Astronomy 501: Radiative Processes

Lecture 26

Oct 24, 2022

Announcements:

- **Problem Set 8 due Friday**
- Midterm exams scores posted
  see Chris to pick up graded exam
  Bonus Round points are *additional*!

Last time: selection rules for atomic transitions

Today:

- spectral line shapes
- awesome application: stellar classification

Atomic Lines: Electric Dipole Radiation

line emission and absorption controlled by dipole maxtrix element

$$\vec{d}_{u\ell} = e \int \psi_u \vec{r} \psi_\ell \, dV$$

forbidden transitions: $\vec{d}_{u\ell} = 0$; otherwise: allowed

Selection Rules

allowed angular momentum change

- for the *jumping electron*, state change obeys
  $\Delta\ell = \pm 1$, $m = 0, \pm 1$
  fixes spectrum of 1-electron atoms
- for *multi-electron atoms*, *total ang. mom.* change obeys
  $\Delta S = 0$, $\Delta L = 0, \pm 1$, $\Delta J = 0, \pm 1$

Mystery Revisited: Hydrogen Lines in Stars

to a good approximation, stellar spectra are:

- blackbody = Planck form, at photospheric T
- with lines (often many!) due to photospheric absorption

History: as part of E. Pickering's team of "computers"
Annie Cannon led the refinement of stellar classification
found "natural" **OBAFGKM** order, but physical origin unknown
`www: examples from different classes`
Q: what controls the OBAFGKM sequence?

Balmer H lines: weak→strong→weak for types O→A→M

Q: why this H behavior?
Q: hint–what are Balmer transitions Hα, Hβ, etc?

Ceclia Payne-Gaposchkin Solves the Mystery

Ceclia Payne-Gaposchkin PhD work: used thermo + QM to show that stellar classes are a sequence in *temperature*

- Sun is G2V
- "early types" hotter than Sun: *OBAF*
- "late types" solar and cooler: *GKMLT*

H line strength reflect populations of bound states and ions

- O stars $T > 30,000$ K: most H is ionized
- A stars $T \sim 10,000$ K: most H neutral, but $n = 2$ populated
- M stars $T \sim 4000$ K: H neutral, tiny $n = 2$ population

by-product: also showed that *stars are mainly made of hydrogen*

Referring to Cecila Payne-Gaposchkin's work:

> undoubtedly the most brilliant Ph.D. thesis
> ever written in astronomy

–Otto Struve, President of American Astronomical Society

Awesome Example: Stellar Lineshapes

`www:  white dwarf spectrum`

`www:  O star spectrum`

Q: similar temperatures, why different?

Q: at fixed T, how can spectrum distinguish main sequence vs giant stars?

Shape of Spectral Lines

consider a transition $u \to \ell$
Q: most naïve guess for line profile $\phi(\nu)$

real astrophysical spectra show wide range profiles
with nonzero observed widths

`www: solar spectrum` try $(\lambda_i, \Delta\lambda) = (6500, 100)$nm; $(4043, 5)$; $(6704, 8)$

`www: spectrum of mystery star`
Q: how is this star different from the Sun?
hint–look at the continuum

`www: spectrum of interstellar matter`
Q: how is this gotten? how do we know the lines are ISM?

Q: reason(s) for nonzero observed linewidths?

Linewidths

naïvely: in transition $u \to \ell$, *energy conservation* requires
$h\nu = E_u - E_\ell \equiv h_{u\ell}$, so $\phi_{\text{naive}}(\nu) = \delta(\nu - \nu_{u\ell})$: *zero width!*

But real observed linewidths are nonzero, for several reasons

• *intrinsic width*
quantum effect, due to nonzero transition probability

• *thermal broadening*
thermal motion of absorbers $\to$ Doppler shifts

• *collisional broadening*
absorber collisions add to transition probability

• *instrumental resolution*
real spectrographs have finite resolving power
$R = \lambda/\Delta\lambda = \nu/\Delta\nu \overset{\text{Keck}}{\sim} 30,000$

Intrinsic Linewidth

in real atoms, any excited state u has nonzero transition rate to lower levels: $\Gamma_u = 1/\tau_u$, with τ_u the state *lifetime*

thus: state u is only populated for timescales $\delta t \sim \tau_u$

but in quantum mechanics, over finite time Δt, *energy* only determined to within finite resolution

$$\Delta E \ \Delta t \gtrsim \frac{\hbar}{2} \tag{1}$$

the **energy-time uncertainty relation**

thus state u, level energy E_u has intrinsic spread $\delta E_u \sim \hbar/\tau_u = \hbar\Gamma_u$

Q: consequence for line profile?

level u energy intrinsic spread $\delta E_u \sim \hbar/\tau_u = \hbar\Gamma_u$
so for $u \to \ell$, transition frequency $\nu_{u\ell} = (E_u - E_\ell)/h$
has natural or *intrinsic width* $\delta\nu_{n\ell} = \Gamma_{u\ell} = \Gamma_u + \Gamma_\ell$

level lifetimes related to Einstein A = decay rates:

$$\Gamma_u = \Gamma_{u \to \text{anything}} = \sum_{u \to \text{allowed } j} A_{uj} \tag{2}$$

where sum is over *all energetically allowed* transitions from u

for *damped classical oscillator*, damping $\Gamma \dot{x}$
leads (PS7) to absorption cross section

$$\sigma_{\ell u}(\nu) = \frac{2\pi e^2}{m_e c} \frac{\Gamma/2}{(\omega - \omega_0)^2 + (\Gamma/2)^2} = \frac{\pi e^2}{m_e c} \frac{4\Gamma}{16\pi^2(\nu - \nu_0)^2 + \Gamma^2}$$

Q: behavior at $\nu = \nu_0$? $\nu \gg \nu_0$? what about a real atomic transition $u \to \ell$?

for a damped classical oscillator, we have

$$\sigma(\nu) = \pi e^2/m_e c \ \phi(\nu) = B_{\mathrm{classical}} \ \phi(\nu) \tag{3}$$

with profile function (normalized to $\int \phi(\nu)\ d\nu = 1$) of

$$\phi(\nu) = \frac{4\Gamma}{16\pi^2(\nu - \nu_0)^2 + \Gamma^2}$$

a real atomic transition $u \to \ell$ has same properties but with overall factor of oscillator strength:

$$\sigma_{u\ell}(\nu) = \pi e^2/m_e c \ f_{u\ell} \ \phi_{u\ell}(\nu) = B_{\mathrm{classical}} \ f_{u\ell} \ \phi(\nu) \tag{4}$$

with *Lorentzian* profile shape

$$\phi_{u\ell}^{\mathrm{intrinsic}}(\nu) = \frac{4\Gamma_{u\ell}}{16\pi^2(\nu - \nu_{u\ell})^2 + \Gamma_{u\ell}^2}$$

full width at half-maximum: $(\Delta\nu)_{\mathrm{FWHM}} = \Gamma_{u\ell}/2\pi$

note that line profiles and linewidths are often expressed in line-of-sight *velocity* units

motivated by the non-relativistic Doppler formula, we have

$$v(\nu) = \frac{\nu - \nu_{u\ell}}{\nu_{u\ell}} \, c \tag{5}$$

so that $v(\nu_{u\ell}) = 0$ at line center

thus the FWHM in velocity units is

$$(\Delta v)_{\mathrm{FWHM}} = \frac{(\Delta\nu)_{\mathrm{FWHM}}}{\nu_{u\ell}} c = \frac{\Gamma_{u\ell}\lambda_{u\ell}}{2\pi} \tag{6}$$

for optical and UV transitions, intrinsic linewidths generally small:
for Lyman-α, $(\Delta v)_{\mathrm{FWHM,Ly}\alpha} = 0.0121$ km/s
Q: implications?

Thermal Linewidth

intrinsic linewidths are generally narrow
so other broadening effects can be important

thermal motion of atoms leads to Doppler shifts
of incident spectra as seen by the atoms
so absorption occurs "off resonance"

a *Gaussian distribution* of line-of-sight velocities
has velocity probability distribution

$$p(v)\ dv = \frac{1}{\sqrt{2\pi}\sigma_v} e^{-(v-v_0)^2/2\sigma_v{}^2} \equiv \frac{1}{\sqrt{\pi} b} e^{-(v-v_0)^2/b^2}\ dv \tag{7}$$

where v_0 is the bulk or "systemic" velocity along sightline
$\sigma_v = b/\sqrt{2}$ is the *velocity dispersion*

Q: v_0, σ_v, and b for thermal gas at rest??

a thermal gas at T of particles with mass m, and *at rest* in bulk, has

$$p_T(v)\ dv = \sqrt{\frac{m}{2\pi kT}} e^{-mv^2/2kT} \tag{8}$$

from which we identify

$$\begin{aligned} v_0 &= 0 & (9) \\ \sigma_v &= v_T \equiv \sqrt{\frac{kT}{m}} = 9.12 \text{ km/s} \left(\frac{T}{10^4 \text{ K}}\right) \left(\frac{1 \text{ amu}}{m}\right) & (10) \\ b &= \sqrt{\frac{2kT}{m}} & (11) \end{aligned}$$

Q: implications of numerical result?

Q: how to combine intrinsic and thermal broadening?

Voigt Profile

in general both intrinsic and thermal broadening present and so resulting line profile includes both effects

observed profile is *weighted average* of natural/intrinsic width with Doppler shifted center

$$\nu'_{u\ell} = \left(1 - \frac{v}{c}\right) \nu_{u\ell} \tag{12}$$

giving the **Voigt profile**

$$\phi_{\mathrm{Voigt}}(\nu) = \frac{1}{\sqrt{\pi}\ b} \int e^{-v^2/b^2} \frac{4\Gamma_{u\ell}}{16\pi^2 \left[\nu - (1 - v/c)\nu_{u\ell}\right]^2 + \Gamma_{u\ell}^2}\, dv$$

integral has no simple analytic result

Q: simple and interesting approximation?

we saw that for astrophysical situations, often
intrinsic linewidths $(\Delta v)_{\mathrm{FWHM}} \ll b$ thermal linewidths

simple approximation: intrinsic absorption is δ-function
$\phi^{\mathrm{intrinsic}}(\nu) \rightarrow \delta[\nu - (1 - v/c)\nu_{u\ell}]$

this gives a thermally-dominated Voigt profile

$$\phi_{\mathrm{Voigt}}(\nu) \;\; \rightarrow \;\; \phi_T(\nu) = \frac{1}{\sqrt{\pi}} \frac{c}{\nu_{u\ell}\, b} \; \exp\left[-\frac{v(\nu)^2}{b^2}\right] \tag{13}$$

valid in the *"thermal core"* $\nu - \nu_{u\ell} \ll \Gamma_{u\ell}$, with

$$v(\nu) \equiv \left(\frac{\nu - \nu_{u\ell}}{\nu_{u\ell}}\right) c \tag{14}$$

for $\nu - \nu_{u\ell} \gg b$, in the *"damping wings,"* we have

$$\phi_{\mathrm{Voigt}}(\nu) \approx \frac{1}{4\pi^2} \frac{\Gamma_{u\ell}}{(\nu - \nu_{u\ell})^2} \tag{15}$$

Q: sketch of $\phi_{\mathrm{Voigt}}(\nu)$? of $\sigma_{u\ell}(\nu)$?

Collisional Linewidth

if particle densities are high, atomic collisions are rapid and can drive transitions $u \leftrightarrow \ell$

thus there is a nonzero collision rate Γ_{coll} per atom where $\Gamma_{\text{coll}} = n\ \sigma_{\text{coll}} v$

heuristically: this decreases excited state lifetimes and thus adds to energy uncertainty

so total transition rate includes both Γ_{int} and Γ_{coll}:
$\rightarrow$ collisions add damping, which depends on photospheric density and temperature via Γ_{coll}

thus collisional broadening measures density and temperature
thus also know as "pressure broadening"

Q: effect of collisions on lineshape?

recall: atomic transition $u \to \ell$ has

$$\sigma_{u\ell}(\nu) = \pi e^2/m_e c \; f_{u\ell} \; \phi_{u\ell}(\nu) = B_{\text{classical}} \; f_{u\ell} \; \phi(\nu) \tag{16}$$

without collisions, intrinsic profile shape that is *Lorentzian*

$$\phi_{u\ell}^{\text{intrinsic}}(\nu) = \frac{4\Gamma_{u\ell}}{16\pi^2(\nu - \nu_{u\ell})^2 + \Gamma_{u\ell}^2}$$

full width at half-maximum: $(\Delta\nu)_{\text{FWHM}} = \Gamma_{u\ell}/2\pi$
set by intrinsic level de-excitation rate $\Gamma_{u\ell}$

With collisions: $\Gamma_{\text{coll}} = n \; \sigma_{\text{coll}} \; v$
still a Lorentzian profile, but with effective transition rate to

$$\frac{\Gamma_{u\ell}}{2} = \frac{\Gamma_{u\ell}^{\text{intrinsic}}}{2} + \Gamma_{\text{coll}} \tag{17}$$

`www: solar` $\mathrm{H}\alpha$ `line`

`www: white dwarf spectrum`

`www: O star spectrum`

Q: similar temperatures, so what causes difference?

relatedly: stars with the same temperature T
can have very different lumiosity L

`www: HR diagram`

Q: what is different about these stars?
Q: hint–think about blackbody flux $F = \sigma T^4$

How does this difference manifest in the spectrum?

Stellar Luminosity Class: I, II, III, IV, V
determined by *shapes of strong lines* at *fixed spectral type*
i.e., at (nearly) fixed temperature
V: line wings broader than intrinsic damping width
I: no additional broadening

physically: damping wings sensitive to *pressure broadening*
i.e., by collision rate $\Gamma_{\text{coll}} = n\sigma_{\text{coll}}v$
at fixed T, this corresponds to different *density* and pressure
but hydrostatic equilibrium: $\nabla P = \rho\vec{g} = G\rho M/R^2$
linewidth set by pressure $\rightarrow$ set by stellar *radius* R

Class I: supergiant
Class II: bright giants
Class III: normal ("red" giants)
Class IV: subgiants
Class V: main sequence (non-giants = "dwarfs"); Sun is G2V

Absorption Lines: Probing the Depths

so far: focused on absorption line *shape*
but important information also in line *depth*
below the continuum level

Q: what is needed to measure line depth?
Q: in high-resolution spectra, what sets line depth at each ν?
Q: as absorber density increases, effect on line?

absorption cross section (line oscillator strength) generally known
`www: online databases`
Q: given this, what quantitative information does line depth give?

so imagine we can resolve a strong absorption line and measure the shape vs ν or λ to high precision

Q: what will we see?

Q: what will we learn?

Q: what if the line is not very strong?

Q: what if we only have moderate spectral resolution?

`www: overview of the optical solar spectrum`
Q: what are we seeing?