Astronomy 501: Radiative Processes

Lecture 28

Oct 28, 2022

Announcements:

- **Problem Set 8 due today**
- **Problem Set 9 due next week**

Last time: absorption lines–curve of growth

Q: equivalent width–what's that?

Q: how does it change if continuum flux goes up?

Q: what's the curve of growth? why useful? what's growing?

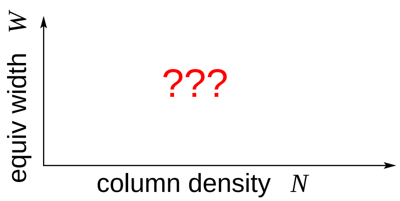
Curve of Growth

in terms of optical depth, equivalent width is

$$W = \int_{\Delta\nu_{\text{line}}} \left[1 - \frac{F_\nu}{F_\nu(0)}\right] \frac{d\nu}{\nu_0} = \int_{\Delta\nu_{\text{line}}} \left(1 - e^{-\tau_\nu}\right) \frac{d\nu}{\nu_0} \quad (1)$$

and thus $W = W(N_\ell)$ via $\tau_\nu = \sigma_\nu N_\ell$

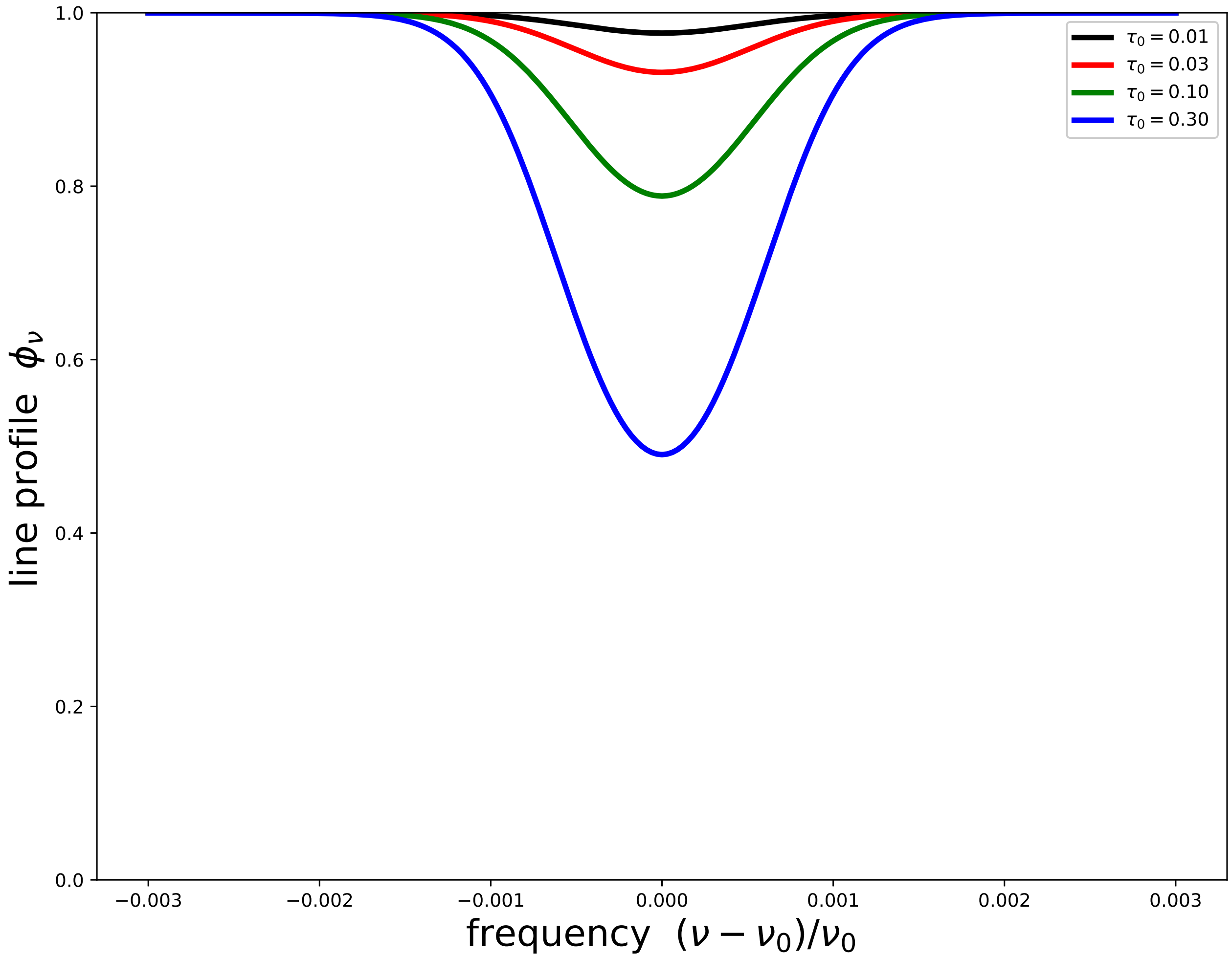
dependence of W vs N_ℓ: **curve of growth**



even if line is unresolved, equivalent width still measures

$\Delta F = W \nu_0 F_\nu(0) =$ *total missing flux* across the line

Q: what is W if absorbers are optically thin? what do we learn?


1.0
0.8
0.6
0.4
0.2
0.0
−0.003
−0.002
−0.001
0.000
0.001
0.002
0.003
frequency $(\nu - \nu_0)/\nu_0$
line profile ϕ_ν
$\tau_0 = 0.01$
$\tau_0 = 0.03$
$\tau_0 = 0.10$
$\tau_0 = 0.30$

Optically Thin Absorption: $\tau_0 \lesssim 1$

for an optically thin line: $\tau_0 \lesssim 1$
and thus maximal flux reduction at line center is $e^{-\tau_0} \gtrsim 1/e$

if $\tau_\nu \ll 1$ then we can put $1 - e^{-\tau_\nu} \approx \tau_\nu$:

$$W \approx \int \tau_\nu \frac{d\nu}{\nu_0} = N_\ell \frac{\int_{\mathrm{line}} \sigma_{\ell u}(\nu)\ d\nu}{\nu_0} \tag{2}$$

so $W \propto N_\ell$: *linear regime* in curve of growth

for Gaussian profile, good fit to second order in τ_0 is

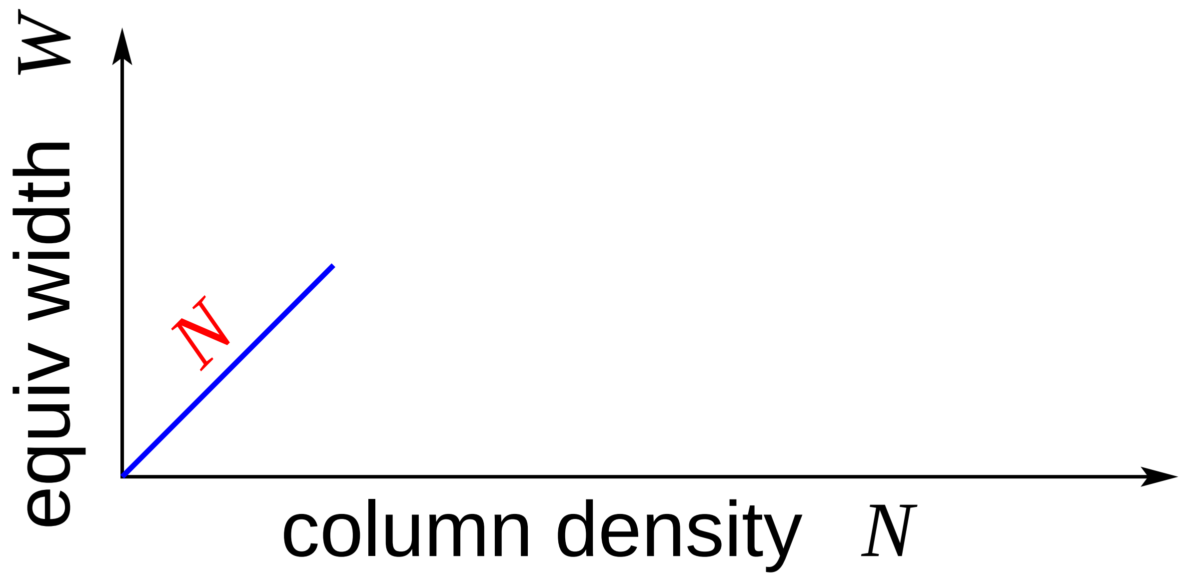
$$W \approx \sqrt{\pi}\ \frac{b}{c} \frac{\tau_0}{1 + \tau_0/(2\sqrt{2})} = \frac{\pi e^2}{m_e c^2}\ N_\ell\ f_{\ell u}\ \lambda_{\ell u} \frac{\tau_0}{1 + \tau_0/(2\sqrt{2})} \tag{3}$$

and thus when $\tau_0 \ll 1$,

$$N_\ell = \frac{m_e c^2}{\pi e^2} \frac{W}{f_{\ell u}\ \lambda_{\ell u}} = 1.130 \times 10^{12}\ \mathrm{cm}^{-2}\ \frac{W}{f_{\ell u}\ \lambda_{\ell u}} \tag{4}$$

if line optically thin, then $W \propto N_\ell$

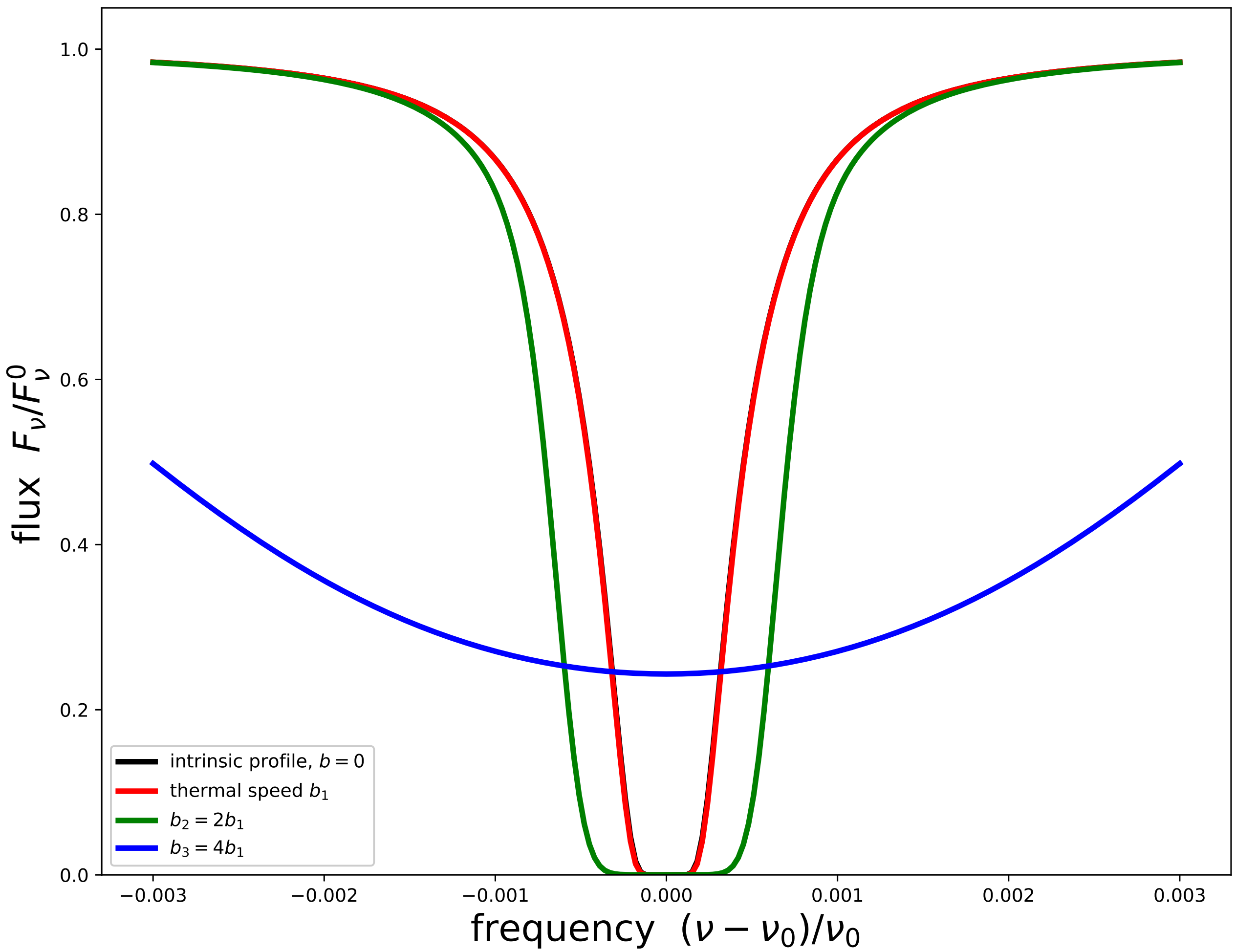
width measures absorber column density



Q: what happens if line is optically thick?

Q: what if line is thick and we assume thin?

Q: how can we use W to check if line is thick or thin?


1.0
0.8
0.6
0.4
0.2
0.0
flux F_ν/F_ν^0
−0.003
−0.002
−0.001
0.000
0.001
0.002
0.003
frequency $(\nu-\nu_0)/\nu_0$
intrinsic profile, $b=0$
thermal speed b_1
$b_2=2b_1$
$b_3=4b_1$

Flat Part of Curve of Growth: $1 \lesssim \tau_0 \lesssim \tau_{\rm damp}$

once $\tau_0 \gtrsim 1$, line center has essentially no flux
$\rightarrow$ line *core* is totally dark and thus *saturated*
true line profile is nearly "*box-shaped*"

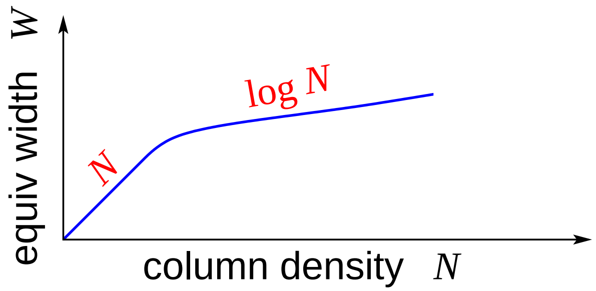
true line shape still has damping wings
but there cross section is small, so if $\tau_0 \lesssim \tau_{\rm damp}$
then wings only "round the edges" of the line "box"

if we treat the *unresolved* line as a box
then width is just Gaussian width

$$W \approx \frac{(\Delta\nu)_{\rm FWHM}}{\nu_0} = \frac{(\Delta v)_{\rm FWHM}}{c} = \frac{2\ b}{c}\sqrt{\frac{\ln \tau_0}{2}} \qquad (5)$$

and thus $W \propto b\ \sqrt{\ln \tau_0}$
Q: implications?

when $1 \lesssim \tau_0 \lesssim \tau_{\rm damp}$ then equivalent width
$W \propto b\sqrt{\ln \tau_0}$ depends very weakly on N_ℓ
$\rightarrow$ "flat part" of curve of growth



$$N_\ell \approx \frac{\ln 2}{\sqrt{\pi}} \frac{m_e c}{e^2} \frac{b}{f_{\ell u} \lambda_{\ell u}} e^{(cW/2b)^2} \qquad (6)$$

column is *exponentially sensitive* to W

Warning! if a line is in this regime:

- difficult to get N_ℓ from measurements of W
- reliable result requires
  - ▷ very accurate measurements of W *and* b
  - ▷ confidence that true line profile is Gaussian

Q: what if absorber column density increases further?

Damped Part of Curve of Growth: $\tau_0 > \tau_{\rm damp}$

if N_ℓ and thus τ_0 very large,
then absorption very strong, then high-res profile
shows *Lorentzian "damping wings"*

away from line center, in "wing" regime $|\nu - \nu_0| \gg \nu_0/b/c$:

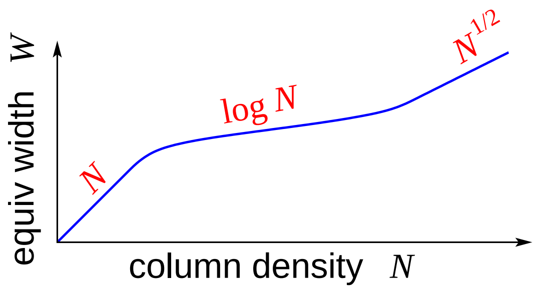
$$\tau_\nu \approx \frac{\pi e^2}{m_e c} N_\ell f_{\ell u} \; \frac{4\Gamma_{\ell u}}{16\pi^2(\nu - \nu_0)^2 + \Gamma_{\ell u}^2} \tag{7}$$

full width at half-max, i.e., width at 50% transmission, is

$$\frac{(\Delta\lambda)_{\rm FWHM}}{\lambda_0} = \frac{(\Delta u)_{\rm FWHM}}{c} = \sqrt{\frac{1}{\pi \ln 2} \frac{e^2}{m_e c} N_\ell f_{\ell u} \lambda_{\ell u}} \; \frac{\Gamma_{\ell u}}{\nu_{\ell u}}$$

thus equivalent width has $W \propto \sqrt{N_\ell}$:

$$W = \sqrt{\pi \ln 2}\frac{(\Delta\lambda)_{\rm FWHM}}{\lambda_0} = \sqrt{\frac{e^2}{m_e c} N_\ell f_{\ell u} \lambda_{\ell u} \frac{\Gamma_{\ell u}}{\nu_{\ell u}}} = \sqrt{\frac{b}{c}\frac{\tau_0}{\sqrt{\pi}}\frac{\Gamma_{\ell u}\lambda_{\ell u}}{c}} \tag{8}$$



www: professional plot of curve of growth

$$N_\ell = \frac{m_e c^3}{e^2}\frac{W^2}{f_{\ell u}\Gamma_{\ell u}\lambda_{\ell u}^2} \tag{9}$$

transition from flat to damped when $W_{\rm flat} \approx W_{\rm dampled}$:

$$\tau_{\rm damp} \approx 4\sqrt{\pi}\,\frac{b}{\Gamma_{\ell u}\lambda_{\ell u}} \ln\left[\frac{4\sqrt{\pi}}{\ln 2}\frac{b}{\Gamma_{\ell u}\lambda_{\ell u}}\right] \tag{10}$$

Awesome Example: Quasar Absorption Lines

Q: let's remind ourselves–what's a quasar?

quasar (QSO) rest-frame optical to UV spectra $F_\lambda(0) = F_\lambda^{\rm qso}$:
- *smooth continuum* *Q: possible origin?*
- *broad peak* at rest-frame *Lyman-α* line *Q: possible origin?*

`www: famous SDSS composite quasar spectrum`

quasars generally at large redshift, *typically* $z_{\rm qso} \sim 3$
- distance very large: $\gtrsim d_H \sim 4000$ Mpc
- observed peak at $\lambda_{\rm peak,obs} = (1+z_{\rm qso})\lambda_{{\rm Ly}\alpha} \sim 3600$ Å: *optical!*

QSO light passes through all intervening material at $z < z_{\rm qso}$

Q: what is intervening material made of?
Q: effect if absorbers have uniform comoving cosmic density?
Q: why can we rule out a uniform density?

Quasar Absorption Line Systems

quasars are distant, high-redshift *backlighting*
to all of the foreground universe

but thanks to big-bang nucleosynthesis, we know:
cosmic *baryonic** matter mostly made of *hydrogen*

if universe *uniformly filled* with H in $1s$ ground state, then:

- *at redshift* z, Lyα $1s \to 2s$ absorption
  at absorber-frame $\lambda_{\mathrm{Ly}\alpha}$, and observer-frame $\lambda_{\mathrm{obs}} = (1+z)\lambda_{\mathrm{Ly}\alpha}$
  absorption should occur at all $\lambda < (1+z_{\mathrm{qso}})\lambda_{\mathrm{Ly}\alpha}$
- absorbers have same comoving density at each z
  so optical depth τ_λ and hence transmission *spectrum*
  should be *smooth* as a function of λ

*in cosmo-practice: a *baryon* = *neutron* or *proton* or combinations of them
= *anything made of atoms* = *ordinary matter* $\neq$ dark matter

Observed quasar spectra:

- *do* show absorption shortwards of the quasar Lyα!
- but transmitted spectrum is not smooth continuum, rather, a series of many separate *lines*

Implications:

- diffuse intervening neutral hydrogen exists!
  $\rightarrow$ there is an **intergalactic medium**
- intergalactic neutral gas is not uniform but *clumped* into "clouds" of atomic hydrogen

note: low-z quasars show few absorption lines
high-z quasars show many: **Lyman-α forest**
a major cosmological probe

Q: what information does each forest line encode?

The Lyman-α Forest: Observables

each forest *line* $\leftrightarrow$ *cloud of neutral hydrogen*

- absorber z_{abs} gives *cloud redshift*
- absorber depth gives cloud *column density* $N(\mathrm{H\ I})$

note that absorbers span wide range in column densities

- most common: optically thin "*forest systems*"
  correspond to *modest overdensities* $\delta\rho/\rho \sim 1$
- rare: optically thick "*damped Lyα systems*"
  damping wings of seen in line profile $\rightarrow N(\mathrm{H\ I}) \gtrsim 10^{20}\ \mathrm{cm}^{-2}$
  correspond to *large overdensities: protogalaxies!*

www: zoom into damped Lyα system

Atomic Hydrogen: the Lyman Series and Lyman Limit

neutral hydrogen in the ground state has transitions
to all excited states; lines form the **Lyman series**

`www: Grotrian diagram`

- Lyman-α: $n = 1 \leftrightarrow 2$; $\lambda_{\mathrm{Ly}\alpha} = 1215.67$ Å
  Q: EM regime? implications?
- Lyβ: $n = 1 \leftrightarrow 3$; Lyγ: $n = 1 \leftrightarrow 4$, etc

Lyman series line $n = 1 \leftrightarrow n_f$ has

$$\lambda_{n_f} = \left(1 - \frac{1}{n_f^2}\right) \frac{hc}{B_{\mathrm{H}}} \tag{11}$$

for $n_f \gg 1$, line *pileup* of lines
at **Lyman limit** $\lambda_{n_f \to \infty} = hc/B_{\mathrm{H}} = 911.75$ Å

but note: real spectra do not show infinite lines near Ly limit
Q: Why not? What will we really see?

The Lyman Limit

there are infinitely many transitions near the Lyman limit
but each line has a *finite width*
typically due to thermal broadening

lines overlap and *blend* when

$$\text{line width} \lesssim \text{line spacing} \tag{12}$$

$$\frac{v_{\mathrm{FWHM}}}{c}\lambda_n \lesssim \lambda_n - \lambda_{n-1} \approx \frac{\partial \lambda_n}{\partial n} \tag{13}$$

occurs when $1/n^3 \lesssim v_{\mathrm{FWHM}}/c$, and thus
quantum number $n \gtrsim 67\ (2\ \mathrm{km\,s^{-1}}/v_{\mathrm{FWHM}})^{1/3}$, and wavelength

$$\lambda \approx 911.75 + 0.2 \left(\frac{v_{\mathrm{FWHM}}}{2\ \mathrm{km/s}}\right)^{2/3}\ \text{Å} \tag{14}$$

`www: Lyman limit QSO absorption line system`

Q: what about atoms from elements with $\lambda(n = 1 \rightarrow 2) <$ 912 Å*?*

Atomic Resonance Lines

for neutral atoms, *permitted* lines due to transitions from the *ground state* are called **resonance lines** – easiest to excite in neutral matter

we saw: neutral H atoms absorb all photons with $\lambda < 912$ Å in *bound-bound transitions* to $n_f \gg 1$ for λ near limit or for smaller λ, in *bound-free transitions* that ionize H

but H is the most abundant element in the Universe!

thus for atoms with all resonance lines $\lambda < 912$ Å
local hydrogen absorbs photons that can drive these transition
$\rightarrow$ *cannot observe these elements in sightlines with H I!*
Q: are any atoms excluded this way? if so, which?

Good news:

the only atoms excluded this way are He and Ne

noble gasses, first excited states at very high energies

all other atoms have resonance lines $\lambda > 912$ Å!

Bad news:

for most atoms and ions, resonance lines have $\lambda < 3000$ Å

Q: why is this bad?

Q: how to get around the badness?

“Metal” Lines

quaint astro-lingo: **“metal”** any element $\neq$ H, He
including, e.g., famous “metals” C, N, O

most metal atoms and ions have resonance lines $\lambda < 3000$ Å
this is in the UV, blocked by Earth’s atmosphere
`www: atmosphere transmittance`
for such species, can only see absorption lines:

- by going to space
- when looking at high-redshift objects
  `www: metal lines in QSO spectrum`

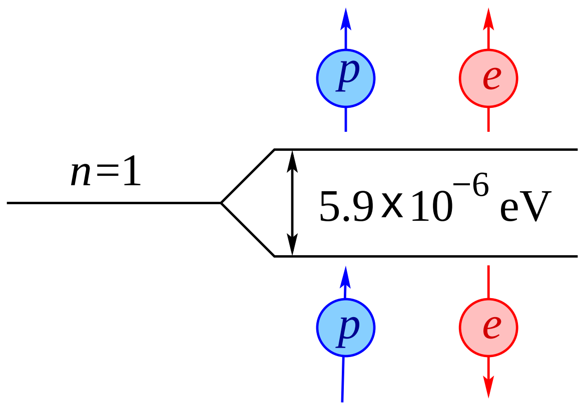
but nature has not been totally unkind
a few atoms and ions have resonance lines with $\lambda > 3000$ Å
examples: Na I D doublet $\lambda = 5891.6, 5897.6$ Å,
Ca II doublet at $3934.8, 3969.6$ Å
`www: solar lines`

Atomic Hydrogen: Hyperfile Splitting

in *hydrogen*, both e and p have spin $S = 1/2$ (fermions!)
coupled via *spin-spin* or *hyperfine* interaction
with Hamiltonian $H_{\text{spin-spin}} = H_{\text{hf}} \; \vec{s}_e \cdot \vec{s}_p$
radiation is *magnetic dipole*

hydrogen ground state has two possible *spin configurations*

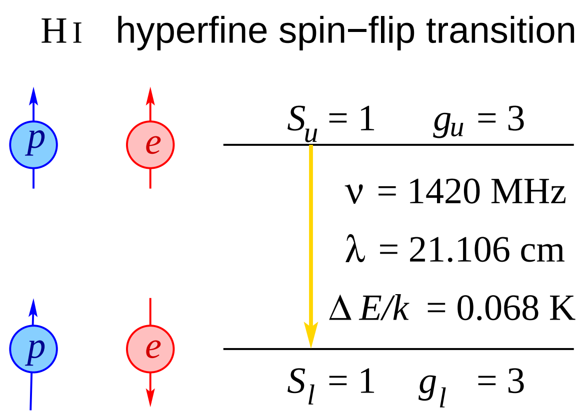
- proton and electron spins *parallel*: $\uparrow_e \uparrow_p$
  excited state: $S_u = 1$, $g_u = 3$
- spins *antiparallel*: $\downarrow_e \uparrow_p$
  ground state: $S_\ell = 0$, $g_\ell = 1$



Q: and so...?

Atomic Hydrogen: the 21 cm Line

transition $u \to \ell$ requires electron *spin flip* $\Delta s \neq 0$, $\Delta n = \Delta \ell = 0$



$$A_{u\ell} \;=\; 2.8843 \times 10^{-15}\ \mathrm{s}^{-1} = (11.0\ \mathrm{Myr})^{-1}$$

$$\Delta E = E_u - E_\ell \;=\; 5.86 \times 10^{-6}\ \mathrm{eV} = k_{\mathrm{B}}\,(0.06816\ \mathrm{K})$$

$$\nu_{u\ell} = 1420.4\ \mathrm{MHZ} \qquad \lambda_{u\ell} = 21.106\ \mathrm{cm}$$

implications of A value? ΔE? λ?

Einstein coefficient:
$A_{u\ell} = (11.0 \text{ Myr})^{-1}$: *very slow rate*

- spontaneous emission only occurs after a ~ 11 Myr if the atoms has been *undisturbed*: no collisions!
- spontaneous emission never observed in the laboratory! can only measure transition via stimulated emission!
- but can occur in low-density astrophysical environment
- excited state lifetime $A^{-1} \ll$ age of Universe $\rightarrow$ need some collisions to replenish excited state

EM regime:
$\nu = 1420.4$ MHZ and $\lambda = 21.106$ cm:
"21 cm radiation" in **radio**

Thermal Properties:
$\Delta E/k_{\mathrm{B}} = 0.06816$ K small splitting
$\rightarrow$ easy to thermally populate excited state
Q: recall that today, $T_{\mathrm{cmb}} = 2.725$ K; implications?

Spin Temperature

the CMB has $T_{\text{CMB}} \gg \Delta E/k \rightarrow$ can populate upper level!

if states in thermal equilibrium at *excitation* or *spin temperature* with $T_{\text{ex}} \equiv T_{\text{spin}} \gg \Delta E/k$, then

$$\frac{n_u}{n_\ell} = \frac{g_u}{g_\ell} e^{-h\nu_{u\ell}/kT_{\text{spin}}} \approx \frac{g_u}{g_\ell} = 3 \tag{15}$$

a nearly fixed ratio *independent of temperature*, so that

$$n_u \approx \frac{3}{4} n(\text{H I}) \ , \quad n_\ell \approx \frac{1}{4} n(\text{H I}) \tag{16}$$

thus: 21-cm emissivity also independent of spin temperature

$$j_\nu = n_u \frac{A_{u\ell}}{4\pi} h\nu_{u\ell} \ \phi_\nu \approx \frac{3}{16\pi} A_{u\ell} \ h\nu_{u\ell} \ n(\text{H I}) \ \phi_\nu \tag{17}$$

Q: absorption coefficient?

21-cm Absorption Coefficient

as usual, absorption coefficient has true and stimulated terms:

$$\alpha_\nu = n_\ell \sigma_{\ell u} - n_u \sigma_{\ell u} \tag{18}$$

$$= n_\ell \frac{g_u}{g_\ell} \frac{A_{u\ell}}{8\pi} \lambda_{u\ell}^2 \ \phi_\nu \ \left[1 - \frac{n_u}{n_\ell} \frac{g_\ell}{g_u} \right] \tag{19}$$

$$= n_\ell \frac{g_u}{g_\ell} \frac{A_{u\ell}}{8\pi} \lambda_{u\ell}^2 \ \phi_\nu \ \left[1 - e^{-h\nu_{u\ell}/kT_{\rm spin}} \right] \tag{20}$$

but in practice we always have $e^{-h\nu_{u\ell}/kT_{\rm spin}} \approx 1$, so
stimulated emission correction is very important!

using $e^{-h\nu_{u\ell}/kT_{\rm spin}} \approx 1 - h\nu_{u\ell}/kT_{\rm spin}$, we have

$$\alpha_\nu \approx n_\ell \frac{3}{32\pi} A_{u\ell} \ \frac{hc\lambda_{u\ell}}{kT_{\rm spin}} n(\text{H I}) \ \phi_\nu \tag{21}$$

and thus $\alpha_\nu \propto 1/T_{\rm spin}$

Q: what determines ϕ_ν in practice?

since $A = \Gamma$ is very small, 21-cm line intrinsically very narrow
$\rightarrow$ width entirely determined by *velocity dispersion* of the emitting hydrogen

for a random, Gaussian velocity distribution

$$\phi_\nu = \frac{1}{\sqrt{2\pi}} \frac{c}{\nu_{u\ell}} \frac{1}{\sigma_v} e^{-u^2/2\sigma_v^2} \tag{22}$$

with $u = c(\nu_{u\ell} - \nu)/\nu_{u\ell}$, we have

$$\alpha_\nu \approx n_\ell \frac{3}{32\pi} \frac{1}{\sqrt{2\pi}} \frac{A_{u\ell}\lambda_{u\ell}^2}{\sigma_v} \frac{hc}{kT_{\rm spin}} n(\text{H I})\ e^{-u^2/2\sigma_v^2} \tag{23}$$

and optical depth

$$\tau_\nu = 2.190\ \left(\frac{N(\text{H I})}{10^{21}\ cm^{-2}}\right) \left(\frac{100\ \text{K}}{T_{\rm spin}}\right) \left(\frac{1\ \text{km/s}}{\sigma_v}\right)\ e^{-u^2/2\sigma_v^2} \tag{24}$$

Q: implications?

21 cm Emission: Optically Thin Case

21 cm optical depth:

$$\tau_\nu = 2.190 \left(\frac{N(\text{H I})}{10^{21}\ cm^{-2}}\right)\left(\frac{100\text{ K}}{T_{\text{spin}}}\right)\left(\frac{1\text{ km/s}}{\sigma_v}\right) e^{-u^2/2\sigma_v^2} \quad (25)$$

real interstellar lines of sight can have $N(\text{H I}) > 10^{21}\ cm^{-2}$
$\rightarrow$ *self-absorption can be important!*

But in the optically thin limit, for $N(\text{H I}) \lesssim 10^{20}\ \text{cm}^{-2}$
then absorption is small and

$$I_\nu \approx I_\nu(0) + \int j_\nu\ ds = I_\nu(0) + \frac{3}{16\pi} A_{u\ell}\ h\nu_{u\ell}\ N(\text{H I})\ \phi_\nu \quad (26)$$

with $N(\text{H I}) = \int n_{\text{H I}}\ ds$

if $I_\nu(0)$ is known *Q: how?*, then

$$\int [I_\nu - I_\nu(0)]\ d\nu = \frac{3}{16\pi} A_{u\ell}\ h\nu_{u\ell}\ N(\text{H I}) \quad (27)$$

in terms of antenna temperature, integrating in velocity space

$$\int [T_{\mathsf{A}} - T_{\mathsf{A}}(0)] \; du = \int \frac{c^2}{2k\nu^2} [T_{\mathsf{A}} - T_{\mathsf{A}}(0)] \; c \, \frac{d\nu}{\nu} = \frac{3}{16\pi} A_{u\ell} \frac{hc\lambda_{u\ell}^2}{k} \, N(\mathrm{H\ I})$$

measures *hydrogen column* $N(\mathrm{H\ I})$ independent of spin temperature!

integrating over solid angles gives flux density

$$F_{\mathrm{obs}} = \int F_\nu \; d\nu = \int I_\nu \cos\theta \; d\Omega \; d\nu \approx \int I_\nu \; d\Omega \; d\nu \qquad (28)$$

and thus the integrated flux

$$F_{\mathrm{obs}} \propto \int N(\mathrm{H\ I}) \; d\Omega = \frac{\int n_{\mathrm{H\ I}} \; ds \; dA}{D_L^2} \propto \frac{M_{\mathrm{H\ I}}}{D_L^2} \qquad (29)$$

measures the *total hydrogen mass* $M_{\mathrm{H\ I}}$
if we know the (luminosity) distance D_L

useful for H I clouds in our own Galaxy,
and measuring H I content of external galaxies

consider cold, diffuse atomic H in a galaxy that has bulk internal motions with speeds $v_{\mathrm{bulk}} > \sigma_v$

Q: how would this arise?

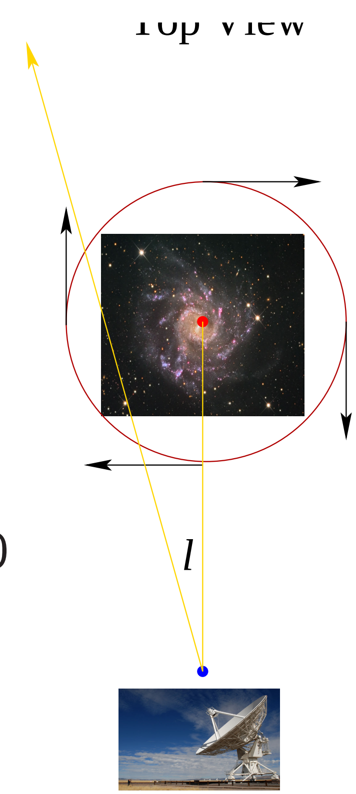
Q: what spectral pattern would uniform rotation give?

Q: what is a more realistic expectation?

Awesome Example: Galaxies in 21 cm

spiral galaxies observed in 21 cm emission, ellipticals are not
$\rightarrow$ spirals are gas rich, ellipticals gas poor `www: THINGS survey`

spiral galaxies also rotate:
bulk line-of-sight motion imprinted on 21 cm
via Doppler shift at different sightlines



spectrum depends on *rotation curve* $V(R)$

- uniform rotation: $V = \omega_0 R \propto R$
  small V near center, only large at edge
  $\rightarrow$21 cm peak near galaxy systemic speed $V = 0$
- "flat" curve: $V(R) \rightarrow V_0$, a constant
  small V only near center, large elsewhere
  $\rightarrow$21 cm peak at $V = \pm V_0$

`www: observed 21 cm spectrum`

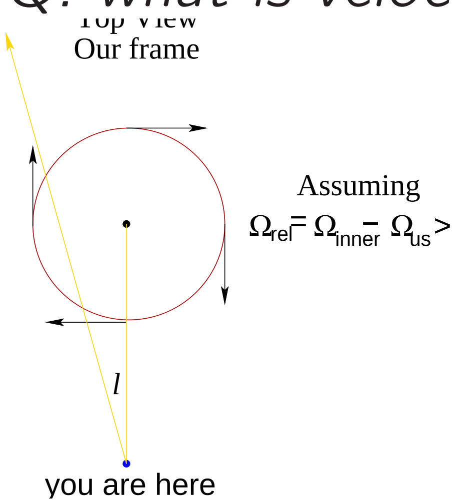
Awesome Example: the 21 cm Milky Way

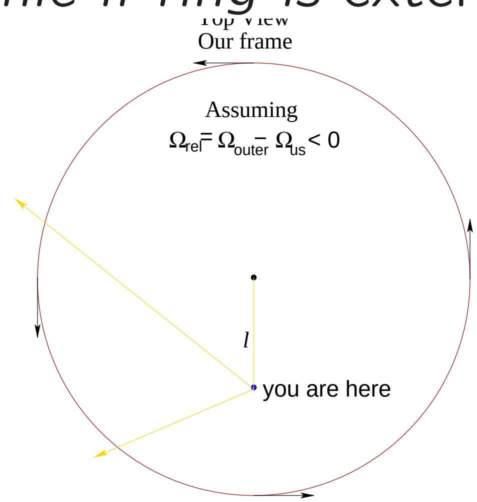
the Galactic plane is well-mapped in 21 cm

Q: what do we expect for the intensity map?

Q: what do we expect for the velocity map?

Hint: imagine single *rings* of rotating gas
Q: what is velocity profile if ring is interior *to us?*
Q: what is velocity profile if ring is exterior *to us?*





`www: observed MW velocity profile`

Awesome Example: Cosmic 21 cm Radiation

CMB today, redshift $z = 0$, has $T_{\mathrm{cmb}}(0) = 2.725\ \mathrm{K} \gg \mathrm{T}_{\mathrm{ex,21\,cm}}$
but what happens over cosmic time?

fun & fundamental cosmological result:
(relativistic) *momentum redshifts*: $p \propto 1/a(t)$, which means

$$p(z) = (1 + z)\ p(0) \tag{30}$$

where $p(0)$ is observed momentum today ($z = 0$)

why? photon or de Broglie wavelength λ is a *length*, so

$$\lambda(t) = a(t)\ \lambda_{\mathrm{emit}} = \frac{\lambda_0}{(1 + z)} \tag{31}$$

and quantum relation $p = h/\lambda$ implies $p \propto (1 + z)$

Q: implications for gas vs radiation after recombination?

Thermal History of Cosmic Gas and Radiation

until recombination (CMB formation) $z \geq z_{\rm rec} \sim 1000$
(mostly) hydrogen gas is ionized, tightly coupled to CMB
via Thomson scattering: $T_{\rm cmb} = T_{\rm gas}$

after recombination, before gas decoupling $z_{\rm dec} \sim 150 \lesssim z \leq z_{\rm rec}$

- most gas in the Universe is *neutral*
  but a small "residual" fraction $x_e \sim 10^{-5}$ of e^- remain ionized

- Thompson scattering off residual free e^- ($x_e \sim 10^{-5}$)
  still couples gas to CMB $\rightarrow$ $T_{\rm cmb} = T_{\rm gas}$ maintained
- until about $z_{\rm dec} \sim 150$, when Thomson scattering ineffective,
gas *decoupled*

Q: after decoupling, net effect of 21 cm transition?

radiation transfer along each sightline:

$$I_\nu = I_\nu^{\text{cmb}} \; e^{-\tau_\nu} + I_\nu^{\text{gas}} \; (1 - e^{-\tau_\nu}) \tag{32}$$

with τ_ν optical depth to CMB
in terms of *brightness or antenna temperature* $T_B = (c^2/2k\nu^2) I_\nu$

$$T_b = T_{\text{cmb}} \; e^{-\tau_\nu} + T_{\text{gas}}(1 - e^{-\tau_\nu}) \tag{33}$$

when $T_{\text{gas}} = T_{\text{cmb}}$ (really, $T_{\text{spin}} = T_{\text{cmb}}$)
gas is in equilibrium with CMB: emission = absorption
$\rightarrow$ $T_b = T_{\text{cmb}}$: no net effect from CMB passage through gas

after gas decoupling, before reionization $z_{\text{reion}} \sim 10 \lesssim z \leq z_{\text{dec}}$
separate thermal evolution: $T_{\text{cmb}} \sim E_{\text{peak}} \propto p_{\text{peak}} \propto (1+z)$
but matter has $T_{\text{gas}} \sim p^2/2m \propto p^2 \propto (1+z)^2$
$\rightarrow$ *gas cools* (thermal motions "redshift") *faster than the CMB!*

Q: net effect of 21 cm transitions in this epoch?

21 cm Radiation in the Dark Ages

before the first stars and quasars: **cosmic dark ages**
first structure forming, but not yet "lit up"

during dark ages: intergalactic gas has $T_{\text{gas}} < T_{\text{cmb}}$

$$\delta T_b \equiv T_b - T_{\text{cmb}} = (T_{\text{gas}} - T_{\text{cmb}})_z \, (1 - e^{-\tau_\nu})_z \tag{34}$$

we have $\delta T_b < 0$: gas seen in 21 cm *absorption*

Q: what cosmic matter will be seen this way?

Q: what will its structure be in 3-D?

Q: how will this structure be encoded in δT_b?

The "21 cm Forest"

what will absorb at 21 cm?
any neutral hydrogen in the universe!
but after recomb., most H is neutral, and most baryons are H
so absorbers are *most of the baryons in the universe*

thus absorber spatial distribution is *3D distribution of baryons*
i.e., intergalactic baryons as well as seeds of galaxies and stars!
baryons fall into potentials of dark matter halos, form galaxies
so *cosmic 21 cm traces formation of structure and galaxies!*

gas at redshift z absorbs at $\lambda(z) = (1 + z)\lambda_{\ell u}$
and o responsible for decrement $\delta T_b[\lambda(z)]$
$\rightarrow$ thus $\delta T_b(\lambda)$ *encodes redshift history* of absorbers
a sort of "21 cm forest"

Q: what about sky pattern of $\delta T_b(\lambda)$ at fixed λ?

and at fixed λ, sky map of $\delta T_b(\lambda)$
gives baryon distribution in "shell" at $1+z=\lambda/\lambda_0$
$\rightarrow$ a radial "slice" of the baryonic Universe!

so by scanning through λ, and at each
making sky maps of $\delta T_b(\lambda)$
$\rightarrow$ we build in "slices" a *3-D map of cosmic structure evolution!*
"cosmic tomography"! a cosmological gold mine!
encodes huge amounts of information

sounds amazing! and it is! but there is a catch!

Q: why is this measurement very difficult to do?
Hint: it hasn't yet been done

21 cm Cosmology: The Challenge

The 21 cm cosmic goldmine lies at redshifts $z \sim 6$ to 150 corresponding to:

- $\lambda_{\rm obs} \sim 1.5 - 30$ m
  enormous wavelengths! `www: LOFAR`
- $\nu_{\rm obs} \sim 200 - 10$ MHz
  but ionosphere opaque $> \nu_{\rm plasma} \sim 20$ MHz
  for highest z (most interesting!) have to go to space! in fact, have to go to far side of the Moon *Q: why?*
  `www: proposed lunar observatories`

But wait! It's worse!
at these wavelengths, dominant emission is *Galactic synchrotron* with brightness $T_{\rm B,synch} \sim 200 - 2000\ {\rm K} \gg T_{\rm cmb} \gg T_{\rm B,21\ cm}$

`www: radio continuum sky`

Q: implications? how to get around this?

sky intensity $T_{\rm B,synch} \sim 200 - 2000 \text{ K} \gg T_{\rm cmb}$
$\rightarrow$ Galactic synchrotron foreground dominates cosmic 21 cm
curse you, cosmic rays!

But there remains hope!
recall: cosmic-ray electron energy spectrum is a power law
so their *synchrotron spectrum is a power law*
i.e., $I_{\nu,\rm synch}$ is *smooth function of* ν

compare 21 cm at high-z: a "forest" of absorption lines
not smooth! full of spectral *lines & features*
$\rightarrow$ can hope to measure with very good spectral coverage
and foreground subtraction

also: can use spatial (i.e., angular) distribution
e.g., consider effect of first stars (likely massive) *Q: namely?*

first stars: likely massive $\rightarrow$ hot $\rightarrow$ large UV sources
ionizing photons carve out "bubble" neutral H
$\rightarrow$ corresponding to a *void* in 21 cm
$\rightarrow$ sharp bubble edges may be detectable
$\rightarrow$ 21 cm can probe *epoch of reionization*

hot, ongoing research area!

stay tuned!

Director's Cut Extras

Absorption Line Spectroscopy: Doublets

what if absorbing level ℓ has transitions to
two "neighboring" excited states u_1 and u_2?
i.e., $\ell \to u_1$ and $\ell \to u_2$ both allowed

Q: how will this imprint on the spectrum?

Q: if both lines optically thin, what is equiv width ratio W_2/W_1*?*
Q: what is notable about this?

Q: how will W_2/W_1 *ratio change with* N_ℓ*?*
Q: what is notable about this?

Doublet Line Ratios

if the states u_1 and u_2 are "neighboring" levels
then spectra will show two neighboring lines: a **doublet**

in the optically thin limit, $W \to (\pi e^2/m_e c^2)\ N_\ell\ f_{\ell u}\ \lambda_{\ell u}$
so if both lines are *optically thin*, then

$$\frac{W_2}{W_1} \approx \frac{f_{\ell u_2}\lambda_{\ell u_2}}{f_{\ell u_1}\lambda_{\ell u_1}} \tag{35}$$

• ratio set entirely by atomic line properties
• transition with *larger* $f_{\ell u_2}\lambda_{\ell u_2}$ has stronger line
sketch curve of growth for both lines

as N_ℓ increases: optical depth increases
stronger line first enters *flat part of curve of growth*
equivalent width set by thermal width: $W = 2b/c\sqrt{\ln \tau_0 / \ln 2}$
$\rightarrow$ W_2/W_1 *drops*, until weaker line also in flat part:

$$\frac{W_2}{W_1} \approx \sqrt{1 + \frac{\ln(f_{\ell u_2}\lambda_{\ell u_2}/f_{\ell u_1}\lambda_{\ell u_1})}{\ln \tau_{\ell u_1}/\ln 2}} \; \rightarrow \; 1 \tag{36}$$

$\rightarrow$ limit is *independent of atomic properties*

as absorber column N_ℓ increases further
the stronger line first enters the damping wing regime
line ratio increases again, approaching

$$\frac{W_2}{W_1} \approx \frac{\lambda_{\ell u_2}}{\lambda_{\ell u_1}} \sqrt{\frac{f_{\ell u_2}\Gamma_{\ell u_2}}{f_{\ell u_1}\Gamma_{\ell u_1}}} \tag{37}$$

$\rightarrow$ set by atomic properties and *total damping width*

Q: why is all of this useful?

thus *doublet ratio is diagnostic of curve-of-growth regime*

Strategy:

compare observed W_2/W_1 value to

optically thin limiting value $f_{\ell u_2}\lambda_{\ell u_2}/f_{\ell u_1}\lambda_{\ell u_1}$

- agreement verifies both lines optically thin
- $W_2/W_1 \approx 1$ is warning: flat part of curve of growth
- $W_2/W_1 \to$ damping wing ratio gives $\Gamma_{\ell u_2}/\Gamma_{\ell u_1}$ and thus collisional broadening & hence pressure/density