Astronomy 501: Radiative Processes

Lecture 17

Sept 30, 2022

Announcements:

- **Problem Set 5 due today**
- Good news: no problem set for next week!
- Bad news: Midterm Exam next Friday
  Info on Canvas

Last time: Thomson scattering

Q: what's that–actors? needed conditions? astrophysical sites?

Q: dependence on incident radiation? ν? viewing angle?

Thomson Scattering Recap

Thomson: **scattering of light by free electrons**

- free electrons needed $\Rightarrow$ *ionized* gas
- sites–*hot!* solar corona, galaxy clusters, early Universe
- scattered power for one electron $dP/d\Omega = d\sigma/d\Omega\ S$:
  $P \propto S$ incident flux, and
  $P \propto \sigma$ cross section
- for linearly polarized incident radiation

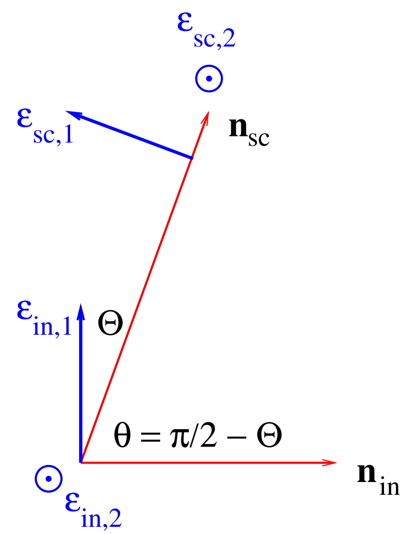
$$\frac{d\sigma}{d\Omega} = \frac{3}{2\pi}\sigma_T \cos^2\theta$$

$$\sigma_T = \frac{8\pi}{3}\frac{e^4}{m_e^2 c^4} = \frac{8\pi}{3} r_0^2$$

- where $\cos\theta = \hat{n}_{\rm incident} \cdot \hat{n}_{\rm scattered}$
- independent of photon frequency ν
- photon frequency ν unchanged: elastic

Q: cross section for unpolarized incident radiation?

Thomson scattering of *unpolarized* incident radiation:



$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{unpol}} = \frac{1}{2}\left(\frac{d\sigma}{d\Omega}\right)_1 + \frac{1}{2}\left(\frac{d\sigma}{d\Omega}\right)_2$$
$$= \frac{r_0^2}{2}\left(\sin^2\Theta + 1\right)$$
$$= \frac{r_0^2}{2}\left(1 + \cos^2\theta\right)$$

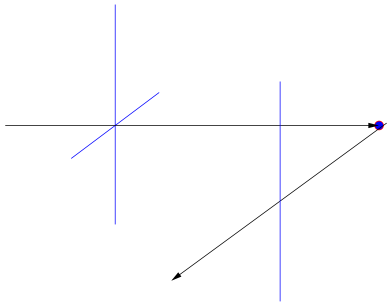
which only depends on angle θ
between incident $\hat{n}_{\text{in}}$ and scattered $\hat{n}_{\text{sc}}$ radiation directions

- **forward-backward symmetry**: $\theta \to -\theta$ invariance
- angular pattern: $\cos^2\theta \propto \cos 2\theta$ term

$\to$ scattered radiation has 180^0 periodicity: **quadrupole**
Thomson-scattered radiation becomes linearly polarized!

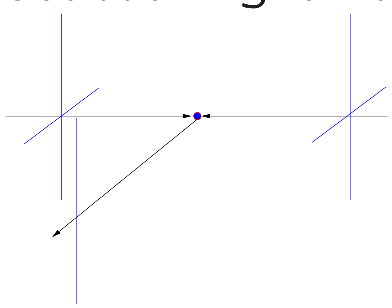
$$\Pi = \frac{1 - \cos^2\theta}{1 + \cos^2\theta} \qquad (1)$$

scattering of one unpolarized beam:



$\rightarrow$ see radiation from e motion in sky plane
$\rightarrow$ linear polarization!

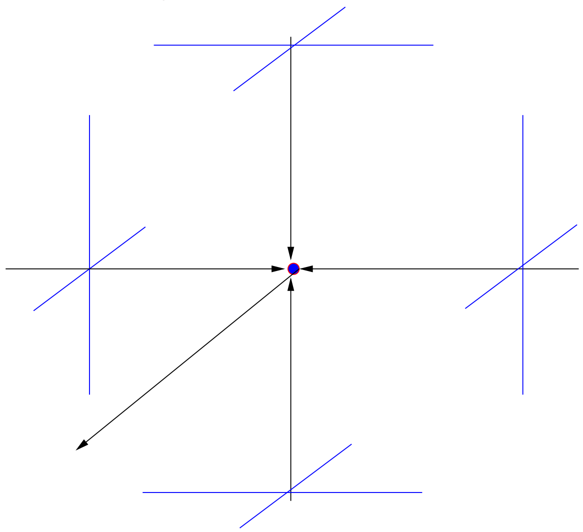
scattering of two unpolarized beams in opposite directions:



$\rightarrow$ the other side only adds to e motion in sky plane
$\rightarrow$ also linear polarization!

Q: what if isotropic initial radiation field?

isotropic initial radiation field:

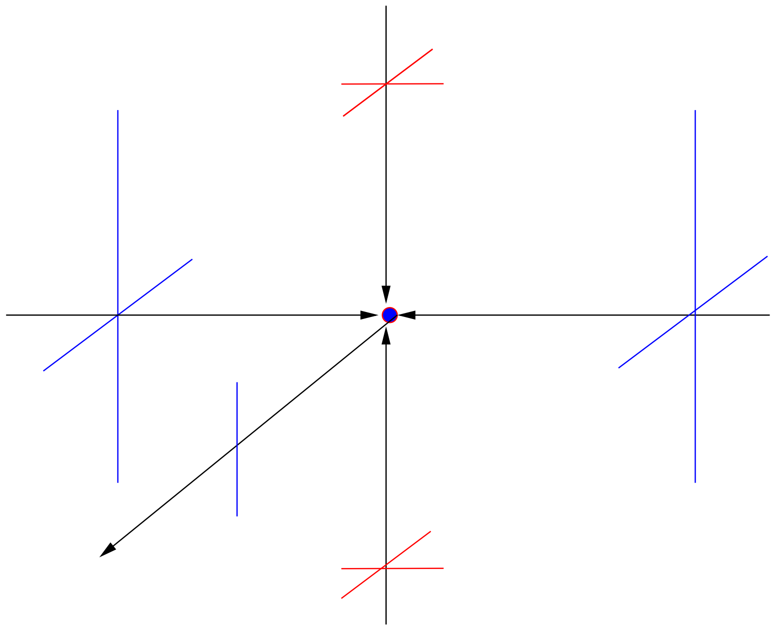


e motions in x and y sky directions cancel
$\rightarrow$ no net polarization

Q: what incident radiation fields do create polarization?

Q: lesson?

if initial radiation field has quadrupole intensity pattern



linear polarization!

lesson: polarization arises from Thomson scattering
when electrons "see" quadrupole anisotropies in radiation field

Q: If Thomson scattering is the only process acting
what is the appropriate transfer equation?

Thomson Scattering in Radiation Transfer

recall: in *coherent scattering*

- photon number and energy preserved
- but directions changed

$$\frac{dI_\nu(\hat{n})}{ds} = -n_e \sigma_\mathrm{T} \; [I_\nu(\hat{n}) - S_\nu(\hat{n})]$$

for scattering of unpolarized radiation, source is not isotropic!

$$S_\nu(\hat{n}) = \frac{1}{\sigma_T} \int I_\nu(\hat{n}') \, \frac{d\sigma}{d\Omega}(\hat{n}, \hat{n}') \, \frac{d\Omega'}{4\pi} = \frac{3}{16\pi} \int I_\nu(\hat{n}') \; \left[1 + (\hat{n} \cdot \hat{n}')^2\right] \; d\Omega'$$

where the *redistribution function*

$$\mathcal{R}(\hat{n}, \widehat{n'}) = \frac{1}{4\pi\sigma_\mathrm{tot}} \frac{d\sigma}{d\Omega}(\hat{n}, \hat{n}') \;\; \overset{\mathrm{Thom}}{=} \; \frac{3}{16\pi} \left[1 + (\hat{n} \cdot \hat{n}')^2\right]$$

$\leadsto$ encodes the scattering directionality

Q: what if scattering is isotropic?

if we *approximate Thomson as isotropic*, then

$$\frac{d\sigma}{d\Omega} \xrightarrow{\text{iso}} \sigma_{\mathsf{T}}/4\pi$$

and we recover our old result

$$S_\nu \xrightarrow{\text{iso}} J_\nu = \frac{1}{4\pi} \int I_\nu \; d\Omega \tag{2}$$

for which the redistribution function is just

$$\mathcal{R}(\hat{n}, \widehat{n'}) = \frac{1}{4\pi} \tag{3}$$

∞

Awesomest Example of Thompson Polarization: the CMB

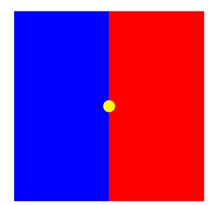
The CMB is nearly isotropic radiation field
arises from $\tau s \sim 1$ "surface of last scattering" at $z \approx 1000$
when free e and protons "re" combined $e + p \to H + \gamma$

- *before recombination:*
  Thomson scattering of CMB photons, Universe opaque
- after recombination: no free e, Universe transparent

the CMB is the cosmic photosphere!

electrons during last scattering see anisotropic radiation field

consider point at hot/cold "wall"
  locally sees *dipole* T anisotropy
  net polarization towards us: zero! *Q: why?*

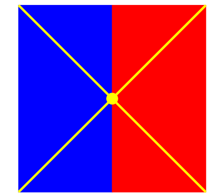


Q: what about edge of circular hot spot? cold spot?

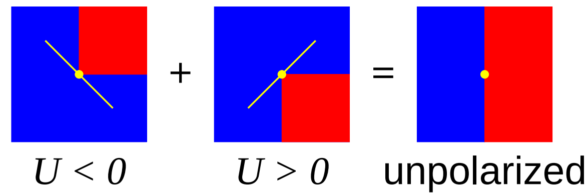
at wall: see local dipole

hot side horizontal and vertical contributions are equal!

$\rightarrow$ no net polarization!



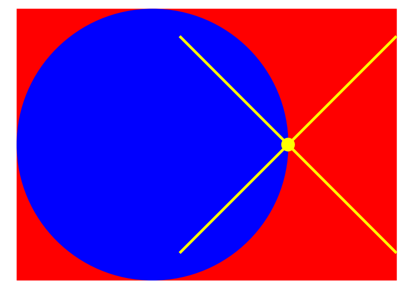
also follows from this superposition



polarization tangential (ring) around hot spots

radial (spokes) around cold spots

(superpose to "$+$" $=$ zero net pol)



`www: WMAP polarization observations of hot and cold spots`

Note: polarization & T anisotropies *linked*

$\rightarrow$ consistency test for CMB theory and hence hot big bang

Polarization Observed

First detection: pre-WMAP!
⋆ DASI (2002) ground-based interferometer
at level predicted based on T anisotropies! Woo hoo!

WMAP (2003): first polarization-T correlation function

Planck (March 2013): much more sensitive to polarization

Build Your Toolbox: Thomson Scattering

microphysics: matter-radiation interactions

Q: physical origin of Thomson scattering?

Q: physical nature of sources?

Q: spectrum characteristics?

Q: frequency range?

real/expected astrophysical sources of Thomson scattering

Q: where do we expect this to be important?

Q: relevant EM bands? temperatures?

Toolbox: Thomson Scattering

emission physics

- physical origin: scattering by non-relativistic free electrons
- physical sources: need free e^- $\rightarrow$ ionized gas
  scattering $\rightarrow$ photons conserved, need incident radiation
  scattering induces polarization even for unpolarized sources
- spectrum: Thomson scattered energy unchanged
  $\Rightarrow$ coherent or elastic scattering
  σ_T indept of ν: spectral shape preserved in scattered radiation

astrophysical sources of Thomson scattering

- sites are illuminated and highly ionized gas: stellar interiors, stellar coronæ, hot nebulæ (Hii regions), early Universe
- EM bands radio to X-ray
  for γ-rays relativistic effects are important $\rightarrow$ Compton
- temperatures up to $\sim 10^6$ K
  above this, relativistic effects are important $\rightarrow$ Compton

Bremsstrahlung

Bremsstrahlung

German lesson for today:
Bremse = brake (as in stopping)
Strahlung = radiation
$\rightarrow$ Bremsstrahlung = "breaking radiation"
= radiation from decelerated charge particles

Consider a **dilute plasma** at temperature T, with

- free ions: charge $+Ze$, number density $n_{\rm i}$
- free electrons: charge $-e$, number density n_e

Q: what conditions needed to realize this?
Q: astrophysical examples? `www: awesome example`
Q: what microphysics what will cause the plasma to emit?
i.e., what interactions will occur?
Q: which particles will radiate more?

dilute plasma = low particle density = typical in astrophysics
$\rightarrow$ three-body collisions unlikely; ignore these
$\rightarrow$ focus on two-body collisions

possible interactions: Coulomb forces between particle pairs

- electron-electron
- ion-ion
- electron-ion

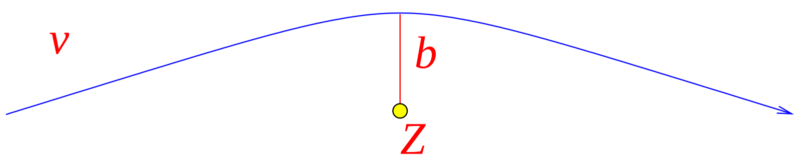
But electrons repel each other!
don't approach closely: electron-electron acceleration weak

electron and ion attracted and scattered by same Coulomb force
But $a_{\mathrm{i}}/a_e = m_e/m_{\mathrm{i}} < 10^{-3}$ $\rightarrow$ ion acceleration negligible
$\rightarrow$ focus electron acceleration in static field of ion
electron-ion radiation dominates

Order of Magnitude Expectations

start with *classical, nonrelativistic* picture

consider a free, unbound electron with asymptotic speed v moving in Coulomb field of stationary ion



let b = *the distance of closest approach* or **impact parameter**

Q: estimate of maximum acceleration?

Q: duration of acceleration? velocity change? radiation frequency

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Recall the *Spirit of Order-of-Magnitude*:

- ignore all dimensionless constants, e.g., "small circle approximation" $2\pi \approx 1$
- lower expectations for precision
- use rough result to guide more careful calculations

maximum acceleration:

Coulomb acceleration at closest approach

$$a_{\text{max}} \sim \frac{Ze^2}{m_e b^2} \tag{4}$$

duration of acceleration: **collision time**

$$\tau \sim \frac{b}{v} \tag{5}$$

velocity change

$$\Delta v \sim a_{\text{max}} \ \tau \sim \frac{Ze^2}{m_e b v} \sim \left(\frac{Ze^2/b}{m_e v^2} \right) v \tag{6}$$

frequency of radiation: use only timescale in problem

$$\omega \sim \frac{1}{\tau} \sim \frac{v}{b} \tag{7}$$

Q: what is maximum radiated power? radiated energy? energy per unit freq?

maximum radiated power is

$$P_{\text{max}} \sim \frac{e^2 a_{\text{max}}^2}{c^3} \sim \frac{e^2 \Delta v^2}{c^3\ \tau^2} \sim \frac{Z^2 e^6}{m_e v^2 b^2\ \tau^2} \tag{8}$$

radiated energy

$$\Delta W \sim P_{\text{max}}\ \tau \sim \frac{Z^2 e^6}{m_e v^2 b^2\ \tau} \tag{9}$$

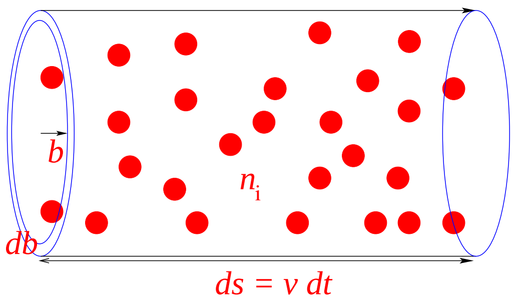
radiated energy per unit frequency

$$\frac{\Delta W}{\Delta \nu} \sim \frac{\Delta W}{\omega} \sim \frac{Z^2 e^6}{m_e v^2 b^2} \tag{10}$$

this energy radiated per electron-ion encounter at distance b

electron with speed v moves encounters ion number density n_{i}

- we want number of ions $d\mathcal{N}_{\text{i}}$ that e encounters out to distance $\sim b$ in time dt *Q: which is?*
- *Q: what is typical rate of energy emitted per electron?*



in cylindrical distance $(b, b + db)$, volume swept is

$$dV = 2\pi \ b \ db \ ds = 2\pi \ v \ b \ db \ dt \tag{11}$$

i.e., $dV \sim b^2 \ v \ dt$

thus number of ions encountered is

$$d\mathcal{N}_{\mathrm{i}} = n_{\mathrm{i}} \ dV \ \ \sim n_{\mathrm{i}} \ b^2 \ v \ dt \tag{12}$$

Thus the rate of energy emitted = *power emitted per e* is

$$\frac{dP_{\mathrm{per}e}}{d\nu} = \frac{\Delta W}{\Delta \nu} \frac{d\mathcal{N}_{\mathrm{i}}}{dt} \sim \frac{e^6 Z^2}{m_e c^3 v} n_{\mathrm{i}} \tag{13}$$

Q: and so what is emission coefficient j_ν?

Our order-of-magnitude estimate for the emission coefficient from nonrelativistic bremsstrahlung:

$$j_\nu = n_e \frac{dP_{\text{per}e}}{d\nu} \sim \frac{e^6 Z^2}{m_e c^3 v} n_e n_{\text{i}} \tag{14}$$

Q: what's the basic physical picture?

Q: notable features? what didn't we get from order of mag?

Q: how can we do the classical calculation more carefully?

Bremsstrahlung: Physical Picture

we are interested in the motion of an electron through a plasma

we approximate this as a series of
- *two-body electron-ion* scattering events
- *unbound Coulomb* trajectories: *hyperbolæ*
  $\rightarrow$ asymptotically free, scattered through small angle
- acceleration maximum at closest approach b
  lasting for scattering time $\tau = b/v$
- burst of radiation over this time, frequency $\nu \sim 1/\tau$

So net effect is
- many scattering events
- a series of small-angle scatterings
- and radiation bursts at different frequencies