Astronomy 501: Radiative Processes
Lecture 30
Nov 2, 2022

Announcements:

- **Problem Set 9 due Friday**
  as always when number crunching: I recommend
  writing a small code or spreadsheet
- Office Hours after class (until 3:30 today only), or by appt
- Proposal: make a A501-Themed Meme
  SFW, uses course topics and memes correctly
  if there is interest: competition, exhibition, prizes, small bonus

Last time: resonance lines `www: atmospheric transmission`

Today: the atomic hydrogen sky at 21 cm

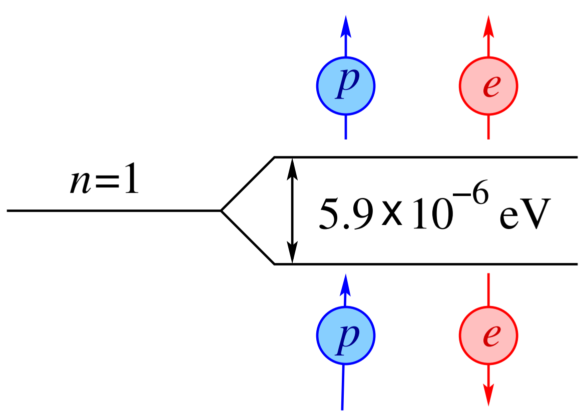
Q: zoom into H ground state, and...?

Atomic Hydrogen: Hyperfile Splitting

in *hydrogen*, both e and p have spin $S = 1/2$ (fermions!)
coupled via *spin-spin* or *hyperfine* interaction
with Hamiltonian $H_{\mathrm{spin-spin}} = H_{\mathrm{hf}}\ \vec{s}_e \cdot \vec{s}_p$
radiation is *magnetic dipole*

hydrogen ground state has two possible *spin configurations*

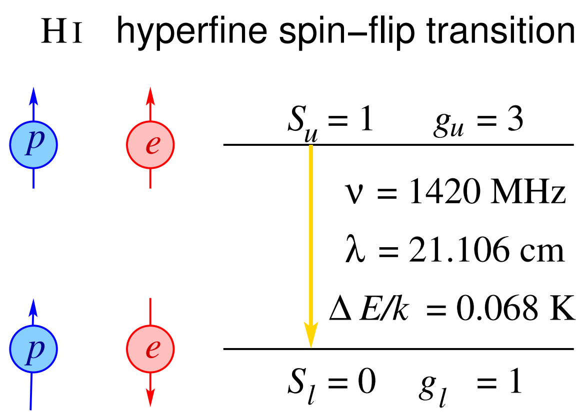
- proton and electron spins *parallel*: $\uparrow_e \uparrow_p$
  excited state: $S_u = 1$, $g_u = 3$
- spins *antiparallel*: $\downarrow_e \uparrow_p$
  ground state: $S_\ell = 0$, $g_\ell = 1$



Q: and so...?

Atomic Hydrogen: the 21 cm Line

transition $u \to \ell$ requires electron *spin flip* $\Delta s \neq 0$, $\Delta n = \Delta \ell = 0$



$$A_{u\ell} = 2.8843 \times 10^{-15}\ \mathrm{s}^{-1} = (11.0\ \mathrm{Myr})^{-1}$$

$$\Delta E = E_u - E_\ell = 5.86 \times 10^{-6}\ \mathrm{eV} = k_{\mathrm{B}}\,(0.06816\ \mathrm{K})$$

$$\nu_{u\ell} = 1420.4\ \mathrm{MHZ} \qquad \lambda_{u\ell} = 21.106\ \mathrm{cm}$$

ω *implications of A value? ΔE? λ?*

Einstein coefficient:

$A_{u\ell} = (11.0\ \text{Myr})^{-1}$: *very slow rate*

- spontaneous emission only occurs after a ~ 11 Myr if the atoms has been *undisturbed*: no collisions!
- spontaneous emission never observed in the laboratory! can only measure transition via stimulated emission!
- but can occur in low-density astrophysical environment
- excited state lifetime $A^{-1} \ll$ age of Universe $\rightarrow$ need some collisions to replenish excited state

EM regime:

$\nu = 1420.4$ MHZ and $\lambda = 21.106$ cm:

"21 cm radiation" in **radio**

Thermal Properties:

$\Delta E/k_{\rm B} = 0.06816$ K small splitting

$\rightarrow$ easy to thermally populate excited state

Q: recall that today, $T_{\rm cmb} = 2.725$ K; implications?

Spin Temperature

the CMB has $T_{\text{CMB}} \gg \Delta E/k \rightarrow$ can populate upper level!

if states in thermal equilibrium at *excitation* or *spin temperature* with $T_{\text{ex}} \equiv T_{\text{spin}} \gg \Delta E/k$, then

$$\frac{n_u}{n_\ell} = \frac{g_u}{g_\ell} e^{-h\nu_{u\ell}/kT_{\text{spin}}} \approx \frac{g_u}{g_\ell} = 3 \tag{1}$$

a nearly fixed ratio *independent of temperature*, so that

$$n_u \approx \frac{3}{4} n(\text{H I}) \ , \quad n_\ell \approx \frac{1}{4} n(\text{H I}) \tag{2}$$

thus: 21-cm emissivity also independent of spin temperature

$$j_\nu = n_u \frac{A_{u\ell}}{4\pi} h\nu_{u\ell} \ \phi_\nu \approx \frac{3}{16\pi} A_{u\ell} \ h\nu_{u\ell} \ n(\text{H I}) \ \phi_\nu \tag{3}$$

`www: 21 cm sky` *Q: coordinates? features?*

Q: absorption coefficient?

21-cm Absorption Coefficient

as usual, absorption coefficient has true and stimulated terms:

$$\begin{aligned}
\alpha_\nu &= n_\ell \sigma_{\ell u} - n_u \sigma_{\ell u} &(4)\\
&= n_\ell \frac{g_u}{g_\ell} \frac{A_{u\ell}}{8\pi} \lambda_{u\ell}^2 \ \phi_\nu \ \left[1 - \frac{n_u}{n_\ell} \frac{g_\ell}{g_u}\right] &(5)\\
&= n_\ell \frac{g_u}{g_\ell} \frac{A_{u\ell}}{8\pi} \lambda_{u\ell}^2 \ \phi_\nu \ \left[1 - e^{-h\nu_{u\ell}/kT_{\rm spin}}\right] &(6)
\end{aligned}$$

but in practice we always have $e^{-h\nu_{u\ell}/kT_{\rm spin}} \approx 1$, so
stimulated emission correction is very important!

using $e^{-h\nu_{u\ell}/kT_{\rm spin}} \approx 1 - h\nu_{u\ell}/kT_{\rm spin}$, we have

$$\alpha_\nu \approx n_\ell \frac{3}{32\pi} A_{u\ell} \ \frac{hc\lambda_{u\ell}}{kT_{\rm spin}} n(\text{H I}) \ \phi_\nu \qquad (7)$$

and thus $\alpha_\nu \propto 1/T_{\rm spin}$

Q: what determines ϕ_ν in practice?

since $A = \Gamma$ is very small, 21-cm line intrinsically very narrow
$\rightarrow$ width entirely determined by *velocity dispersion*
of the emitting hydrogen

for a random, Gaussian velocity distribution

$$\phi_\nu = \frac{1}{\sqrt{2\pi}} \frac{c}{\nu_{u\ell}} \frac{1}{\sigma_v} e^{-u^2/2\sigma_v^2} \tag{8}$$

with $u = c(\nu_{u\ell} - \nu)/\nu_{u\ell}$, we have

$$\alpha_\nu \approx n_\ell \frac{3}{32\pi} \frac{1}{\sqrt{2\pi}} \frac{A_{u\ell}\lambda_{u\ell}^2}{\sigma_v} \frac{hc}{kT_{\rm spin}} n(\text{H I})\ e^{-u^2/2\sigma_v^2} \tag{9}$$

and optical depth

$$\tau_\nu = 2.190\ \left(\frac{N(\text{H I})}{10^{21}\ cm^{-2}}\right) \left(\frac{100\text{ K}}{T_{\rm spin}}\right) \left(\frac{1\text{ km/s}}{\sigma_v}\right)\ e^{-u^2/2\sigma_v^2} \tag{10}$$

Q: implications?

21 cm Emission: Optically Thin Case

21 cm optical depth:

$$\tau_\nu = 2.190 \left(\frac{N(\text{H I})}{10^{21}\ cm^{-2}}\right)\left(\frac{100\ \text{K}}{T_\text{spin}}\right)\left(\frac{1\ \text{km/s}}{\sigma_v}\right)\ e^{-u^2/2\sigma_v^2} \tag{11}$$

real interstellar lines of sight can have $N(\text{H I}) > 10^{21}\ cm^{-2}$
$\rightarrow$ *self-absorption can be important!*

But in the optically thin limit, for $N(\text{H I}) \lesssim 10^{20}\ \text{cm}^{-2}$
then absorption is small and

$$I_\nu \approx I_\nu(0) + \int j_\nu\ ds = I_\nu(0) + \frac{3}{16\pi} A_{u\ell}\ h\nu_{u\ell}\ N(\text{H I})\ \phi_\nu \tag{12}$$

with $N(\text{H I}) = \int n_{\text{H I}}\ ds$

if $I_\nu(0)$ is known *Q: how?*, then

$$\int [I_\nu - I_\nu(0)]\ d\nu = \frac{3}{16\pi} A_{u\ell}\ h\nu_{u\ell}\ N(\text{H I}) \tag{13}$$

in terms of antenna temperature, integrating in velocity space

$$\int [T_{\mathsf{A}} - T_{\mathsf{A}}(0)] \; du = \int \frac{c^2}{2k\nu^2} [T_{\mathsf{A}} - T_{\mathsf{A}}(0)] \; c \, \frac{d\nu}{\nu} = \frac{3}{16\pi} A_{u\ell} \frac{hc\lambda_{u\ell}^2}{k} \, N(\mathrm{H\ I})$$

measures *hydrogen column* $N(\mathrm{H\ I})$ independent of spin temperature!

integrating over solid angles gives flux density

$$F_{\mathrm{obs}} = \int F_\nu \; d\nu = \int I_\nu \cos\theta \; d\Omega \; d\nu \approx \int I_\nu \; d\Omega \; d\nu \qquad (14)$$

and thus the integrated flux

$$F_{\mathrm{obs}} \propto \int N(\mathrm{H\ I}) \; d\Omega = \frac{\int n_{\mathrm{H\ I}} \; ds \; dA}{D_L^2} \propto \frac{M_{\mathrm{H\ I}}}{D_L^2} \qquad (15)$$

measures the *total hydrogen mass* $M_{\mathrm{H\ I}}$
if we know the (luminosity) distance D_L

useful for H I clouds in our own Galaxy,
and measuring H I content of external galaxies

consider cold, diffuse atomic H in a galaxy that has bulk internal motions with speeds $v_{\text{bulk}} > \sigma_v$

Q: how would this arise?

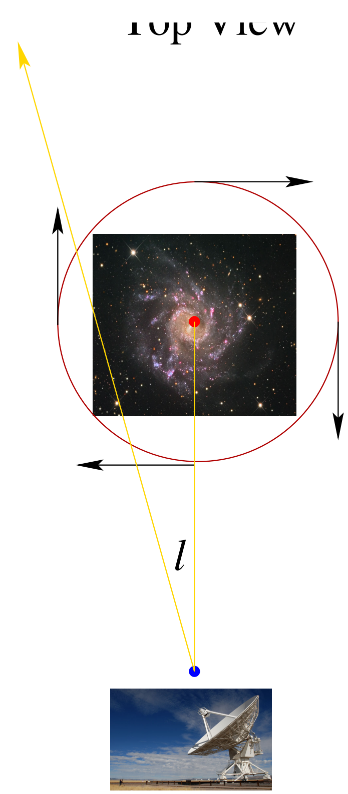
Q: what spectral pattern would uniform rotation give?

Q: what is a more realistic expectation?

Awesome Example: Galaxies in 21 cm

spiral galaxies observed in 21 cm emission, ellipticals are not
$\rightarrow$ spirals are gas rich, ellipticals gas poor `www: THINGS survey`

spiral galaxies also rotate:
bulk line-of-sight motion imprinted on 21 cm
via Doppler shift at different sightlines



spectrum depends on *rotation curve* $V(R)$

- uniform rotation: $V = \omega_0 R \propto R$
  small V near center, only large at edge
  $\rightarrow$21 cm peak near galaxy systemic speed $V = 0$
- "flat" curve: $V(R) \rightarrow V_0$, a constant
  small V only near center, large elsewhere
  $\rightarrow$21 cm peak at $V = \pm V_0$

`www: observed 21 cm spectrum`

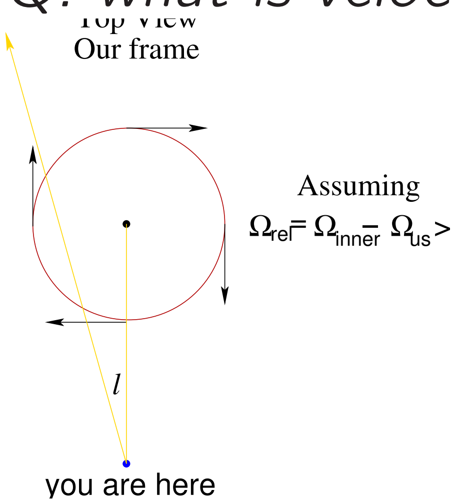
Awesome Example: the 21 cm Milky Way

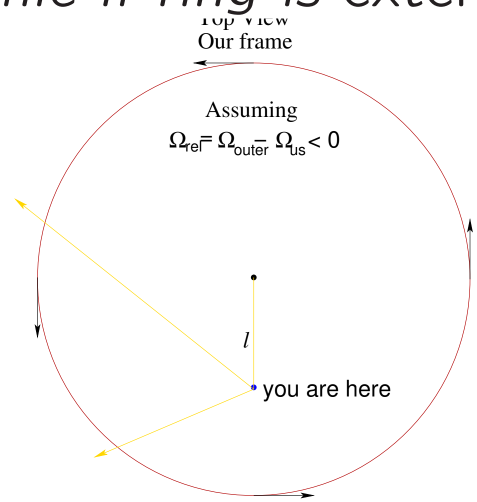
the Galactic plane is well-mapped in 21 cm

Q: what do we expect for the intensity map?

Q: what do we expect for the velocity map?

Hint: imagine single *rings* of rotating gas
Q: what is velocity profile if ring is interior *to us?*
Q: what is velocity profile if ring is exterior *to us?*





`www: observed MW velocity profile`

Awesome Example: Cosmic 21 cm Radiation

radiation transfer along each sightline, at rest-frame 21cm:

$$I = B_\nu(T_{\mathrm{cmb}})\ e^{-\tau_\nu} + B_\nu(T_{\mathrm{gas}})\ (1 - e^{-\tau_\nu}) \tag{16}$$

in terms of antenna temperature

$$T_b = T_{\mathrm{cmb}}\ e^{-\tau_\nu} + T_{\mathrm{gas}}(1 - e^{-\tau_\nu}) \tag{17}$$

so: if $T_{\mathrm{gas}} = T_{\mathrm{cmb}}$ - no signal!

otherwise: emission or absorpbtion
depending on $T_{\mathrm{gas}} - T_{\mathrm{cmb}}$ sign

$$T_b = T_{\text{cmb}}\ e^{-\tau_\nu} + T_{\text{gas}}(1 - e^{-\tau_\nu}) \tag{18}$$

after gas decoupling, before reionization $z_{\text{reion}} \sim 10 \lesssim z \leq z_{\text{dec}}$
before the first stars and quasars: **cosmic dark ages**
first structure forming, but not yet "lit up"

during dark ages: intergalactic gas has $T_{\text{gas}} < T_{\text{cmb}}$

$$\delta T_b \equiv T_b - T_{\text{cmb}} = (T_{\text{gas}} - T_{\text{cmb}})_z\,(1 - e^{-\tau_\nu})_z \tag{19}$$

we have $\delta T_b < 0$: gas seen in 21 cm *absorption*

Q: what cosmic matter will be seen this way?
Q: what will its structure be in 3-D?
Q: how will this structure be encoded in δT_b*?*

The "21 cm Forest"

what will absorb at 21 cm?
any neutral hydrogen in the universe!
but after recomb., most H is neutral, and most baryons are H
so absorbers are *most of the baryons in the universe*

thus absorber spatial distribution is *3D distribution of baryons*
i.e., intergalactic baryons as well as seeds of galaxies and stars!
baryons fall into potentials of dark matter halos, form galaxies
so *cosmic 21 cm traces formation of structure and galaxies!*

gas at redshift z absorbs at $\lambda(z) = (1+z)\lambda_{\ell u}$
and o responsible for decrement $\delta T_b[\lambda(z)]$
$\rightarrow$ thus $\delta T_b(\lambda)$ *encodes redshift history* of absorbers
a sort of "21 cm forest"

Q: what about sky pattern of $\delta T_b(\lambda)$ at fixed λ?

and at fixed λ, sky map of $\delta T_b(\lambda)$
gives baryon distribution in "shell" at $1 + z = \lambda/\lambda_0$
$\rightarrow$ a radial "slice" of the baryonic Universe!

so by scanning through λ, and at each
making sky maps of $\delta T_b(\lambda)$
$\rightarrow$ we build in "slices" a *3-D map of cosmic structure evolution!*
"cosmic tomography"! a cosmological gold mine!
encodes huge amounts of information

sounds amazing! and it is! but there is a catch!

Q: why is this measurement very difficult to do?
Hint: it hasn't yet been done

21 cm Cosmology: The Challenge

The 21 cm cosmic goldmine lies at redshifts $z \sim 6$ to 150 corresponding to:

- $\lambda_{\rm obs} \sim 1.5 - 30$ m
  enormous wavelengths! `www: LOFAR`
- $\nu_{\rm obs} \sim 200 - 10$ MHz
  but ionosphere opaque $> \nu_{\rm plasma} \sim 20$ MHz
  for highest z (most interesting!) have to go to space! in fact, have to go to far side of the Moon *Q: why?*
  `www: proposed lunar observatories`

But wait! It's worse!
at these wavelengths, dominant emission is *Galactic synchrotron* with brightness $T_{\rm B,synch} \sim 200 - 2000\ {\rm K} \gg T_{\rm cmb} \gg T_{\rm B,21\ cm}$

`www: radio continuum sky`

Q: implications? how to get around this?

sky intensity $T_{\rm B,synch} \sim 200 - 2000\ {\rm K} \gg T_{\rm cmb}$
$\rightarrow$ Galactic synchrotron foreground dominates cosmic 21 cm
curse you, cosmic rays!

But there remains hope!
recall: cosmic-ray electron energy spectrum is a power law
so their *synchrotron spectrum is a power law*
i.e., $I_{\nu,\rm synch}$ is *smooth function of* ν

compare 21 cm at high-z: a "forest" of absorption lines
not smooth! full of spectral *lines & features*
$\rightarrow$ can hope to measure with very good spectral coverage
and foreground subtraction

also: can use spatial (i.e., angular) distribution
e.g., consider effect of first stars (likely massive) *Q: namely?*

first stars: likely massive → hot → large UV sources
ionizing photons carve out “bubble” neutral H
→ corresponding to a *void* in 21 cm
→ sharp bubble edges may be detectable
→ 21 cm can probe *epoch of reionization*

hot, ongoing research area!

stay tuned!

Director's Cut Extras

Awesome Example: Cosmic 21 cm Radiation

CMB today, redshift $z = 0$, has $T_{\rm cmb}(0) = 2.725\ {\rm K} \gg {\rm T}_{\rm ex,21\,cm}$ but what happens over cosmic time?

fun & fundamental cosmological result:
(relativistic) *momentum redshifts*: $p \propto 1/a(t)$, which means

$$p(z) = (1 + z)\ p(0) \tag{20}$$

where $p(0)$ is observed momentum today ($z = 0$)

why? photon or de Broglie wavelength λ is a *length*, so

$$\lambda(t) = a(t)\ \lambda_{\rm emit} = \frac{\lambda_0}{(1+z)} \tag{21}$$

and quantum relation $p = h/\lambda$ implies $p \propto (1 + z)$

Q: implications for gas vs radiation after recombination?

Thermal History of Cosmic Gas and Radiation

until recombination (CMB formation) $z \geq z_{\rm rec} \sim 1000$
(mostly) hydrogen gas is ionized, tightly coupled to CMB
via Thomson scattering: $T_{\rm cmb} = T_{\rm gas}$

after recombination, before gas decoupling $z_{\rm dec} \sim 150 \lesssim z \leq z_{\rm rec}$

- most gas in the Universe is *neutral*
  but a small "residual" fraction $x_e \sim 10^{-5}$ of e^- remain ionized

- Thompson scattering off residual free e^- $(x_e \sim 10^{-5})$
  still couples gas to CMB $\rightarrow$ $T_{\rm cmb} = T_{\rm gas}$ maintained
- until about $z_{\rm dec} \sim 150$, when Thomson scattering ineffective, gas *decoupled*

Q: after decoupling, net effect of 21 cm transition?

radiation transfer along each sightline:

$$I_\nu = I_\nu^{\rm cmb}\ e^{-\tau_\nu} + I_\nu^{\rm gas}\ (1 - e^{-\tau_\nu}) \tag{22}$$

with τ_ν optical depth to CMB

in terms of *brightness or antenna temperature* $T_B = (c^2/2k\nu^2) I_\nu$

$$T_b = T_{\rm cmb}\ e^{-\tau_\nu} + T_{\rm gas}(1 - e^{-\tau_\nu}) \tag{23}$$

when $T_{\rm gas} = T_{\rm cmb}$ (really, $T_{\rm spin} = T_{\rm cmb}$)

gas is in equilibrium with CMB: emission = absorption

$\rightarrow$ $T_b = T_{\rm cmb}$: no net effect from CMB passage through gas

after gas decoupling, before reionization $z_{\rm reion} \sim 10 \lesssim z \leq z_{\rm dec}$

separate thermal evolution: $T_{\rm cmb} \sim E_{\rm peak} \propto p_{\rm peak} \propto (1+z)$

but matter has $T_{\rm gas} \sim p^2/2m \propto p^2 \propto (1+z)^2$

$\rightarrow$ *gas cools* (thermal motions "redshift") *faster than the CMB!*

Q: net effect of 21 cm transitions in this epoch?

21 cm Radiation in the Dark Ages

before the first stars and quasars: **cosmic dark ages**
first structure forming, but not yet "lit up"

during dark ages: intergalactic gas has $T_{\rm gas} < T_{\rm cmb}$

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