Theta$ pattern!
total radiated power is

$$P = \frac{2}{3}\,\frac{q^2}{c^3} a^2$$

for a **non-relativistic dipole**, we have

$$\frac{dP}{d\Omega} = \frac{\ddot{d}^{\,2}}{4\pi c^3} \sin^2 \Theta \qquad (2)$$

$$P = \; = \frac{2}{3}\frac{\ddot{d}^{\,2}}{c^3} \qquad (3)$$

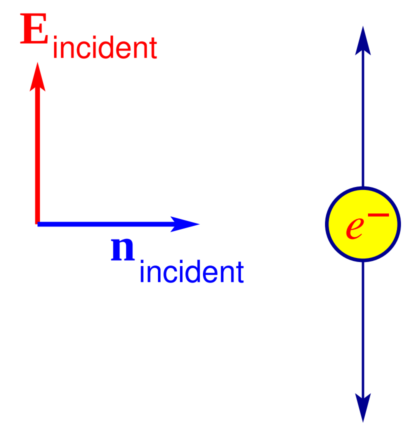
radiated energy per solid angle and frequency is

ω

$$\frac{dW}{d\Omega d\omega} = \frac{1}{c^3}\omega^4 \;\left|\tilde{d}(\omega)\right|^2 \sin^2 \Theta \qquad (4)$$

Thomson Scattering

Consider *monochromatic* radiation
linearly polarized in direction $\hat{\epsilon}_{\rm init}$
incident on a free, non-relativistic **electron**
magnetic/electric force ratio
$F_B/F_E \sim (v/c)B/E = v/c \ll 1$: ignore F_B



thus the force on the electron is

$$\vec{F} \approx -eE_0 \, \hat{\epsilon}_{\rm init} \, \cos\omega_0 t \tag{5}$$

and thus the electron has

$$\ddot{\vec{r}} = -\frac{e}{m_e} E_0 \, \hat{\epsilon}_{\rm init} \, \cos\omega_0 t \tag{6}$$

Q: dipole moment $\vec{d}$? scattered radiation spectrum?

and so the dipole moment $\vec{d} = -e\vec{r}$ has

$$\ddot{\vec{d}} = \frac{e^2}{m_e} E_0 \hat{\epsilon}_{\rm init} \ \cos \omega_0 t \tag{7}$$

and thus the time-averaged power radiated by each electron is

$$\left\langle \frac{dP}{d\Omega} \right\rangle = \frac{e^4 E_0^2}{8\pi m_e^2 c^3} \sin^2 \Theta \tag{8}$$

$$\langle P \rangle = \frac{e^4 E_0^2}{3 m_e^2 c^3} \tag{9}$$

were Θ is angle between $\hat{n}$ and $\hat{a} = \hat{\epsilon}_{\rm init}$
in terms of time-averaged incident flux is $\langle S \rangle = c E_0^2 / 8\pi$

$$\left\langle \frac{dP}{d\Omega} \right\rangle = \frac{e^4 E_0^2}{m_e^2 c^4} \sin^2 \Theta \ \langle S \rangle \tag{10}$$

$$\langle P \rangle = \frac{8\pi}{3} \frac{e^4}{m_e^2 c^4} \ \langle S \rangle \tag{11}$$

Q: and so?

Thomson Cross Section

time-averaged power

$$\left\langle \frac{dP}{d\Omega} \right\rangle = \frac{e^4 E_0^2}{8\pi m_e^2 c^3} \sin^2 \Theta = \frac{e^4}{m_e^2 c^4} \sin^2 \Theta \ \ \langle S \rangle \tag{12}$$

where time-averaged incident flux is

recall: **differential scattering cross section** can be defined as

$$\frac{d\sigma}{d\Omega} = \frac{\text{scattered power}}{\text{incident flux}} = \frac{dP/d\Omega}{\langle S \rangle} \tag{13}$$

$$= \frac{e^4}{m_e^2 c^4} \sin^2 \Theta \tag{14}$$

integral over solid angle gives total **Thomson cross section**

$$\sigma_{\mathrm{T}} \equiv \int \frac{d\sigma}{d\Omega} d\Omega = \frac{8\pi}{3} \frac{e^4}{m_e^2 c^4} = \frac{8\pi}{3} r_0^2 = 0.665 \times 10^{-24} \ \mathrm{cm}^2 \tag{15}$$

with the *classical electron radius* $r_0 \equiv e^2/m_e c^2$

Thomson Appreciation

We have found the cross section for scattering of **monochromatic, linearly polarized radiation** on **free electrons:**

$$\text{differential cross section} \quad \frac{d\sigma}{d\Omega} = \frac{e^4}{m_e^2 c^4} \sin^2 \Theta \tag{16}$$

$$\text{total cross section} \quad \sigma = \sigma_{\mathrm{T}} = \frac{8\pi}{3} \frac{e^4}{m_e^2 c^4} \tag{17}$$

Q: notable features?

Q: dependence (or lack thereof) on incident radiation?

plasmas will generally have ions as well as free electrons
Q: which is more important for Thomson scattering?

Q: under what conditions might our assumptions break down?

The Charms of Thomson

Thomson scattering is

- *independent of radiation frequency*
  implicitly assumes electron recoil negligible

$\to$ initial spectral *shape vs* ν is *unchanged*!

- example: Solar corona highly ionized, Thomson dominates
  Q: implications: spectrum/color? angular distribution?
  Q: how observe? `www: corona`

- $\sigma \propto 1/m^2$: *electron scattering larger than ions*
  by factor $(m_{\text{ion}}/m_e)^2 \gg 10^6$!

- if electron recoil large, and/or electron relativistic
  assumptions break down, will have to revisit

∞

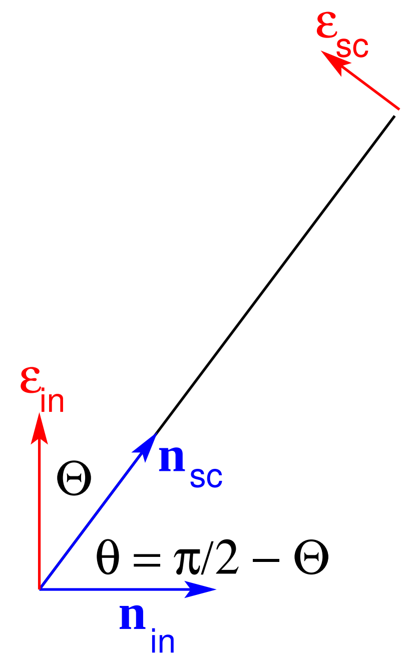
if we measure polarization state $\hat{\epsilon}$,
Q: what is angular pattern of scattered radiation?

sky projection of electron acceleration:

- linear oscillation
- for each initial polarization state scattered radiation 100% linearly polarized

when measuring polarization state $\hat{\epsilon}_{\rm sc}$, find

$$\frac{d\sigma}{d\Omega} = \frac{e^4}{m_e^2 c^4} |\hat{\epsilon}^*_{\rm sc} \cdot \hat{\epsilon}_{\rm in}|^2 \qquad (18)$$



Q: what if incident radiation is superposition of two polarzation states?

Thomson Scattering: Electron Dipole Radiation

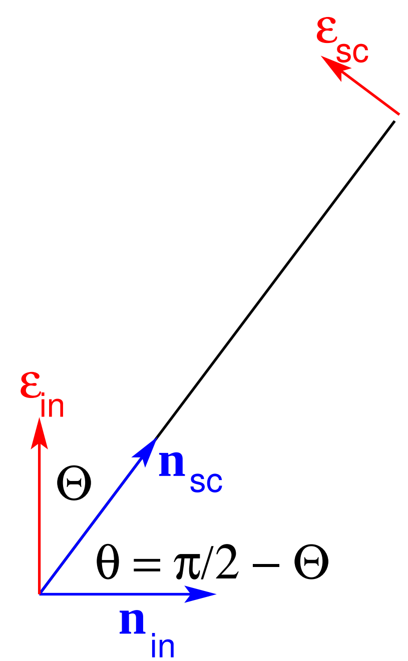
- Thomson = **scattering by non-relativistic free electrons**
- no change in photon λ, ν: **coherent scattering**

- electron acts as **dipole antenna**

$$\frac{dP}{d\Omega} = \frac{d\sigma}{d\Omega} \, \langle S_{\rm in} \rangle$$

i.e., scattered power $\propto$ incident flux

proportionality is **Thomson cross section**



$$\frac{d\sigma}{d\Omega} = \frac{e^4}{m_e^2 c^4} \sin^2\Theta = \frac{3}{2\pi}\sigma_T \cos^2\theta$$

$$\sigma_{\rm T} = \frac{8\pi}{3}\frac{e^4}{m_e^2 c^4} = \frac{8\pi}{3} r_0^2$$

maximum at $\hat{\epsilon}_{\rm in} \cdot \hat{n}_{\rm sc} = \cos\Theta = 0$

which is also $\hat{n}_{\rm in} \cdot \hat{n}_{\rm sc} = \cos\theta = 1$, with $\theta = \pi/2 - \Theta$

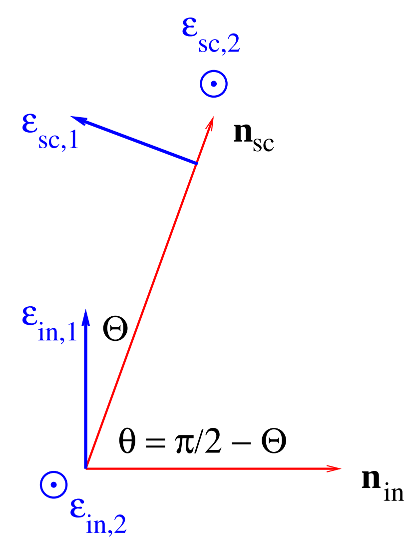
$\rightarrow$ *forward and backward scattering*

Thomson Scattering of Unpolarized Radiation

Using result for linear polarization
we can construct result for unpolarized radiation
by *averaging results for two orthogonal linear polarizations*

Geometry:

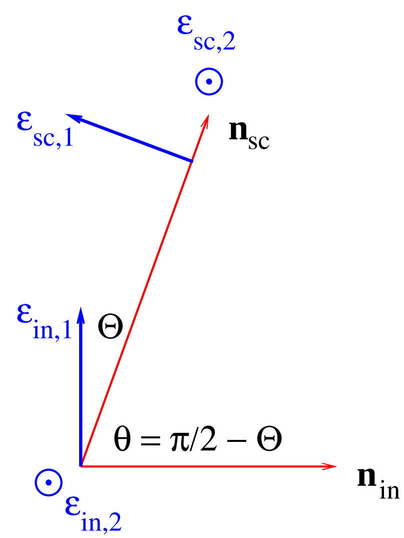
- $\hat{n}_{\rm in}$ direction of incident radiation
- $\hat{n}_{\rm sc}$ direction of scattered radiation
- initial polarizations are both $\perp \hat{n}_{\rm in}$
- choose one polarization $\hat{\epsilon}_{\rm in,1}$ in $\hat{n}_{\rm in} - \hat{n}_{\rm sc}$ plane
- and the other $\hat{\epsilon}_{\rm in,2}$ orthogonal
  to this plane and to $\hat{n}_{\rm in}$



scattering angle of pol 1 has $\cos\Theta_1 = \hat{\epsilon}_1 \cdot \hat{n}_{\rm sc}$ *Q: which means?*
scattering angle of pol 2 has $\cos\Theta_2 = \hat{\epsilon}_2 \cdot \hat{n}_{\rm sc}$ *Q: which means?*

thus scatter polarization 1 by angle $\Theta = \pi/2 - \theta$
and polarization 2 by angle $\pi/2$, and so

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{unpol}} = \frac{1}{2}\left(\frac{d\sigma}{d\Omega}\right)_1 + \frac{1}{2}\left(\frac{d\sigma}{d\Omega}\right)_2$$
$$= \frac{r_0^2}{2}\left(\sin^2\Theta + 1\right)$$
$$= \frac{r_0^2}{2}\left(1 + \cos^2\theta\right)$$



which only depends on angle θ
between incident $\hat{n}_{\text{in}}$ and scattered $\hat{n}_{\text{sc}}$ radiation directions

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{unpol}} = \frac{r_0^2}{2}\left(1 + \cos^2\theta\right) \tag{19}$$

- **forward-backward symmetry**: $\theta \to -\theta$ invariance

- angular pattern: $\cos^2\theta \propto \cos 2\theta$ term

$\to$ scattered radiation has 180^0 periodicity

$\to$ 4 extrema = "poles": **quadrupole** pattern!

- total cross section $\sigma_{\text{unpol}} = \sigma_{\text{pol}} = \sigma_{\text{T}}$

$\to$ electron at rest has no preferred direction

- Polarization degree of scattered radiation

$$\Pi = \frac{\sqrt{Q^2 + U^2 + V^2}}{I} = \frac{1 - \cos^2\theta}{1 + \cos^2\theta} \tag{20}$$

Q: what does this mean?

Thomson Scattering Creates Polarization

Thomson scattering of *initially unpolarized* radiation has

$$\Pi = \frac{1 - \cos^2\theta}{1 + \cos^2\theta} \tag{21}$$

i.e., degree of polarization $\Pi \neq 0$!

Thomson-scattered radiation is linearly polarized!

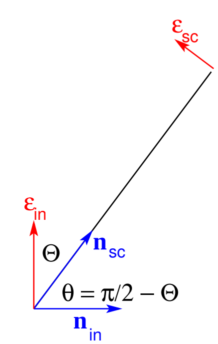
quadrupole pattern in angle θ between $\hat{n}_{\rm init}$ and $\hat{n}_{\rm scattered}$

- 100% polarized at $\theta = \pi/2$
- 0% polarized at $\theta = 0, \pi$

classical picture: e^- as dipole antenna

incident linearly polarized wave accelerates e^-

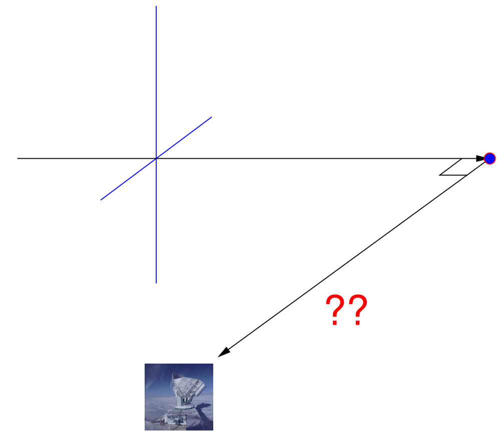
$\to \sin^2\Theta = \cos^2\theta$ pattern, peaks at $\theta = 0, \pi$



Thompson Scattering: A Gut Feeling

Discussion swiped from Wayne Hu's website

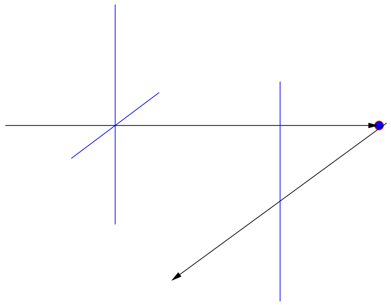
Consider a beam of unpolarized radiation
propagating in plane of sky, incident on an electron
think of as superposition of linear polarizations
one along sightline, one in sky



Q: why is scattered radiation polarized? in which direction?

Q: now what if unpolarized beams from opposite directions?

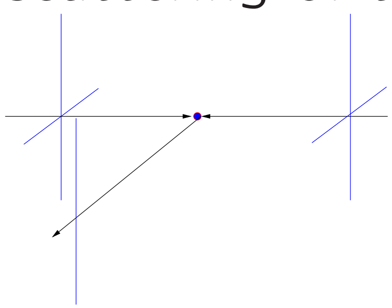
scattering of one unpolarized beam:



$\rightarrow$ see radiation from e motion in sky plane

$\rightarrow$ linear polarization!

scattering of two unpolarized beams in opposite directions:

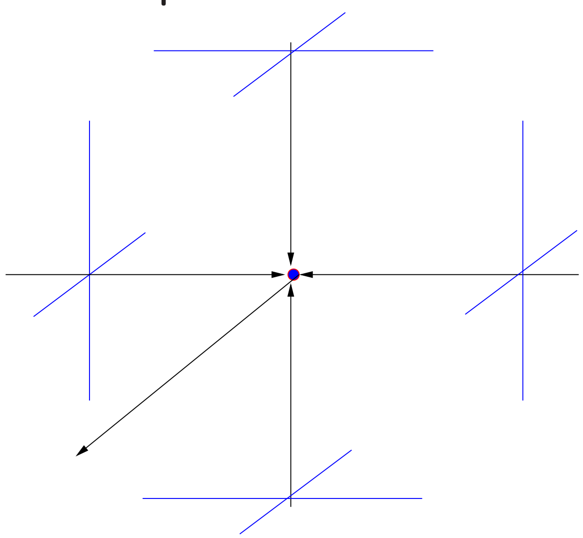


$\rightarrow$ the other side only adds to e motion in sky plane

$\rightarrow$ also linear polarization!

Q: what if isotropic initial radiation field?

isotropic initial radiation field:



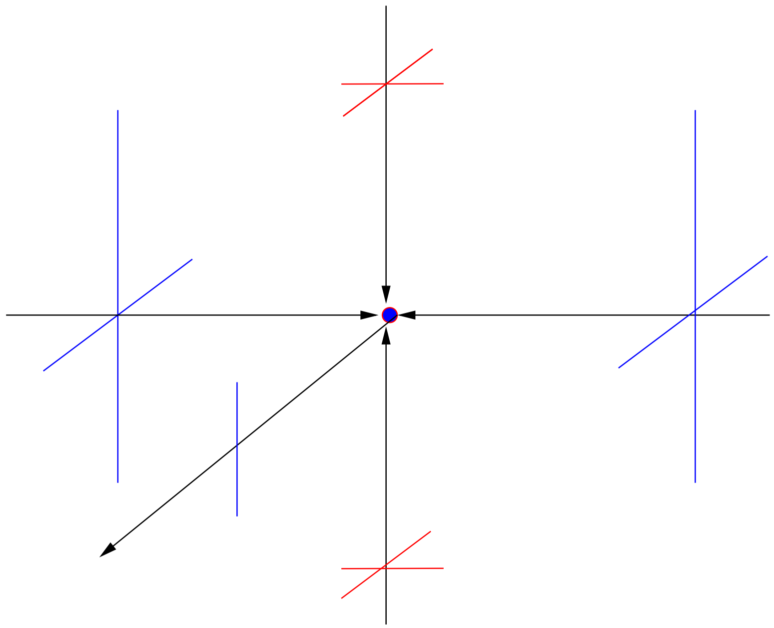
e motions in x and y sky directions cancel

$\rightarrow$ no net polarization

Q: what incident radiation fields do create polarization?

Q: lesson?

if initial radiation field has quadrupole intensity pattern



linear polarization!

lesson: polarization arises from Thomson scattering
when electrons “see” quadrupole anisotropies in radiation field

Q: If Thomson scattering is the only process acting
what is the appropriate transfer equation?

Thomson Scattering in Radiation Transfer

recall: in *coherent scattering*

- photon number and energy preserved
- but directions changed

$$\frac{dI_\nu(\hat{n})}{ds} = -n_e \sigma_{\mathrm{T}} \; [I_\nu(\hat{n}) - S_\nu(\hat{n})]$$

for scattering of unpolarized radiation, source is not isotropic!

$$S_\nu(\hat{n}) = \frac{1}{\sigma_T} \int I_\nu(\hat{n}') \, \frac{d\sigma}{d\Omega}(\hat{n}, \hat{n}') \, \frac{d\Omega'}{4\pi} = \frac{3}{16\pi} \int I_\nu(\hat{n}') \; \left[1 + (\hat{n} \cdot \hat{n}')^2\right] \; d\Omega'$$

where the *redistribution function*

$$\mathcal{R}(\hat{n}, \hat{n'}) = \frac{1}{4\pi\sigma_{\mathrm{tot}}} \frac{d\sigma}{d\Omega}(\hat{n}, \hat{n}') \overset{\mathrm{Thom}}{=} \frac{3}{16\pi} \left[1 + (\hat{n} \cdot \hat{n}')^2\right]$$

encodes the scattering directionality

Q: what if scattering is isotropic?

if we *approximate Thomson as isotropic*, then

$$\frac{d\sigma}{d\Omega} \xrightarrow{\text{iso}} \sigma_{\mathsf{T}}/4\pi$$

and we recover our old result

$$S_\nu \xrightarrow{\text{iso}} J_\nu = \frac{1}{4\pi}\int I_\nu \ d\Omega \tag{22}$$

for which the redistribution function is just

$$\mathcal{R}(\hat{n}, \widehat{n'}) = \frac{1}{4\pi} \tag{23}$$

Awesomest Example of Thompson Polarization: the CMB

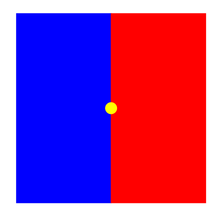
The CMB is nearly isotropic radiation field
arises from $\tau s \sim 1$ "surface of last scattering" at $z \approx 1000$
when free e and protons "re" combined $e + p \rightarrow H + \gamma$

- *before recombination:*
  Thomson scattering of CMB photons, Universe opaque
- after recombination: no free e, Universe transparent

the CMB is the cosmic photosphere!

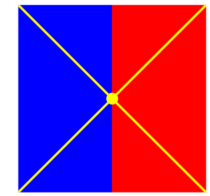
electrons during last scattering see anisotropic radiation field

consider point at hot/cold "wall"
locally sees *dipole* T anisotropy
net polarization towards us: zero! *Q: why?*



Q: what about edge of circular hot spot? cold spot?

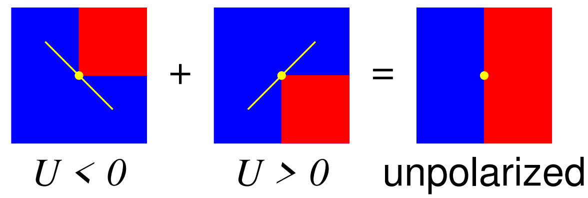
at wall: see local dipole



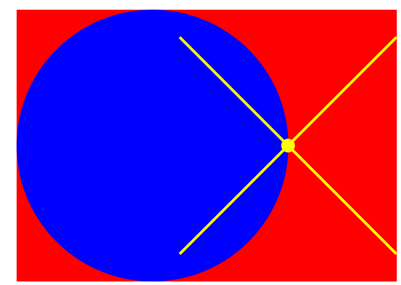
hot side horizontal and vertical contributions are equal!

$\rightarrow$ no net polarization!

also follows from this superposition



polarization tangential (ring) around hot spots
radial (spokes) around cold spots
(superpose to "$+$" $=$ zero net pol)



`www: WMAP polarization observations of hot and cold spots`

Note: polarization & T anisotropies *linked*
$\rightarrow$ consistency test for CMB theory and hence hot big bang

Polarization Observed

First detection: pre-WMAP!
⋆ DASI (2002) ground-based interferometer
at level predicted based on T anisotropies! Woo hoo!

WMAP (2003): first polarization-T correlation function

Planck (March 2013): much more sensitive to polarization

Build Your Toolbox: Thomson Scattering

microphysics: matter-radiation interactions

Q: physical origin of Thomson scattering?

Q: physical nature of sources?

Q: spectrum characteristics?

Q: frequency range?

real/expected astrophysical sources of Thomson scattering

Q: where do we expect this to be important?

Q: relevant EM bands? temperatures?

Toolbox: Thomson Scattering

emission physics

- physical origin: scattering by *non-relativistic free electrons*
- physical sources: need free e^- $\rightarrow$ ionized gas
  scattering $\rightarrow$ *photons conserved, need incident radiation*
  scattering induces *polarization* even for unpolarized sources
- spectrum: Thomson coherent *scattered energy unchanged*
  σ_T indept of ν: *spectral shape preserved in scattered radiation*

astrophysical sources of Thomson scattering

- sites are illuminated and highly ionized gas: stellar interiors, stellar coronæ, hot nebulae (HII regions), early Universe
- EM bands *radio to X-ray*
  for γ-rays relativistic effects are important $\rightarrow$ Compton
- temperatures *up to* $\sim 10^6$ K
  above this, relativistic effects are important $\rightarrow$ Compton