Astronomy 501: Radiative Processes

Lecture 27

Oct 26, 2022

Announcements:

- **Problem Set 8 due Friday**
- Office hours today after class or by appointment

Last time: spectral line profiles

Q: sources of line width?

Linewidths

naïvely: in transition $u \to \ell$, *energy conservation* requires
$h\nu = E_u - E_\ell \equiv h_{u\ell}$, so $\phi_{\text{naive}}(\nu) = \delta(\nu - \nu_{u\ell})$: *zero width!*

But real observed linewidths are nonzero, for several reasons

- *intrinsic width* $\Gamma_{u\ell}$
quantum effect, due to nonzero transition probability

- *thermal broadening*
thermal motion of absorbers $\to$ Doppler shifts

- *collisional broadening* Γ_{coll}
absorber collisions add to transition probability

- *instrumental resolution*
real spectrographs have finite resolving power
$R = \lambda/\Delta\lambda = \nu/\Delta\nu \stackrel{\text{Keck}}{\sim} 30,000$

Voigt Profile

generally *intrinsic* and *thermal* broadening are present
and so real line profiles includes both effects

observed profile is *weighted average*
of natural/intrinsic width with Doppler broadened center (“core”)

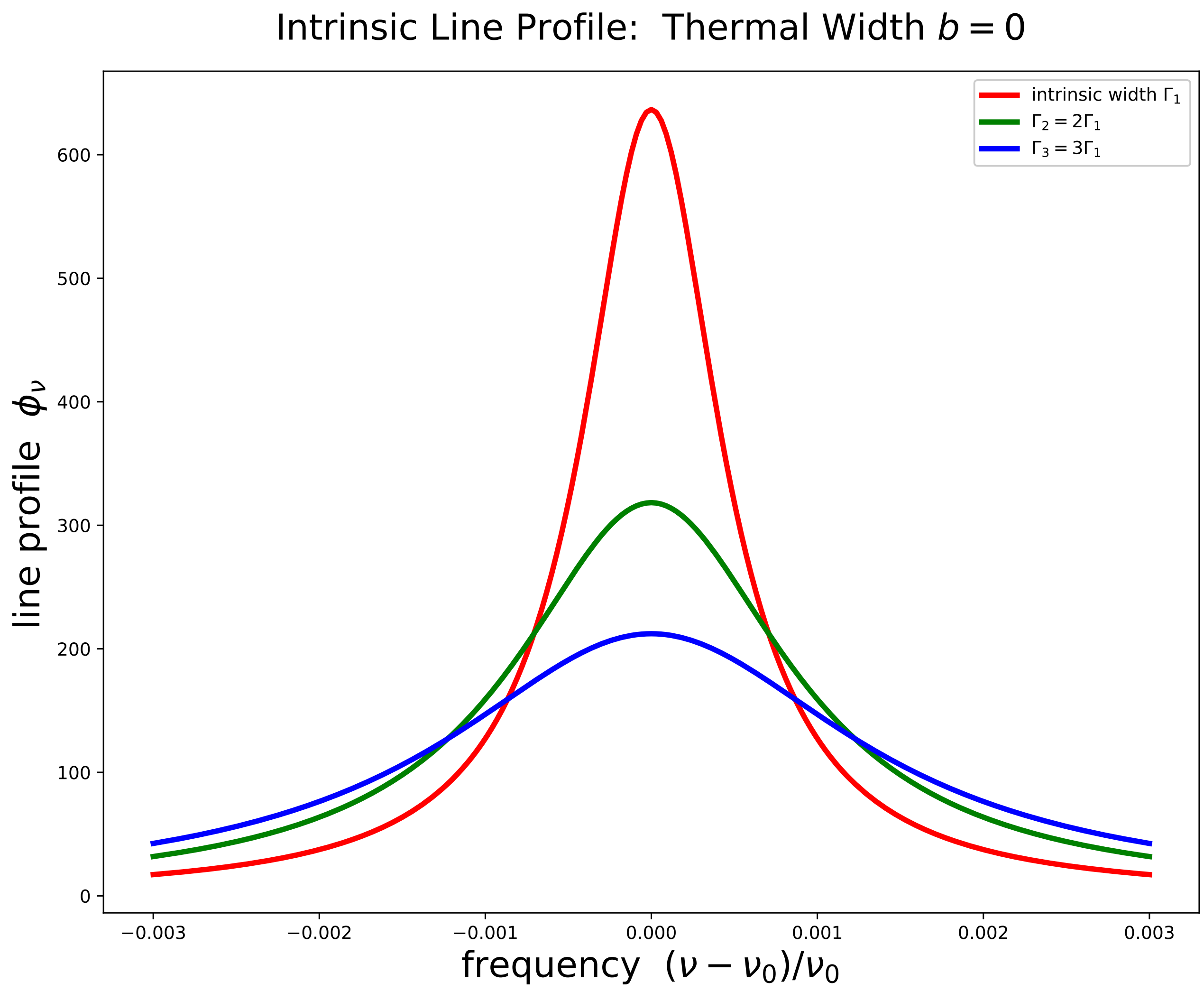
$$\nu'_{u\ell} = \left(1 - \frac{v}{c}\right) \nu_{u\ell} \tag{1}$$

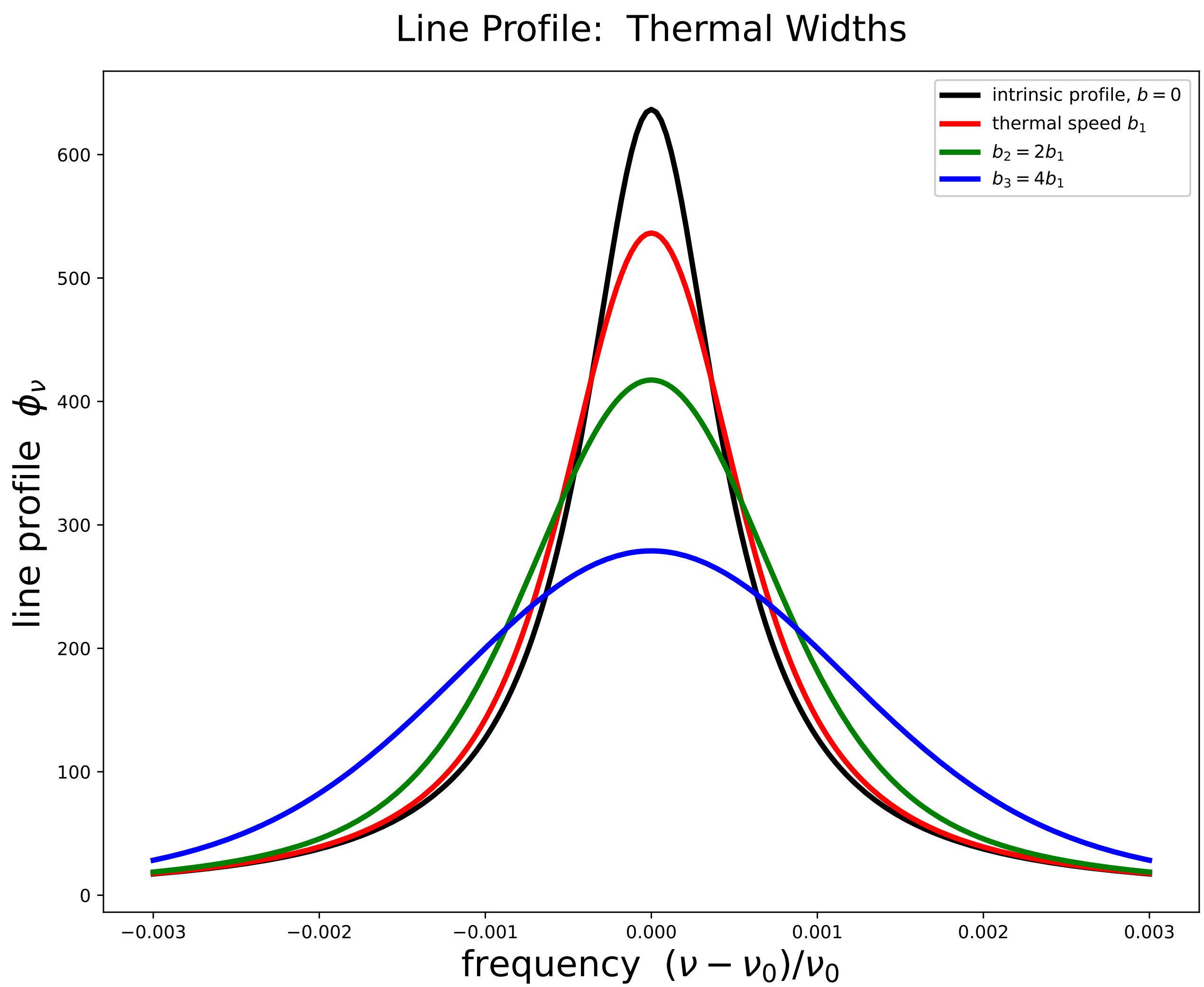
giving the **Voigt profile**

$$\phi_{\rm Voigt}(\nu) = \frac{1}{\sqrt{\pi}\ b} \int e^{-v^2/b^2} \ \frac{4\Gamma_{u\ell}}{16\pi^2 \left[\nu - (1 - v/c)\nu_{u\ell}\right]^2 + \Gamma_{u\ell}^2} \ dv$$

convolution: *thermal gaussian* $\otimes$ *Lorentzian*

Q: what if thermal width $b\nu_{u\ell}/c \ll \Gamma_{u\ell}$*? the reverse?*


Intrinsic Line Profile: Thermal Width $b = 0$
intrinsic width Γ_1
$\Gamma_2 = 2\Gamma_1$
$\Gamma_3 = 3\Gamma_1$
line profile ϕ_ν
0
100
200
300
400
500
600
−0.003
−0.002
−0.001
0.000
0.001
0.002
0.003
frequency $(\nu - \nu_0)/\nu_0$


Line Profile: Thermal Widths
intrinsic profile, $b = 0$
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`www: white dwarf spectrum`

`www: O star spectrum`

Q: similar temperatures, so what causes difference?

Stellar Luminosity Class: I, II, III, IV, V
determined by *shapes of strong lines* at *fixed spectral type*
i.e., at (nearly) fixed temperature
V: line wings broader than intrinsic damping width
I: no additional broadening

physically: damping wings sensitive to *pressure broadening*
i.e., by collision rate $\Gamma_{\rm coll} = n\sigma_{\rm coll} v$
at fixed T, this corresponds to different *density* and pressure
but hydrostatic equilibrium: $\nabla P = \rho \vec{g} = G\rho M/R^2$
linewidth set by pressure $\rightarrow$ set by stellar *radius* R

Class I: supergiant
Class II: bright giants
Class III: normal ("red" giants)
Class IV: subgiants
Class V: main sequence (non-giants = "dwarfs"); Sun is G2V

Absorption Lines: Probing the Depths

so far: focused on absorption line *shape*
but important information also in line *depth*
below the continuum level

Q: what is needed to measure line depth?
Q: in high-resolution spectra, what sets line depth at each ν*?*
Q: as absorber density increases, effect on line?

absorption cross section (line oscillator strength) generally known
`www: online databases`
Q: given this, what quantitative information does line depth give?

so imagine we can resolve a strong absorption line
and measure the shape vs ν or λ to high precision

Q: what will we see?

Q: what will we learn?

Q: what if the line is not very strong?

Q: what if we only have moderate spectral resolution?

`www: overview of the optical solar spectrum`
Q: what are we seeing?

Absorption Lines: Radiation Transfer

consider a (spatially) unresolved source, with angular area $\Delta\Omega$
if no material in foreground, observed flux $F_\nu(0) \approx I_\nu(0)\ \Delta\Omega$

with intervening absorbers of density n at T, observed flux is

$$F_\nu = e^{-\tau_\nu}\ F_\nu(0) + (1 - e^{-\tau_\nu}) S_\nu(T)\ \Delta\Omega \tag{2}$$

but usually for bright sources, $S_\nu(T)\ \Delta\Omega \ll F_\nu(0)$
and we have $F_\nu \approx e^{-\tau_\nu}\ F_\nu(0)$

near $\nu_{u\ell}$ for absorber transition $\ell \to u$, optical depth is

$$\tau_\nu = \sigma_\nu\ N_\ell\ \left(1 - \frac{g_u N_u}{g_\ell N_\ell}\right) \tag{3}$$

where $N_i \equiv \int n_i\ ds$ is absorber *column density* for level i

the last factor accounts for stimulated emission
but often $g_u N_u \ll g_\ell N_\ell$ *Q: why?*, so that $\tau_\nu \approx \sigma_\nu\ N_\ell$

So *if we assume we know the spectral shape* $F_\nu(0)$
of the background source across the line profile
then the observed deviation from this continuum
i.e., line *profile* $F_\nu/F_\nu(0) = e^{-\tau_\nu}$
directly measures optical depth $\tau_\nu \approx \sigma_{\ell u} N_\ell$

but the absorption cross section is

$$\sigma_{\ell u}(\nu) = \pi e^2/m_e c \; f_{\ell u} \; \phi_{\ell u}(\nu) \tag{4}$$

oscillator strength $f_{\ell u}$ usually known (i.e., measured in lab)
so at high resolution:

- line profile *depth* $\rightarrow$ absorber *column density* N_ℓ
- line profile *shape* $\rightarrow$ absorber profile function $\phi_{\ell u}(\nu)$
  which encodes, e.g., temperature via core width $b = \sqrt{2kT/m}$,
  and collisional broadening via wing with Γ

Depth of Line Center

if the absorbers have a Gaussian velocity distribution then the optical depth profile is $\tau_\nu = \tau_0 \; e^{-v^2/b^2}$ with the Doppler velocity $v = (\nu_0 - \nu)/\nu_0 \; c$, and thus τ_ν is also Gaussian in ν

the optical depth a the line center is

$$\tau_0 = \sqrt{\pi} \left(\frac{e^2}{m_e c} \right) \; \frac{N_\ell f_{\ell u} \lambda_{\ell u}}{b} \; \left[1 - \frac{g_u N_u}{g_\ell N_\ell} \right] \tag{5}$$

ignoring the stimulated emission term $[\cdots]$, for H Lyman α

$$\tau_0 = 0.7580 \; \left(\frac{N_\ell}{10^{13} \text{ cm}^{-2}} \right) \; \left(\frac{f_{\ell u}}{0.4164} \right) \; \left(\frac{\lambda_{\ell u}}{1215.7 \text{ Å}} \right) \; \left(\frac{10 \text{ km/s}}{b} \right)$$

so if we can measure τ_0, we get column N_ℓ

Q: in low-resolution spectra, what information is lost?

Q: what information remains?

Equivalent Width

if instrumental resolution $R = \Delta\lambda_{\rm inst}/\lambda$ low: $\Delta\lambda_{\rm inst} \ll$ line shape
$\rightarrow$ all information about true astrophysical line profile is lost!
and observed profile is just instrumental artifact

yet flux is still removed by the absorption line
so that we still can measure *integrated* effect of line
i.e., the total flux "lost" due to absorbers
$\Delta F_{\rm line} = \int_{\Delta\nu_{\rm line}} [F_\nu(0) - F_\nu] \, d\nu$
where ν_0 is frequency of *line center*

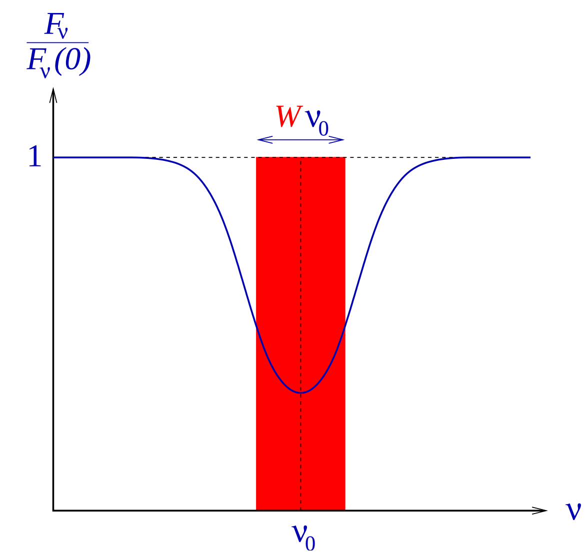
useful to define a dimensionless **equivalent width**

$$W \equiv \frac{\Delta F_{\rm line}}{\nu_0 \, F_\nu(0)} = \int_{\Delta\nu_{\rm line}} \frac{F_\nu(0) - F_\nu}{F_\nu(0)} \, \frac{d\nu}{\nu_0} \qquad (6)$$

Q: what does this correspond to physically?

equivalent width

$$W = \int_{\Delta\nu_{\rm line}} \frac{F_\nu(0) - F_\nu}{F_\nu(0)} \frac{d\nu}{\nu_0}$$



so $W\nu_0$ equivalent to

width of 100% absorbed line

i.e., *saturated* line with "rectangular" profile

and W is width as fraction of ν_0

note: many authors use *dimensionful equivalent* with

$$W \equiv \frac{W_\lambda}{\lambda_0} = \int_{\Delta\lambda_{\rm line}} \frac{F_\lambda(0) - F_\lambda}{F_\lambda(0)} \frac{d\lambda}{\lambda_0} \qquad (7)$$

so that $W_\lambda \approx \Delta\lambda \approx \lambda_0 W$

or the *velocity equivalent width* $W_v = c\, W$

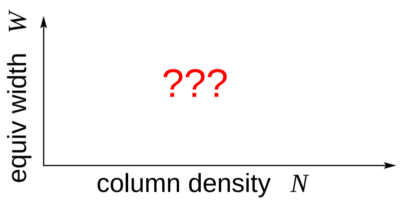
Curve of Growth

in terms of optical depth, equivalent width is

$$W = \int_{\Delta\nu_{\rm line}} \left[1 - \frac{F_\nu}{F_\nu(0)}\right] \frac{d\nu}{\nu_0} = \int_{\Delta\nu_{\rm line}} \left(1 - e^{-\tau_\nu}\right) \frac{d\nu}{\nu_0} \quad (8)$$

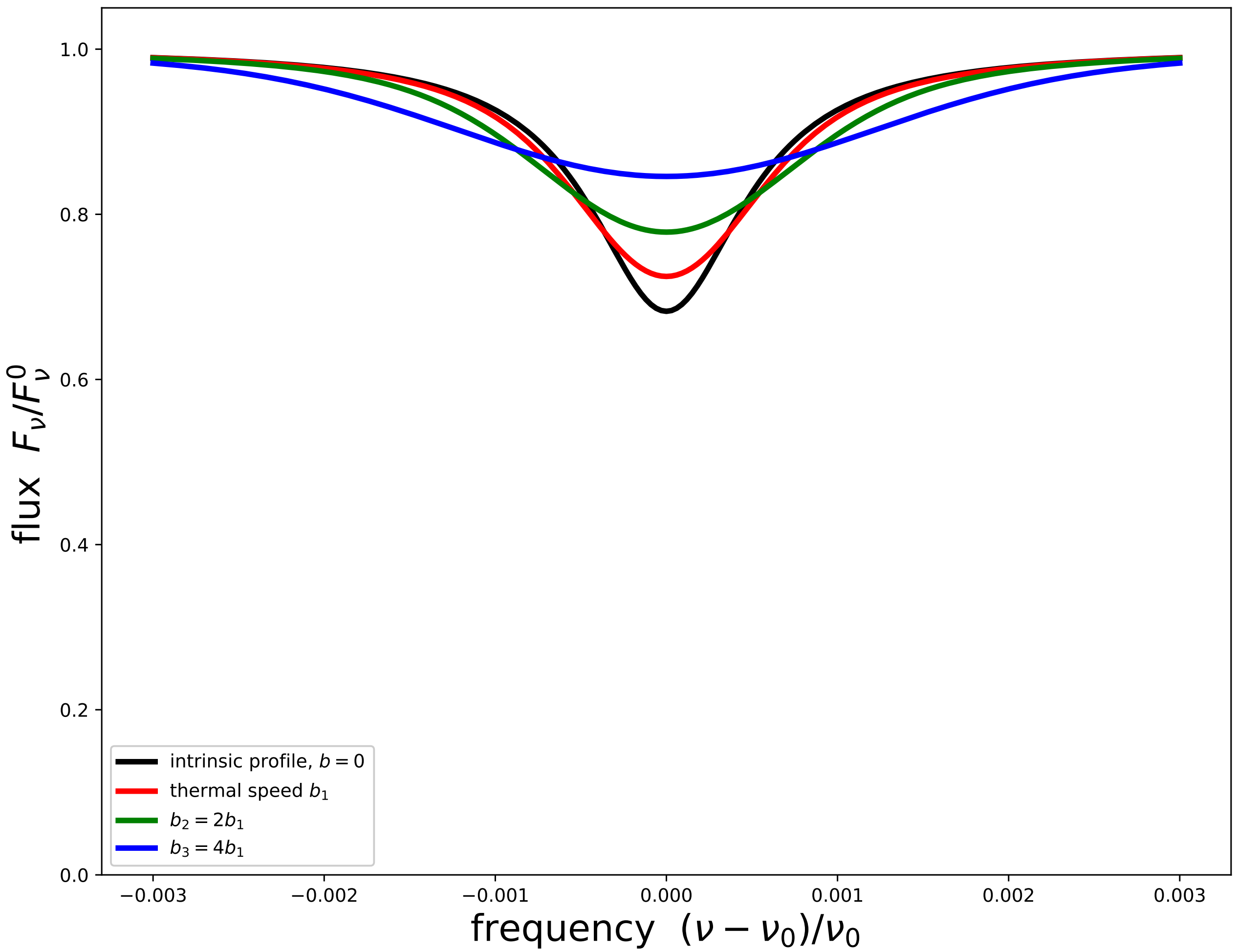
and thus $W = W(N_\ell)$ via $\tau_\nu = \sigma_\nu N_\ell$

dependence of W vs N_ℓ: **curve of growth**



even if line is unresolved, equivalent width still measures
$\Delta F = W \nu_0 F_\nu(0)$ = *total missing flux* across the line

Q: what is W if absorbers are optically thin? what do we learn?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Optically Thin Absorption: $\tau_0 \lesssim 1$

for an optically thin line: $\tau_0 \lesssim 1$
and thus maximal flux reduction at line center is $e^{-\tau_0} \gtrsim 1/e$

if $\tau_\nu \ll 1$ then we can put $1 - e^{-\tau_\nu} \approx \tau_\nu$:

$$W \approx \int \tau_\nu \frac{d\nu}{\nu_0} = N_\ell \frac{\int_{\text{line}} \sigma_{\ell u}(\nu)\ d\nu}{\nu_0} \quad (9)$$

so $W \propto N_\ell$: *linear regime* in curve of growth

for Gaussian profile, good fit to second order in τ_0 is

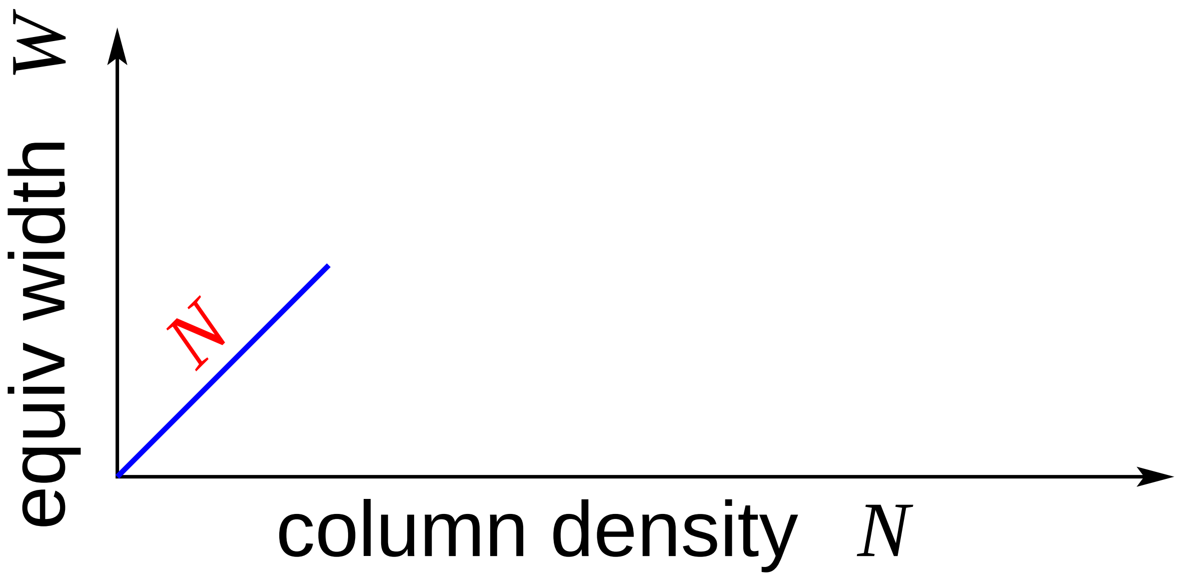
$$W \approx \sqrt{\pi}\ \frac{b}{c} \frac{\tau_0}{1 + \tau_0/(2\sqrt{2})} = \frac{\pi e^2}{m_e c^2}\ N_\ell\ f_{\ell u}\ \lambda_{\ell u} \frac{\tau_0}{1 + \tau_0/(2\sqrt{2})} \quad (10)$$

and thus when $\tau_0 \ll 1$,

$$N_\ell = \frac{m_e c^2}{\pi e^2} \frac{W}{f_{\ell u}\ \lambda_{\ell u}} = 1.130 \times 10^{12}\ \text{cm}^{-2}\ \frac{W}{f_{\ell u}\ \lambda_{\ell u}} \quad (11)$$

if line optically thin, then $W \propto N_\ell$

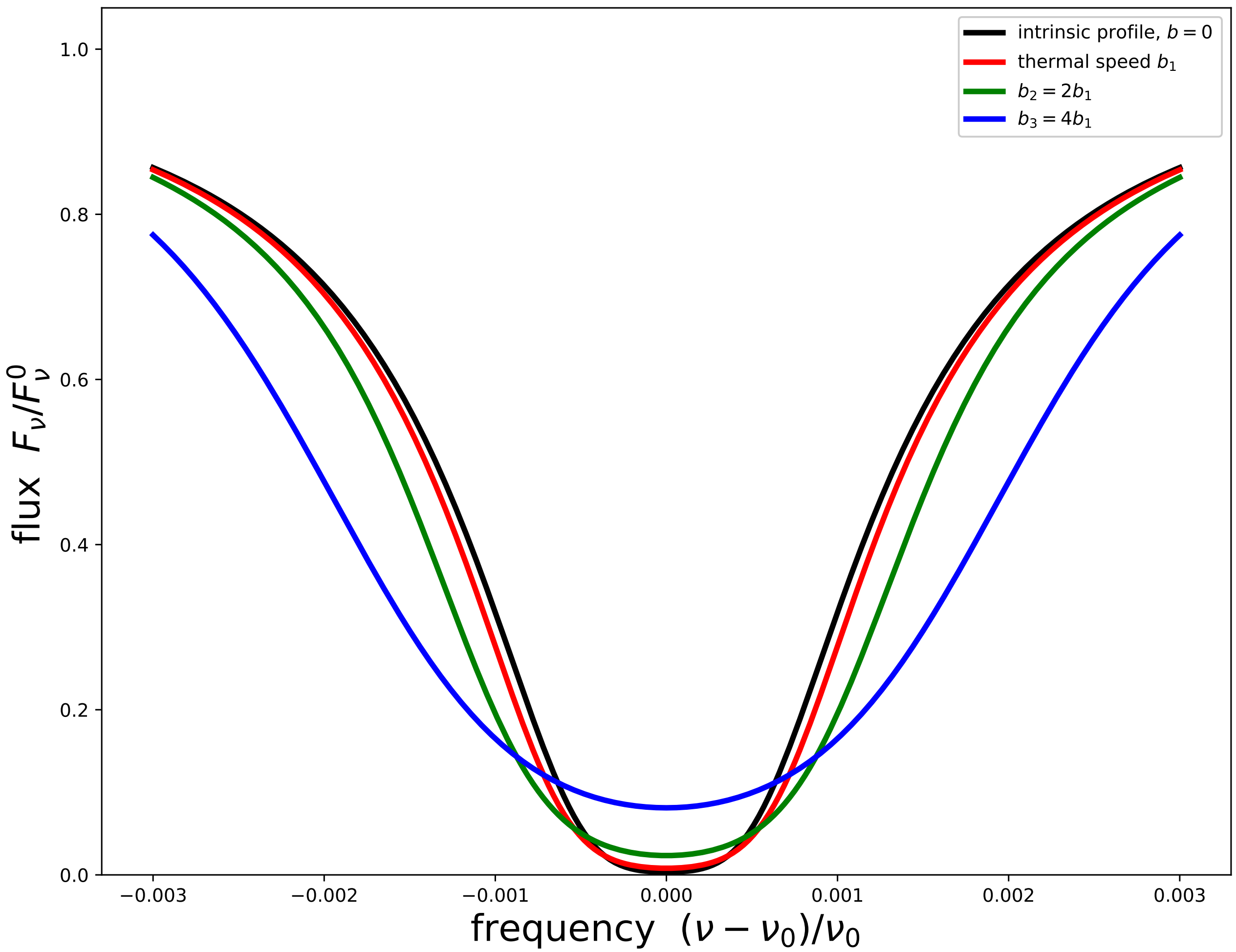
width measures absorber column density



Q: what happens if line is optically thick?

Q: what if line is thick and we assume thin?

Q: how can we use W to check if line is thick or thin?



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Flat Part of Curve of Growth: $1 \lesssim \tau_0 \lesssim \tau_{\rm damp}$

once $\tau_0 \gtrsim 1$, line center has essentially no flux
$\rightarrow$ line *core* is totally dark and thus *saturated*
true line profile is nearly "*box-shaped*"

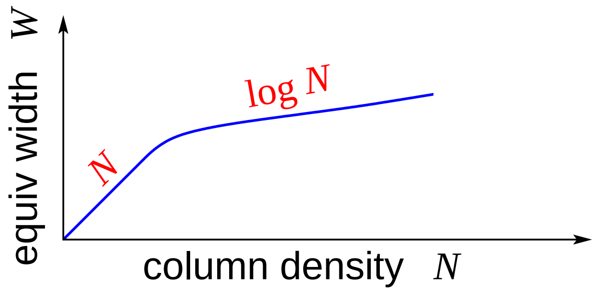
true line shape still has damping wings
but there cross section is small, so if $\tau_0 \lesssim \tau_{\rm damp}$
then wings only "round the edges" of the line "box"

if we treat the *unresolved* line as a box
then width is just Gaussian width

$$W \approx \frac{(\Delta\nu)_{\rm FWHM}}{\nu_0} = \frac{(\Delta v)_{\rm FWHM}}{c} = \frac{2\ b}{c}\sqrt{\frac{\ln \tau_0}{2}} \qquad (12)$$

and thus $W \propto b\ \sqrt{\ln \tau_0}$
Q: implications?

when $1 \lesssim \tau_0 \lesssim \tau_{\rm damp}$ then equivalent width
$W \propto b\sqrt{\ln \tau_0}$ depends very weakly on N_ℓ
$\rightarrow$ "flat part" of curve of growth



$$N_\ell \approx \frac{\ln 2}{\sqrt{\pi}} \frac{m_e c}{e^2} \frac{b}{f_{\ell u} \lambda_{\ell u}} e^{(cW/2b)^2} \tag{13}$$

column is *exponentially sensitive* to W

Warning! if a line is in this regime:

- difficult to get N_ℓ from measurements of W
- reliable result requires
  - ▷ very accurate measurements of W *and* b
  - ▷ confidence that true line profile is Gaussian

Q: what if absorber column density increases further?

Damped Part of Curve of Growth: $\tau_0 > \tau_{\rm damp}$

if N_ℓ and thus τ_0 very large,
then absorption very strong, then high-res profile
shows *Lorentzian "damping wings"*

away from line center, in "wing" regime $|\nu - \nu_0| \gg \nu_0/b/c$:

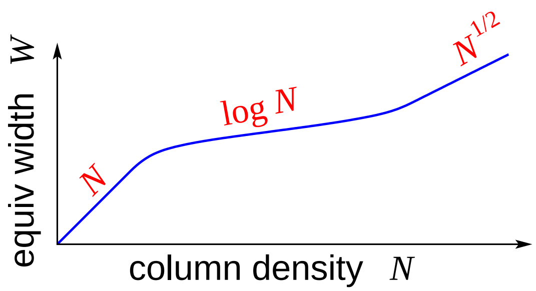
$$\tau_\nu \approx \frac{\pi e^2}{m_e c} N_\ell f_{\ell u} \; \frac{4\Gamma_{\ell u}}{16\pi^2(\nu-\nu_0)^2 + \Gamma_{\ell u}^2} \tag{14}$$

full width at half-max, i.e., width at 50% transmission, is

$$\frac{(\Delta\lambda)_{\rm FWHM}}{\lambda_0} = \frac{(\Delta u)_{\rm FWHM}}{c} = \sqrt{\frac{1}{\pi \ln 2} \frac{e^2}{m_e c} N_\ell f_{\ell u} \lambda_{\ell u} \; \frac{\Gamma_{\ell u}}{\nu_{\ell u}}}$$

thus equivalent width has $W \propto \sqrt{N_\ell}$:

$$W = \sqrt{\pi \ \ln 2}\frac{(\Delta\lambda)_{\rm FWHM}}{\lambda_0} = \sqrt{\frac{e^2}{m_e c} N_\ell f_{\ell u} \lambda_{\ell u} \frac{\Gamma_{\ell u}}{\nu_{\ell u}}} = \sqrt{\frac{b}{c}\frac{\tau_0}{\sqrt{\pi}}\frac{\Gamma_{\ell u}\lambda_{\ell u}}{c}} \qquad (15)$$



www: professional plot of curve of growth

$$N_\ell = \frac{m_e c^3}{e^2}\frac{W^2}{f_{\ell u}\Gamma_{\ell u}\lambda_{\ell u}^2} \qquad (16)$$

transition from flat to damped when $W_{\rm flat} \approx W_{\rm dampled}$:

$$\tau_{\rm damp} \approx 4\sqrt{\pi}\ \frac{b}{\Gamma_{\ell u}\lambda_{\ell u}} \ln\left[\frac{4\sqrt{\pi}}{\ln 2}\frac{b}{\Gamma_{\ell u}\lambda_{\ell u}}\right] \qquad (17)$$

Awesome Example: Quasar Absorption Lines

Q: let's remind ourselves–what's a quasar?

quasar (QSO) rest-frame optical to UV spectra $F_\lambda(0) = F_\lambda^{\rm qso}$:

- *smooth continuum* *Q: possible origin?*
- *broad peak* at rest-frame *Lyman-α* line *Q: possible origin?*

`www: famous SDSS composite quasar spectrum`

quasars generally at large redshift, *typically* $z_{\rm qso} \sim 3$

- distance very large: $\gtrsim d_H \sim 4000$ Mpc
- observed peak at $\lambda_{\rm peak,obs} = (1 + z_{\rm qso})\lambda_{{\rm Ly}\alpha} \sim 3600$ Å: *optical!*

QSO light passes through all intervening material at $z < z_{\rm qso}$

Q: what is intervening material made of?
Q: effect if absorbers have uniform comoving cosmic density?
Q: why can we rule out a uniform density?

Quasar Absorption Line Systems

quasars are distant, high-redshift *backlighting*
to all of the foreground universe

but thanks to big-bang nucleosynthesis, we know:
cosmic *baryonic** matter mostly made of *hydrogen*

if universe *uniformly filled* with H in $1s$ ground state, then:

- *at redshift* z, Lyα $1s \to 2s$ absorption
  at absorber-frame $\lambda_{\mathrm{Ly}\alpha}$, and observer-frame $\lambda_{\mathrm{obs}} = (1+z)\lambda_{\mathrm{Ly}\alpha}$
  absorption should occur at all $\lambda < (1+z_{\mathrm{qso}})\lambda_{\mathrm{Ly}\alpha}$
- absorbers have same comoving density at each z
  so optical depth τ_λ and hence transmission *spectrum*
  should be *smooth* as a function of λ

*in cosmo-practice: a *baryon* $=$ *neutron* or *proton* or combinations of them
$=$ *anything made of atoms* $=$ *ordinary matter* $\neq$ dark matter

Observed quasar spectra:

- *do* show absorption shortwards of the quasar Lyα!
- but transmitted spectrum is not smooth continuum, rather, a series of many separate *lines*

Implications:

- diffuse intervening neutral hydrogen exists! $\rightarrow$ there is an **intergalactic medium**
- intergalactic neutral gas is not uniform but *clumped* into "clouds" of atomic hydrogen

note: low-z quasars show few absorption lines
high-z quasars show many: **Lyman-α forest**
a major cosmological probe

Q: what information does each forest line encode?

The Lyman-α Forest: Observables

each forest *line* $\leftrightarrow$ *cloud of neutral hydrogen*

- absorber z_{abs} gives *cloud redshift*
- absorber depth gives cloud *column density* $N(\mathrm{H\ I})$

note that absorbers span wide range in column densities

- most common: optically thin "*forest systems*"
  correspond to *modest overdensities* $\delta\rho/\rho \sim 1$
- rare: optically thick "*damped Lyα systems*"
  damping wings of seen in line profile $\rightarrow N(\mathrm{H\ I}) \gtrsim 10^{20}\ \mathrm{cm}^{-2}$
  correspond to *large overdensities: protogalaxies!*

`www:  zoom into damped Ly`α` system`