Astronomy 501: Radiative Processes

Lecture 31

Nov 4, 2022

Announcements:

- **Problem Set 9 due today**
- **Problem Set 10–penultimate!–next Friday**
- **Radiative Meme Submission now open on Canvas**

Last time: 21 cm astronomy

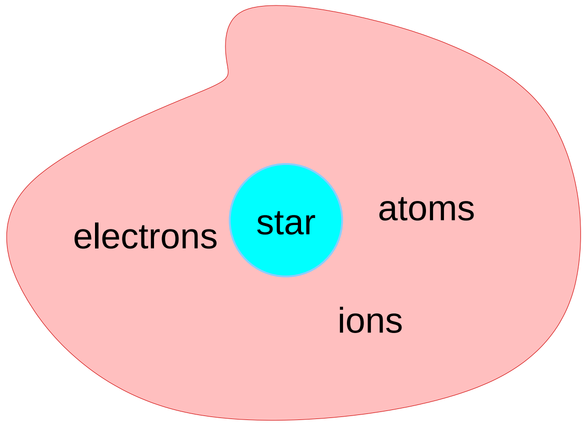
Director's Cut: 21 cm cosmology

Today: Nebular Diagnostics

Nebulae

nebulae are

- interstellar gaseous regions
- illuminated by stars and their own radiation
- made of atoms, ions, and electrons



Our goal today:

- understand how atomic states are populated leading to line emission
- and how observing nebular lines probes the properties of the nebula

Nebular Diagnostics

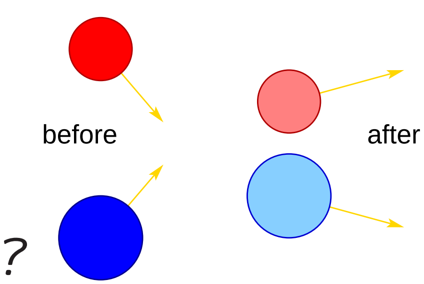
Collisional Excitation

so far we have considered atomic line transitions
due to emission or absorption of radiation
but atom *collisions* can also drive transitions

$\star$ collisions can place atoms in excited states
de-excite radiatively (line emission) $\rightarrow$ cooling source

$\star$ collisions populate atomic levels
observing line emission can diagnose density, temperature, radiation field

key physical input: *collision rates*
consider inelastic collisions $a + c \rightarrow a' + c'$
of species a with "collision partner" c
Q: what is collision rate per volume? per a *atom?*



for collisions $a + c$, collision rate per volume is

$$\frac{d\mathcal{N}_{\text{collisions}}}{dV\ dt} \equiv \dot{n}_{ac\to a'c'} = \langle \sigma_{ac} v \rangle\ n_a\ n_c \tag{1}$$

where **collision rate coefficient** $\langle \sigma_{ac} v \rangle$
averages over collision *cross section* σ_{ac}
and relative velocity v between a and c

Q: order-of-magnitude estimate for σ_{ac}?
Q: what sets typical v?

collision rate *per a* is

$$\Gamma_{ac\to a'c'} = \frac{\dot{n}_{ac\to a'c'}}{n_a} = \langle \sigma_{ac} v \rangle\ n_c \tag{2}$$

Two-Level Atom: No Radiation

instructive simple case: a *two-level atom*
denote *ground state* 0, *excited state* 1
with atomic number densities n_0 and n_1

consider effect of collisions with partner c
when radiation effects are unimportant:

$$\dot{n}_1 = -\Gamma_{10} n_1 + \Gamma_{01} n_0 = -\langle \sigma_{10} v \rangle n_c n_1 + \langle \sigma_{01} v \rangle n_c n_0 \qquad (3)$$

Q: what is n_1/n_0 *ratio in equilibrium* ($\dot{n}_1 = 0$)*?*
Q: what does this imply?

without radiation, in *equilibrium*:

$$\dot{n}_1 = -\langle \sigma_{10} v \rangle \, n_c n_1 + \langle \sigma_{01} v \rangle \, n_c n_0 = 0 \tag{4}$$

which gives $(n_1/n_0)_{\rm eq} = \langle \sigma_{01} v \rangle \, / \, \langle \sigma_{10} v \rangle$

but in thermal equilibrium $(n_1/n_0)_{\rm eq} = (g_1/g_0) \; e^{-E_{10}/kT}$

so we have the *detailed balance* result

$$\langle \sigma_{10} v \rangle = \frac{g_1}{g_0} \; e^{-E_{10}/kT} \; \langle \sigma_{01} v \rangle \tag{5}$$

- links "forward" and "reverse" reaction cross sections
- excitation is suppressed by Boltzmann factor $e^{-E_{10}/kT}$

Q: how do we add radiation effects?

Two-Level Atom with Radiation

if atoms in excited states exist, they can spontaneously emit
$\rightarrow$ radiation must be present

volume rate of: *spontaneous emission* is $A_{10} n_1$

volume rate of: *stimulated emission*

$$B_{10} J_\nu n_1 = A_{10} \frac{c^2 J_\nu}{2h\nu^3} n_1 \equiv A_{10} \ f_\nu \ n_1 \tag{6}$$

where for isotropic radiation $J_\nu = 2 \ \nu^3/c^2 \ f_\nu$, with
f_ν the *photon distribution function* or occupation number

volume rate of: *true absorption*

$$B_{01} J_\nu n_0 \equiv \frac{g_1}{g_0} A_{10} \ f_\nu \ n_1 \tag{7}$$

putting it all together, the two-level atom
in the presence of collisions and radiation has

$$\dot{n}_1 = \left[\langle \sigma_{01} v \rangle \, n_c + f_\nu \frac{g_1}{g_0} A_{10} \right] n_0 - \left[\langle \sigma_{10} v \rangle \, n_c + (1 + f_\nu) A_{10} \right] n_1 \tag{8}$$

this will seek an equilibrium or *steady state* $\dot{n}_1 = 0$
giving the ratio

$$\left(\frac{n_1}{n_0} \right)_{\text{eq}} = \frac{\langle \sigma_{01} v \rangle \, n_c + (g_1/g_0) f_\nu A_{10}}{\langle \sigma_{10} v \rangle \, n_c + (1 + f_\nu) A_{10}} \tag{9}$$

consider the limits of low- and high-density collision partners

$$\left(\frac{n_1}{n_0} \right)_{\text{eq}} \rightarrow \begin{cases} (g_1/g_0) \; f_\nu/(1 + f_\nu) \; , & n_c \rightarrow 0 \\ \langle \sigma_{01} v \rangle \, / \, \langle \sigma_{10} v \rangle & n_c \rightarrow \infty \end{cases} \tag{10}$$

Q: implications of limits if $T_{\text{rad}} \neq T_{\text{gas}}$?

in steady state:

$$\left(\frac{n_1}{n_0}\right)_{\rm eq} = \frac{\langle \sigma_{01} v \rangle\, n_c + f_\nu (g_1/g_0) A_{10}}{\langle \sigma_{10} v \rangle\, n_c + (1 + f_\nu) A_{10}} \tag{11}$$

at *low density* of collision partners: $n_c \to 0$ and thus

$$\left(\frac{n_1}{n_0}\right)_{\rm eq} \to \frac{g_1}{g_0} \frac{f_\nu}{1 + f_\nu} \stackrel{\rm therm}{=} \frac{g_1}{g_0} e^{-E_{10}/kT_{\rm rad}} \tag{12}$$

$\to$ *level population set by radiation temperature* $T_{\rm rad}$

at *high density* of collision partners: $n_c \to \infty$, and

$$\left(\frac{n_1}{n_0}\right)_{\rm eq} \to \frac{\langle \sigma_{01} v \rangle}{\langle \sigma_{10} v \rangle} = \frac{g_1}{g_0} e^{-E_{10}/kT_{\rm gas}} \tag{13}$$

$\to$ *level population set by gas temperature* $T_{\rm gas}$

Q: characteristic density scale?

Critical Density

for each collision partner c, excited state de-excitation by emission and by collisions are equal when

$$\langle \sigma_{10} v \rangle \, n_c = (1 + f_\nu) A_{10} \tag{14}$$

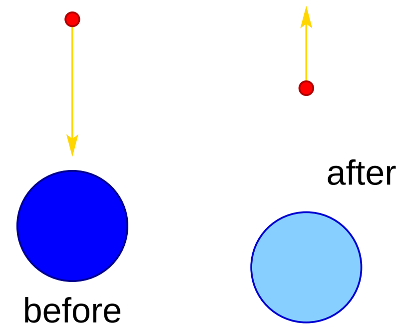
this defines a **critical density**

$$n_{c,\mathrm{crit}} = \frac{(1 + f_\nu) A_{10}}{\langle \sigma_{10} v \rangle} \tag{15}$$

- if $f_\nu \ll 1$: stimulated emission not important
  $n_{c,\mathrm{crit}} \to A_{10} / \langle \sigma_{10} v \rangle$ depends only on T and atomic properties
- but if f_ν not small, critical density depends on local radiation field

so when partner density $n_c \gg n_{c,\mathrm{crit}}$:
  state population set by $T \to T_{\mathrm{gas}}$
and when $n_c \ll n_{c,\mathrm{crit}}$:
  state population set by $T \to T_{\mathrm{rad}}$

Electron-Atom Collisions



consider a partially ionized gas: has atoms, free e^-, and ions

atoms can collide inelastically (change state) with thermal *electrons* at T

Q: geometric cross section of electron?

Q: quantum mechanical lengthscale for non-relativistic e?

Q: collision cross section, reaction rate estimate for e at T?

electrons are quantum particles
with de Broglie wavelength $\lambda_{\rm deB} = h/p_e = h/m_e v$
so thermal electrons have a *thermal de Broglie wavelength*

$$\lambda_{{\rm deB},e} \sim \frac{h}{m_e v_T} = \frac{h}{\sqrt{m_e\, kT}} = 52\ \text{Å}\ \left(\frac{1000\ \rm K}{T}\right)^{1/2} \tag{16}$$

so for T of interest, $\lambda_{{\rm deB},e} \gg r_{\rm atom} \sim a_0$

so to order-of-magnitude, atom-electron cross section is

$$\sigma_{ae} \sim \pi \lambda_{{\rm deB},e}^2 = \pi \frac{h^2}{m_e\, kT} \tag{17}$$

and thermal collision rate coefficient is

$$\langle \sigma_{ae} v \rangle \sim v_T\ \sigma_{ae} \sim \frac{h^2}{m_e \sqrt{m_e kT}} \propto \frac{1}{\sqrt{T}} \tag{18}$$

to order of magnitude,

$$\langle \sigma_{ae} v \rangle \sim v_T \; \sigma_{ae} \sim \frac{h^2}{m_e \sqrt{m_e k T}} \propto \frac{1}{\sqrt{T}} \tag{19}$$

useful to define dimensionless **collision strength** $\Omega_{u\ell}$
for electron-atom transition $u \to \ell$:

$$\langle \sigma_{ae} v \rangle \equiv \frac{h^2}{(2\pi m_e)^{3/2} (kT)^{1/2}} \frac{\Omega_{u\ell}}{g_u} \tag{20}$$

in principle, $\Omega_{u\ell}(T)$ depends on T
but in practice, nearly constant with T,
and values are in range $\Omega_{u\ell} \sim 1 - 10$

Awesome Example: C^+ 158μm

singly ionized carbon: C^+ or C ii

ground state hyperfine splitting $J = 3/2, 1/2$

$$\Delta E/k = 91.21 \text{ K} \tag{21}$$

$$\lambda = 158 \ \mu\text{m} \tag{22}$$

$$A_{10} = 2.4 \times 10^{-6} \text{ s}^{-1} = (4.8 \text{ days})^{-1} \tag{23}$$

Q: waveband? appropriate telescopes?

critical densities

$$n_{\text{crit}}(\text{H}) \sim 3000 \text{ cm}^{-3} \tag{24}$$

$$n_{\text{crit}}(e^-) \sim 50 \sqrt{T/10^4\,\text{K}} \text{ cm}^{-3} \tag{25}$$

consider a low density, optically thin region with C^+

Q: what are the level populations?

Q: if upper level collisionally excited, what happens?

Q: where is this likely to occur?

C^+ Hyperfine Emission as a Star-Formation Coolant

low density parts of star-forming regions

- contain C II,
- but are below critical densities
- and optically thin: not radiatively excited

so: *upper level "subthermal" → collisions can excite*

and if collisional excitation occurs

- radiative de-excitation is the most likely
- [C II] 158 μm photon emitted
- usually optically thin, lost from system: *observable!*
- removes energy: *coolant*

This line is a major tracer of diffuse star-forming regions!

Q: what should an all-sky map of 158 μm look like?

`www: all-sky`, `www: external galaxies`

Nebular Diagnostics

consider a *diffuse nebula*: low-density gas generally irradiated by stars

Q: expected optical spectrum?

`www: example spectra`

Q: how to use spectra to measure T? density?

Nebular Temperature Diagnostic

diffuse nebulae: usually optically thin in visible band
continuum radiation is not blackbody
and reprocesses stellar radiation with $T \sim 3000 - 30,000$ K
spectra dominated by *emission lines*
$\rightarrow$ need to use these to determine T, density

temperature diagnostics: *pairs of lines* that are

- energetically accessible: $E_{u\ell} \lesssim kT$
- widely spaced: $\Delta E \sim kT$

consider an idealized *3-level atom*

- **ground state** $n = 1$, **excited states** $n = 2, 3$
- excited states populated by *electron collisions*
  at volume rate $dn_{13}/dt = \langle \sigma_{e1\rightarrow 3} v \rangle \ n_1 \ n_e \propto \Omega_{13} e^{-E_{13}/kT} n_1 n_e$
- probability for $3 \rightarrow 1$ transition: $A_{31}/(A_{31} + A_{32})$ *Q: why?*

if electron density $n_e \ll n_{e,\mathrm{crit}}$
then de-excitation occurs via spontaneous emission
and integrated emissivity from the $3 \to 1$ transition is

$$j_{31} = E_{31}\dot{n}_{13}\frac{A_{31}}{A_{31}+A_{32}} = E_{31}\langle\sigma_{31}v\rangle\frac{A_{31}}{A_{31}+A_{32}}n_1 n_e \tag{26}$$

and from the $3 \to 1$ transition is

$$j_{21} = E_{21}\left(\langle\sigma_{12}v\rangle + \langle\sigma_{13}v\rangle\frac{A_{32}}{A_{31}+A_{32}}\right)n_1 n_e \tag{27}$$

thus the emissivity ratio and hence line ratio is

$$\frac{j_{31}}{j_{21}} = \frac{A_{31}E_{31}}{A_{32}E_{32}}\,\frac{(A_{31}+A_{32})\langle\sigma_{31}v\rangle}{(A_{31}+A_{32})\langle\sigma_{21}v\rangle + A_{31}\langle\sigma_{31}v\rangle} \tag{28}$$

$$= \frac{A_{31}E_{31}}{A_{32}E_{32}}\,\frac{(A_{31}+A_{32})\Omega_{31}e^{-E_{32}/kT}}{(A_{31}+A_{32})\Omega_{21} + A_{31}\Omega_{31}e^{-E_{32}/kT}} \tag{29}$$

excellent! Q: Why?

3-level atom line ratio

$$\frac{j_{31}}{j_{21}} = \frac{A_{31}E_{31}}{A_{32}E_{32}} \frac{(A_{31}+A_{32})\Omega_{31}e^{-E_{32}/kT}}{(A_{31}+A_{32})\Omega_{21}+A_{31}\Omega_{31}e^{-E_{32}/kT}} \tag{30}$$

depends only on T and atomic properties

so: for appropriate systems

- measure line ratio
- look up the atomic properties
- use observed ratio to solve for T!

Director's Cut Extras

Awesome Example: Cosmic 21 cm Radiation

CMB today, redshift $z = 0$, has $T_{\rm cmb}(0) = 2.725$ K $\gg$ $T_{\rm ex,21\,cm}$ but what happens over cosmic time?

fun & fundamental cosmological result:
(relativistic) *momentum redshifts*: $p \propto 1/a(t)$, which means

$$p(z) = (1+z)\ p(0) \tag{31}$$

where $p(0)$ is observed momentum today ($z = 0$)

why? photon or de Broglie wavelength λ is a *length*, so

$$\lambda(t) = a(t)\ \lambda_{\rm emit} = \frac{\lambda_0}{(1+z)} \tag{32}$$

and quantum relation $p = h/\lambda$ implies $p \propto (1+z)$

Q: implications for gas vs radiation after recombination?

Thermal History of Cosmic Gas and Radiation

until recombination (CMB formation) $z \geq z_{\rm rec} \sim 1000$
(mostly) hydrogen gas is ionized, tightly coupled to CMB
via Thomson scattering: $T_{\rm cmb} = T_{\rm gas}$

after recombination, before gas decoupling $z_{\rm dec} \sim 150 \lesssim z \leq z_{\rm rec}$

- most gas in the Universe is *neutral*
  but a small "residual" fraction $x_e \sim 10^{-5}$ of e^- remain ionized

- Thompson scattering off residual free e^- ($x_e \sim 10^{-5}$)
  still couples gas to CMB $\rightarrow$ $T_{\rm cmb} = T_{\rm gas}$ maintained
- until about $z_{\rm dec} \sim 150$, when Thomson scattering ineffective,
gas *decoupled*

Q: after decoupling, net effect of 21 cm transition?

radiation transfer along each sightline:

$$I_\nu = I_\nu^{\text{cmb}}\ e^{-\tau_\nu} + I_\nu^{\text{gas}}\ (1 - e^{-\tau_\nu}) \tag{33}$$

with τ_ν optical depth to CMB

in terms of *brightness or antenna temperature* $T_B = (c^2/2k\nu^2) I_\nu$

$$T_b = T_{\text{cmb}}\ e^{-\tau_\nu} + T_{\text{gas}}(1 - e^{-\tau_\nu}) \tag{34}$$

when $T_{\text{gas}} = T_{\text{cmb}}$ (really, $T_{\text{spin}} = T_{\text{cmb}}$)

gas is in equilibrium with CMB: emission = absorption

$\rightarrow$ $T_b = T_{\text{cmb}}$: no net effect from CMB passage through gas

after gas decoupling, before reionization $z_{\text{reion}} \sim 10 \lesssim z \leq z_{\text{dec}}$

separate thermal evolution: $T_{\text{cmb}} \sim E_{\text{peak}} \propto p_{\text{peak}} \propto (1+z)$

but matter has $T_{\text{gas}} \sim p^2/2m \propto p^2 \propto (1+z)^2$

$\rightarrow$ *gas cools* (thermal motions "redshift") *faster than the CMB!*

Q: net effect of 21 cm transitions in this epoch?

Awesome Example: Cosmic 21 cm Radiation

radiation transfer along each sightline, at rest-frame 21cm:

$$I = B_\nu(T_{\text{cmb}})\ e^{-\tau_\nu} + B_\nu(T_{\text{gas}})\ (1 - e^{-\tau_\nu}) \tag{35}$$

in terms of antenna temperature

$$T_b = T_{\text{cmb}}\ e^{-\tau_\nu} + T_{\text{gas}}(1 - e^{-\tau_\nu}) \tag{36}$$

so: if $T_{\text{gas}} = T_{\text{cmb}}$ - no signal!

otherwise: emission or absorpbtion
depending on $T_{\text{gas}} - T_{\text{cmb}}$ sign

$$T_b = T_{\rm cmb}\ e^{-\tau_\nu} + T_{\rm gas}(1 - e^{-\tau_\nu}) \tag{37}$$

after gas decoupling, before reionization $z_{\rm reion} \sim 10 \lesssim z \leq z_{\rm dec}$
before the first stars and quasars: **cosmic dark ages**
first structure forming, but not yet "lit up"

during dark ages: intergalactic gas has $T_{\rm gas} < T_{\rm cmb}$

$$\delta T_b \equiv T_b - T_{\rm cmb} = (T_{\rm gas} - T_{\rm cmb})_z\, (1 - e^{-\tau_\nu})_z \tag{38}$$

we have $\delta T_b < 0$: gas seen in 21 cm *absorption*

Q: what cosmic matter will be seen this way?
Q: what will its structure be in 3-D?
Q: how will this structure be encoded in δT_b*?*

The “21 cm Forest”

what will absorb at 21 cm?
any neutral hydrogen in the universe!
but after recomb., most H is neutral, and most baryons are H
so absorbers are *most of the baryons in the universe*

thus absorber spatial distribution is *3D distribution of baryons*
i.e., intergalactic baryons as well as seeds of galaxies and stars!
baryons fall into potentials of dark matter halos, form galaxies
so *cosmic 21 cm traces formation of structure and galaxies!*

gas at redshift z absorbs at $\lambda(z) = (1+z)\lambda_{\ell u}$
and o responsible for decrement $\delta T_b[\lambda(z)]$
$\rightarrow$ thus $\delta T_b(\lambda)$ *encodes redshift history* of absorbers
a sort of “21 cm forest”

Q: what about sky pattern of $\delta T_b(\lambda)$ at fixed λ?

and at fixed λ, sky map of $\delta T_b(\lambda)$
gives baryon distribution in "shell" at $1 + z = \lambda/\lambda_0$
$\rightarrow$ a radial "slice" of the baryonic Universe!

so by scanning through λ, and at each
making sky maps of $\delta T_b(\lambda)$
$\rightarrow$ we build in "slices" a *3-D map of cosmic structure evolution!*
"cosmic tomography"! a cosmological gold mine!
encodes huge amounts of information

sounds amazing! and it is! but there is a catch!

Q: why is this measurement very difficult to do?
Hint: it hasn't yet been done

21 cm Cosmology: The Challenge

The 21 cm cosmic goldmine lies at redshifts $z \sim 6$ to 150
corresponding to:

- $\lambda_{\rm obs} \sim 1.5 - 30$ m
  enormous wavelengths! `www: LOFAR`
- $\nu_{\rm obs} \sim 200 - 10$ MHz
  but ionosphere opaque $> \nu_{\rm plasma} \sim 20$ MHz
  for highest z (most interesting!) have to go to space! in fact, have to go to far side of the Moon *Q: why?*
  `www: proposed lunar observatories`

But wait! It's worse!
at these wavelengths, dominant emission is *Galactic synchrotron*
with brightness $T_{\rm B,synch} \sim 200 - 2000 \text{ K} \gg T_{\rm cmb} \gg T_{\rm B,21\ cm}$

`www: radio continuum sky`

Q: implications? how to get around this?

sky intensity $T_{\mathrm{B,synch}} \sim 200 - 2000\ \mathrm{K} \gg T_{\mathrm{cmb}}$
$\rightarrow$ Galactic synchrotron foreground dominates cosmic 21 cm
curse you, cosmic rays!

But there remains hope!
recall: cosmic-ray electron energy spectrum is a power law
so their *synchrotron spectrum is a power law*
i.e., $I_{\nu,\mathrm{synch}}$ is *smooth function of* ν

compare 21 cm at high-z: a "forest" of absorption lines
not smooth! full of spectral *lines & features*
$\rightarrow$ can hope to measure with very good spectral coverage
and foreground subtraction

also: can use spatial (i.e., angular) distribution
e.g., consider effect of first stars (likely massive) *Q: namely?*

first stars: likely massive $\rightarrow$ hot $\rightarrow$ large UV sources

ionizing photons carve out “bubble” neutral H

$\rightarrow$ corresponding to a *void* in 21 cm

$\rightarrow$ sharp bubble edges may be detectable

$\rightarrow$ 21 cm can probe *epoch of reionization*

hot, ongoing research area!

stay tuned!