Astronomy 501: Radiative Processes

Lecture 28

Oct 31, 2022

Announcements:

- **Problem Set 9 due Friday**
  as always when number crunching: I recommend writing a small code or spreadsheet

Last time: began applications for spectral lines

Today:

- hydrogen spectral features and stellar spectra
- how to measure interstellar atomic hydrogen

Atomic Hydrogen: the Lyman Series and Lyman Limit

neutral hydrogen in the ground state has transitions to all excited states; lines form the **Lyman series**

`www: Grotrian diagram`

- Lyman-α: $n = 1 \leftrightarrow 2$; $\lambda_{\mathrm{Ly}\alpha} = 1215.67$ Å
  Q: EM regime? implications?
- Lyβ: $n = 1 \leftrightarrow 3$; Lyγ: $n = 1 \leftrightarrow 4$, etc

Lyman series line $n = 1 \leftrightarrow n_f$ has

$$\lambda_{n_f} = \left(1 - \frac{1}{n_f^2}\right) \frac{hc}{B_{\mathrm{H}}} \quad (1)$$

for $n_f \gg 1$, line *pileup* of lines
at **Lyman limit** $\lambda_{n_f \to \infty} = hc/B_{\mathrm{H}} = 911.75$ Å

but note: real spectra do not show infinite lines near Ly limit
Q: Why not? What will we really see?

The Lyman Limit

there are infinitely many transitions near the Lyman limit
but each line has a *finite width*
typically due to thermal broadening

lines overlap and *blend* when

$$\text{line width} \lesssim \text{line spacing} \tag{2}$$

$$\frac{v_{\rm FWHM}}{c}\lambda_n \lesssim \lambda_n - \lambda_{n-1} \approx \frac{\partial \lambda_n}{\partial n} \tag{3}$$

occurs when $1/n^3 \lesssim v_{\rm FWHM}/c$, and thus
quantum number $n \gtrsim 67\ (2\ \text{km s}^{-1}/v_{\rm FWHM})^{1/3}$, and wavelength

$$\lambda \approx 911.75 + 0.2 \left(\frac{v_{\rm FWHM}}{2\ \text{km/s}}\right)^{2/3}\ \text{Å} \tag{4}$$

`www: Lyman limit QSO absorption line system`

Q: what about atoms from elements with $\lambda(n = 1 \rightarrow 2) <$ 912 Å?

The Balmer Jump

Balmer series: hydrogen transtions between $n_i = 2$ and $n_f > 2$

$$\lambda(\mathrm{H}_{n_f}) = \frac{hc}{\Delta E} = \left(1 - \frac{4}{n_f^2}\right) \frac{4hc}{B_\mathrm{H}} \tag{5}$$

to see in stellar absorption requires $n = 2$ population
and thus high T, since $N_2/N_1 = 8e^{-3B/4kT}$
PS8: but if $T \gtrsim B$, more likely to ionize H entirely!
so Balmer series prominent around $T \sim 10^4$K

where Balmer lines present
photons blueward of "Balmer limit" $4hc/B = 4\lambda_{\mathrm{Ly\,limit}} = 3647$ Å
ionize the $n = 2$ atoms - readily absorbed, "bound-free"
so stellar opacity increases below this limit: **Balmer jump**
less severe cousin of Lyman limit
`www: stellar spectra`

Atomic Resonance Lines

for neutral atoms, *permitted* lines due to transitions from the *ground state* are called **resonance lines** – easiest to excite in neutral matter

we saw: neutral H atoms absorb all photons with $\lambda < 912$ Å in *bound-bound transitions* to $n_f \gg 1$ for λ near limit or for smaller λ, in *bound-free transitions* that ionize H

but H is the most abundant element in the Universe!

thus for atoms with all resonance lines $\lambda < 912$ Å
local hydrogen absorbs photons that can drive these transition
$\rightarrow$ *cannot observe these elements in sightlines with H I!*
Q: are any atoms excluded this way? if so, which?

Good news:
the only atoms excluded this way are He and Ne
noble gasses, first excited states at very high energies
all other atoms have resonance lines $\lambda > 912$ Å!

Bad news:
for most atoms and ions, resonance lines have $\lambda < 3000$ Å
Q: why is this bad?
Q: how to get around the badness?

"Metal" Lines

quaint astro-lingo: **"metal"** any element $\neq$ H, He
including, e.g., famous "metals" C, N, O

most metal atoms and ions have resonance lines $\lambda < 3000$ Å
this is in the UV, blocked by Earth's atmosphere
`www: atmosphere transmittance`
for such species, can only see absorption lines:

- by going to space
- when looking at high-redshift objects
  `www: metal lines in QSO spectrum`

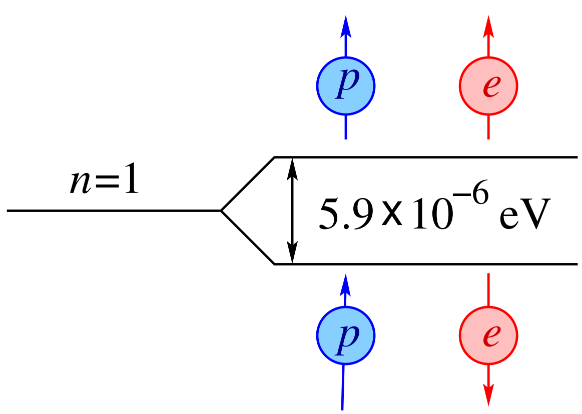
but nature has not been totally unkind
a few atoms and ions have resonance lines with $\lambda > 3000$ Å
examples: Na I D doublet $\lambda = 5891.6, 5897.6$ Å,
Ca II doublet at $3934.8, 3969.6$ Å
`www: solar lines`

Atomic Hydrogen: Hyperfile Splitting

in *hydrogen*, both e and p have spin $S = 1/2$ (fermions!)
coupled via *spin-spin* or *hyperfine* interaction
with Hamiltonian $H_{\text{spin-spin}} = H_{\text{hf}}\ \vec{s}_e \cdot \vec{s}_p$
radiation is *magnetic dipole*

hydrogen ground state has two possible *spin configurations*

- proton and electron spins *parallel*: $\uparrow_e \uparrow_p$
  excited state: $S_u = 1$, $g_u = 3$
- spins *antiparallel*: $\downarrow_e \uparrow_p$
  ground state: $S_\ell = 0$, $g_\ell = 1$

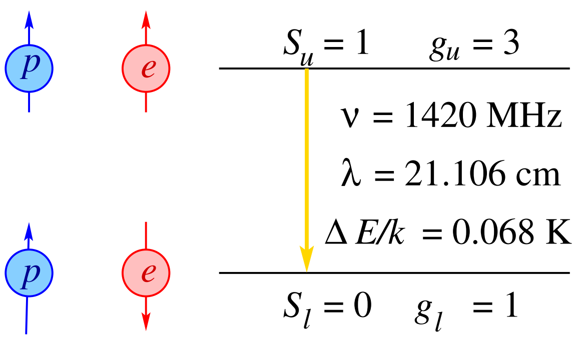


Q: and so...?

Atomic Hydrogen: the 21 cm Line

transition $u \to \ell$ requires electron *spin flip* $\Delta s \neq 0$, $\Delta n = \Delta \ell = 0$

H I hyperfine spin–flip transition



$$A_{u\ell} = 2.8843 \times 10^{-15}\ \text{s}^{-1} = (11.0\ \text{Myr})^{-1}$$

$$\Delta E = E_u - E_\ell = 5.86 \times 10^{-6}\ \text{eV} = k_\text{B}\,(0.06816\ \text{K})$$

$$\nu_{u\ell} = 1420.4\ \text{MHZ} \qquad \lambda_{u\ell} = 21.106\ \text{cm}$$

implications of A value? ΔE? λ?

Einstein coefficient:
$A_{u\ell} = (11.0\ \text{Myr})^{-1}$: *very slow rate*

- spontaneous emission only occurs after a ~ 11 Myr if the atoms has been *undisturbed*: no collisions!
- spontaneous emission never observed in the laboratory! can only measure transition via stimulated emission!
- but can occur in low-density astrophysical environment
- excited state lifetime $A^{-1} \ll$ age of Universe
  $\rightarrow$ need some collisions to replenish excited state

EM regime:
$\nu = 1420.4$ MHZ and $\lambda = 21.106$ cm:
"21 cm radiation" in **radio**

Thermal Properties:
$\Delta E/k_{\mathrm{B}} = 0.06816$ K small splitting
$\rightarrow$ easy to thermally populate excited state
Q: recall that today, $T_{\mathrm{cmb}} = 2.725$ *K; implications?*

Spin Temperature

the CMB has $T_{\text{CMB}} \gg \Delta E/k \rightarrow$ can populate upper level!

if states in thermal equilibrium at *excitation* or *spin temperature* with $T_{\text{ex}} \equiv T_{\text{spin}} \gg \Delta E/k$, then

$$\frac{n_u}{n_\ell} = \frac{g_u}{g_\ell} e^{-h\nu_{u\ell}/kT_{\text{spin}}} \approx \frac{g_u}{g_\ell} = 3 \tag{6}$$

a nearly fixed ratio *independent of temperature*, so that

$$n_u \approx \frac{3}{4} n(\text{H I}) \ , \quad n_\ell \approx \frac{1}{4} n(\text{H I}) \tag{7}$$

thus: 21-cm emissivity also independent of spin temperature

$$j_\nu = n_u \frac{A_{u\ell}}{4\pi} h\nu_{u\ell} \ \phi_\nu \approx \frac{3}{16\pi} A_{u\ell} \ h\nu_{u\ell} \ n(\text{H I}) \ \phi_\nu \tag{8}$$

Q: absorption coefficient?

21-cm Absorption Coefficient

as usual, absorption coefficient has true and stimulated terms:

$$\alpha_\nu = n_\ell \sigma_{\ell u} - n_u \sigma_{\ell u} \quad (9)$$

$$= n_\ell \frac{g_u}{g_\ell} \frac{A_{u\ell}}{8\pi} \lambda_{u\ell}^2 \ \phi_\nu \ \left[1 - \frac{n_u}{n_\ell} \frac{g_\ell}{g_u} \right] \quad (10)$$

$$= n_\ell \frac{g_u}{g_\ell} \frac{A_{u\ell}}{8\pi} \lambda_{u\ell}^2 \ \phi_\nu \ \left[1 - e^{-h\nu_{u\ell}/kT_{\rm spin}} \right] \quad (11)$$

but in practice we always have $e^{-h\nu_{u\ell}/kT_{\rm spin}} \approx 1$, so
stimulated emission correction is very important!

using $e^{-h\nu_{u\ell}/kT_{\rm spin}} \approx 1 - h\nu_{u\ell}/kT_{\rm spin}$, we have

$$\alpha_\nu \approx n_\ell \frac{3}{32\pi} A_{u\ell} \ \frac{hc\lambda_{u\ell}}{kT_{\rm spin}} n(\text{H I}) \ \phi_\nu \quad (12)$$

12 and thus $\alpha_\nu \propto 1/T_{\rm spin}$

Q: what determines ϕ_ν in practice?

since $A = \Gamma$ is very small, 21-cm line intrinsically very narrow
$\rightarrow$ width entirely determined by *velocity dispersion* of the emitting hydrogen

for a random, Gaussian velocity distribution

$$\phi_\nu = \frac{1}{\sqrt{2\pi}} \frac{c}{\nu_{u\ell}} \frac{1}{\sigma_v} e^{-u^2/2\sigma_v^2} \tag{13}$$

with $u = c(\nu_{u\ell} - \nu)/\nu_{u\ell}$, we have

$$\alpha_\nu \approx n_\ell \frac{3}{32\pi} \frac{1}{\sqrt{2\pi}} \frac{A_{u\ell}\lambda_{u\ell}^2}{\sigma_v} \frac{hc}{kT_{\rm spin}} n(\text{H I})\ e^{-u^2/2\sigma_v^2} \tag{14}$$

and optical depth

$$\tau_\nu = 2.190\ \left(\frac{N(\text{H I})}{10^{21}\ cm^{-2}}\right) \left(\frac{100\ \text{K}}{T_{\rm spin}}\right) \left(\frac{1\ \text{km/s}}{\sigma_v}\right)\ e^{-u^2/2\sigma_v^2} \tag{15}$$

Q: implications?

21 cm Emission: Optically Thin Case

21 cm optical depth:

$$\tau_\nu = 2.190 \left(\frac{N(\text{H I})}{10^{21}\ cm^{-2}}\right)\left(\frac{100\text{ K}}{T_{\text{spin}}}\right)\left(\frac{1\text{ km/s}}{\sigma_v}\right)\ e^{-u^2/2\sigma_v^2} \tag{16}$$

real interstellar lines of sight can have $N(\text{H I}) > 10^{21}\ cm^{-2}$
$\rightarrow$ *self-absorption can be important!*

But in the optically thin limit, for $N(\text{H I}) \lesssim 10^{20}\ \text{cm}^{-2}$
then absorption is small and

$$I_\nu \approx I_\nu(0) + \int j_\nu\ ds = I_\nu(0) + \frac{3}{16\pi} A_{u\ell}\ h\nu_{u\ell}\ N(\text{H I})\ \phi_\nu \tag{17}$$

with $N(\text{H I}) = \int n_{\text{H I}}\ ds$

if $I_\nu(0)$ is known *Q: how?*, then

$$\int \left[I_\nu - I_\nu(0)\right]\ d\nu = \frac{3}{16\pi} A_{u\ell}\ h\nu_{u\ell}\ N(\text{H I}) \tag{18}$$

in terms of antenna temperature, integrating in velocity space

$$\int [T_\mathrm{A} - T_\mathrm{A}(0)] \; du = \int \frac{c^2}{2k\nu^2} [T_\mathrm{A} - T_\mathrm{A}(0)] \; c \frac{d\nu}{\nu} = \frac{3}{16\pi} A_{u\ell} \frac{hc\lambda_{u\ell}^2}{k} N(\mathrm{H\ I})$$

measures *hydrogen column* $N(\mathrm{H\ I})$ independent of spin temperature!

integrating over solid angles gives flux density

$$F_\mathrm{obs} = \int F_\nu \; d\nu = \int I_\nu \cos\theta \; d\Omega \; d\nu \approx \int I_\nu \; d\Omega \; d\nu \tag{19}$$

and thus the integrated flux

$$F_\mathrm{obs} \propto \int N(\mathrm{H\ I}) \; d\Omega = \frac{\int n_\mathrm{H\ I} \; ds \; dA}{D_L^2} \propto \frac{M_\mathrm{H\ I}}{D_L^2} \tag{20}$$

measures the *total hydrogen mass* $M_\mathrm{H\ I}$
if we know the (luminosity) distance D_L

useful for H I clouds in our own Galaxy,
and measuring H I content of external galaxies

consider cold, diffuse atomic H in a galaxy that has bulk internal motions with speeds $v_{\mathrm{bulk}} > \sigma_v$

Q: how would this arise?

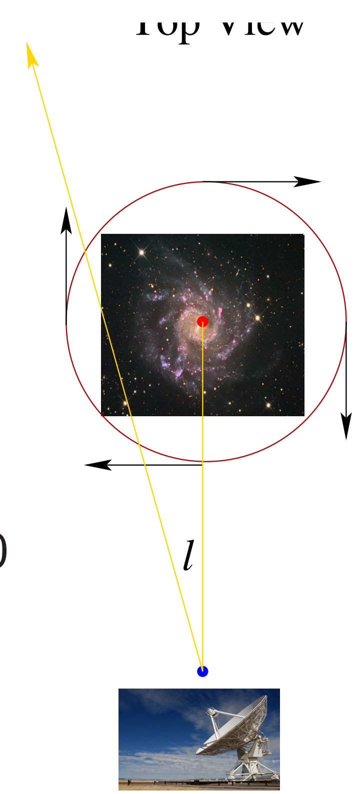
Q: what spectral pattern would uniform rotation give?

Q: what is a more realistic expectation?

Awesome Example: Galaxies in 21 cm

spiral galaxies observed in 21 cm emission, ellipticals are not
$\rightarrow$ spirals are gas rich, ellipticals gas poor `www: THINGS survey`

spiral galaxies also rotate:
bulk line-of-sight motion imprinted on 21 cm
via Doppler shift at different sightlines



spectrum depends on *rotation curve* $V(R)$

- uniform rotation: $V = \omega_0 R \propto R$
  small V near center, only large at edge
  $\rightarrow$21 cm peak near galaxy systemic speed $V = 0$
- "flat" curve: $V(R) \rightarrow V_0$, a constant
  small V only near center, large elsewhere
  $\rightarrow$21 cm peak at $V = \pm V_0$

`www: observed 21 cm spectrum`

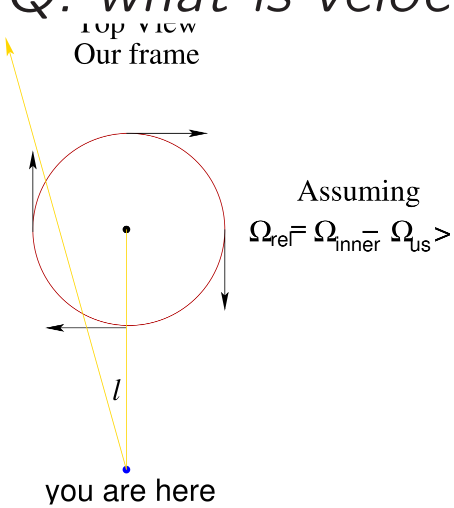
Awesome Example: the 21 cm Milky Way

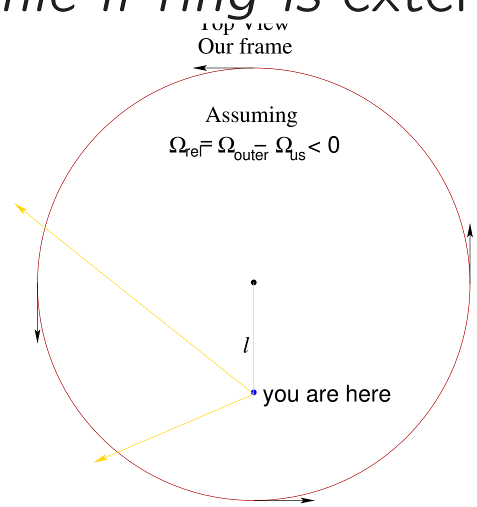
the Galactic plane is well-mapped in 21 cm

Q: what do we expect for the intensity map?

Q: what do we expect for the velocity map?

Hint: imagine single *rings* of rotating gas
Q: what is velocity profile if ring is interior *to us?*
Q: what is velocity profile if ring is exterior *to us?*





`www: observed MW velocity profile`

Awesome Example: Cosmic 21 cm Radiation

CMB today, redshift $z = 0$, has $T_{\rm cmb}(0) = 2.725\ {\rm K} \gg {\rm T}_{\rm ex,21\,cm}$
but what happens over cosmic time?

fun & fundamental cosmological result:
(relativistic) *momentum redshifts*: $p \propto 1/a(t)$, which means

$$p(z) = (1 + z)\ p(0) \tag{21}$$

where $p(0)$ is observed momentum today ($z = 0$)

why? photon or de Broglie wavelength λ is a *length*, so

$$\lambda(t) = a(t)\ \lambda_{\rm emit} = \frac{\lambda_0}{(1 + z)} \tag{22}$$

and quantum relation $p = h/\lambda$ implies $p \propto (1 + z)$

Q: implications for gas vs radiation after recombination?

Thermal History of Cosmic Gas and Radiation

until recombination (CMB formation) $z \geq z_{\text{rec}} \sim 1000$
(mostly) hydrogen gas is ionized, tightly coupled to CMB
via Thomson scattering: $T_{\text{cmb}} = T_{\text{gas}}$

after recombination, before gas decoupling $z_{\text{dec}} \sim 150 \lesssim z \leq z_{\text{rec}}$

- most gas in the Universe is *neutral*
  but a small "residual" fraction $x_e \sim 10^{-5}$ of e^- remain ionized

- Thompson scattering off residual free e^- ($x_e \sim 10^{-5}$)
  still couples gas to CMB $\rightarrow$ $T_{\text{cmb}} = T_{\text{gas}}$ maintained
- until about $z_{\text{dec}} \sim 150$, when Thomson scattering ineffective,

gas *decoupled*

Q: after decoupling, net effect of 21 cm transition?

radiation transfer along each sightline:

$$I_\nu = I_\nu^{\text{cmb}} \ e^{-\tau_\nu} + I_\nu^{\text{gas}} \ (1 - e^{-\tau_\nu}) \tag{23}$$

with τ_ν optical depth to CMB
in terms of *brightness or antenna temperature* $T_B = (c^2/2k\nu^2) I_\nu$

$$T_b = T_{\text{cmb}} \ e^{-\tau_\nu} + T_{\text{gas}}(1 - e^{-\tau_\nu}) \tag{24}$$

when $T_{\text{gas}} = T_{\text{cmb}}$ (really, $T_{\text{spin}} = T_{\text{cmb}}$)
gas is in equilibrium with CMB: emission = absorption
$\rightarrow$ $T_b = T_{\text{cmb}}$: no net effect from CMB passage through gas

after gas decoupling, before reionization $z_{\text{reion}} \sim 10 \lesssim z \leq z_{\text{dec}}$
separate thermal evolution: $T_{\text{cmb}} \sim E_{\text{peak}} \propto p_{\text{peak}} \propto (1+z)$
but matter has $T_{\text{gas}} \sim p^2/2m \propto p^2 \propto (1+z)^2$
$\rightarrow$ *gas cools* (thermal motions "redshift") *faster than the CMB!*

Q: net effect of 21 cm transitions in this epoch?

21 cm Radiation in the Dark Ages

before the first stars and quasars: **cosmic dark ages**
first structure forming, but not yet "lit up"

during dark ages: intergalactic gas has $T_{\rm gas} < T_{\rm cmb}$

$$\delta T_b \equiv T_b - T_{\rm cmb} = (T_{\rm gas} - T_{\rm cmb})_z \, (1 - e^{-\tau_\nu})_z \tag{25}$$

we have $\delta T_b < 0$: gas seen in 21 cm *absorption*

Q: what cosmic matter will be seen this way?

Q: what will its structure be in 3-D?

Q: how will this structure be encoded in δT_b?

The "21 cm Forest"

what will absorb at 21 cm?
any neutral hydrogen in the universe!
but after recomb., most H is neutral, and most baryons are H
so absorbers are *most of the baryons in the universe*

thus absorber spatial distribution is *3D distribution of baryons*
i.e., intergalactic baryons as well as seeds of galaxies and stars!
baryons fall into potentials of dark matter halos, form galaxies
so *cosmic 21 cm traces formation of structure and galaxies!*

gas at redshift z absorbs at $\lambda(z) = (1+z)\lambda_{\ell u}$
and o responsible for decrement $\delta T_b[\lambda(z)]$
$\rightarrow$ thus $\delta T_b(\lambda)$ *encodes redshift history* of absorbers
a sort of "21 cm forest"

Q: what about sky pattern of $\delta T_b(\lambda)$ *at fixed* λ*?*

and at fixed λ, sky map of $\delta T_b(\lambda)$
gives baryon distribution in "shell" at $1+z=\lambda/\lambda_0$
$\rightarrow$ a radial "slice" of the baryonic Universe!

so by scanning through λ, and at each
making sky maps of $\delta T_b(\lambda)$
$\rightarrow$ we build in "slices" a *3-D map of cosmic structure evolution!*
"cosmic tomography"! a cosmological gold mine!
encodes huge amounts of information

sounds amazing! and it is! but there is a catch!

Q: why is this measurement very difficult to do?
Hint: it hasn't yet been done

21 cm Cosmology: The Challenge

The 21 cm cosmic goldmine lies at redshifts $z \sim 6$ to 150 corresponding to:

- $\lambda_{\text{obs}} \sim 1.5 - 30$ m
  enormous wavelengths! `www: LOFAR`
- $\nu_{\text{obs}} \sim 200 - 10$ MHz
  but ionosphere opaque $> \nu_{\text{plasma}} \sim 20$ MHz
  for highest z (most interesting!) have to go to space! in fact, have to go to far side of the Moon *Q: why?*
  `www: proposed lunar observatories`

But wait! It's worse!
at these wavelengths, dominant emission is *Galactic synchrotron* with brightness $T_{\text{B,synch}} \sim 200 - 2000 \text{ K} \gg T_{\text{cmb}} \gg T_{\text{B,21 cm}}$

`www: radio continuum sky`

Q: implications? how to get around this?

sky intensity $T_{\rm B,synch} \sim 200 - 2000\ {\rm K} \gg T_{\rm cmb}$
$\rightarrow$ Galactic synchrotron foreground dominates cosmic 21 cm
curse you, cosmic rays!

But there remains hope!
recall: cosmic-ray electron energy spectrum is a power law
so their *synchrotron spectrum is a power law*
i.e., $I_{\nu,\rm synch}$ is *smooth function of* ν

compare 21 cm at high-z: a "forest" of absorption lines
not smooth! full of spectral *lines & features*
$\rightarrow$ can hope to measure with very good spectral coverage
and foreground subtraction

also: can use spatial (i.e., angular) distribution
e.g., consider effect of first stars (likely massive) *Q: namely?*

first stars: likely massive $\rightarrow$ hot $\rightarrow$ large UV sources

ionizing photons carve out "bubble" neutral H

$\rightarrow$ corresponding to a *void* in 21 cm

$\rightarrow$ sharp bubble edges may be detectable

$\rightarrow$ 21 cm can probe *epoch of reionization*

hot, ongoing research area!

stay tuned!