

# Astronomy 501: Radiative Processes

Lecture 33

Nov 9, 2022

Announcements:

- **Problem Set 10—penultimate!—on Friday**
- **Radiative Meme Submission on Canvas**  
more great submissions have arrived

## Last time: deciphering dust

We are stardust  
Billion year old carbon  
We are golden  
Caught in the devil's bargain  
And we've got to get ourselves  
Back to the garden  
– J. Mitchell (1970)

*Q: what are grain sizes? how do we know?*

*Q: grain composition? how do we know?*

## Dust: Composition

interstellar *atomic absorption lines* trace  
element in *atomic* form

→ measure interstellar *gas-phase abundances* only!

but *dust* particles are in *solid phase*! “*grains*”  
do not give atomic lines!

nearby ISM likely nearly has *nearly solar composition*  
but some elements mostly in gas phase, others mostly in grains

Depletion pattern correlated with **condensation temperature**  
i.e., temperature at which a dilute vapor  $\rightarrow$  50% solid

www: observed depletion pattern

- *low  $T_{\text{cond}}$  elements: volatile* (Kr, Ar, C, O, ...)  
*small/no depletion*
- *high  $T_{\text{cond}}$  elements: refractory* (Fe, Ni, Ti, Ca, ...)  
*large depletion*

*Q: what is this trying to tell us?*

depletion correlated with condensation temperature

suggests **physical picture:**

- *dust formed out of high-temperature material*  
e.g., ejecta from dying stars  
note: AGB stars have dusty shells
- *as this vapor cools, refractory elements form dust first*
- small depletion for  $T_{\text{cond}} \lesssim 700 - 800 \text{ K}$   
either gas never gets this cool,  
or more likely, density becomes too low to form dust  
by collisional processes

### **Warning!**

when using *interstellar abundances from lines*, never forget:  
these include elements in the *gas phase only!*

# Dust and Starlight

We have seen: *interstellar dust absorbs starlight*

**For local sightlines in the Milky Way plane:**

photon mean free path against extinction

$$\frac{1}{l_V} = \sigma_V n_{\text{dust}} = \frac{d\tau_V}{ds} \sim \frac{dA_V}{ds} \sim 2^{\text{mag}} \text{ kpc}^{-1} \quad (1)$$

Q: *implications for visible MW? shorter wavelengths? longer?*

www: the multiwavelength sky

Q: *what happens to dust as it absorbs starlight?*

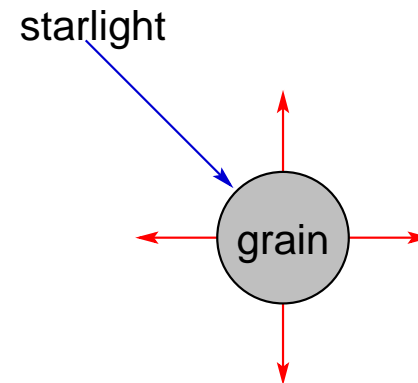
Q: *what dominates the stellar luminosity in a disk galaxy?*

o Q: *so what does  $L_{\text{far-IR}}$  probe?*

# Dust Temperature

interstellar dust: radiative equilibrium

- dust grains absorb starlight particularly in optical/UV
- photon energy *thermalized* in grains
- which *radiate* roughly as black bodies



dust temperature set by absorption/emission equilibrium

at  $T_{\text{dust}} \approx T_{\text{eff,ISM}} \sim 10 - 100 \text{ K}$ , depending on environment  
observable in the **infrared** www: mid-far IR sky

Lesson: *dust reprocesses starlight into thermal IR!*

Implication:

- ✓ starlight dominated by luminous (and short-lived) massive stars
- far-IR emission traces extincted UV/optical from these stars
- $L_{\text{far-IR}} \propto dM_{\star}/dt$  *probes star formation rate!*

## Build Your Toolbox–Dust

emission physics: matter-radiation interactions

*Q: physical conditions for dust emission? absorption?*

*Q: physical nature of sources?*

*Q: spectrum characteristics?*

*Q: frequency range?*

real/expected astrophysical sources of dust radiation

*Q: where do we expect emission? absorption?*

*Q: relevant temperatures? EM bands?*



# Toolbox: Radiation and Dust

## emission physics

- **physical conditions:** light interaction with solid bodies
- **physical sources:** microscopic grains with range of sizes (nm to mm) and compositions
- **spectrum:** strong absorption at  $\lambda \leq$  dust size  
heated grains re-radiate thermally

## astrophysical sources of dust

- **emitters:** present in interstellar medium  
produced in stars, processed in ISM
- **temperatures:** depends on environment,  $\sim 10 - 300$  K
- ● **EM bands:** absorption important from X rays to near IR  
emission in radio to infrared

# The Relativistic Cosmos

## Shifting to High Gear

Thus far we have only considered radiative properties of **non-relativistic matter**, which means

- speeds  $v \ll c$
- kinetic energies  $E \ll mc^2$
- temperatures  $kT \ll mc^2$

*Q: what astrophysical sources satisfy these criteria?*

*Q: what sources do not?*

# The Relativistic Cosmos

**most of cosmic matter is non-relativistic**

stars, planets, interstellar gas all have  $T \ll m_p c^2 / k \sim 10^{10}$  K  
and bulk motions of stars, planets, ISM have  $v \ll c$

**but: cosmic accelerators exist, create relativistic motions!**

- bulk **jets** launched by: **gamma-ray bursts**  
supermassive black holes in **active galaxies** www: M87 jet
- **high-energy particles: cosmic rays**

to describe these objects and their radiation  
we need Special Relativity

# Special Relativity for the Impatient

# Spacetime

see S. Carroll, *Spacetime and Geometry*; R. Geroch, *General Relativity from A to B*

evolving view of space, time, and motion:

Aristotle → Galileo → Einstein

Key basic concept: **event**

occurrence localized in space and time

e.g., firecracker, finger snap

idealized → no spatial extent, no duration in time

a goal (*the* goal?) of physics:

describe relationships among events

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*Q: consider collection of all possible events—what's included?*

## Spacetime Coordinates

Each event specifies a unique point in space and time  
collection of all possible events = **spacetime**

lay down coordinate system: 3 space coords, one time  
4-dimensional, but as yet time & space unrelated

e.g., time  $t$ , Cartesian  $x, y, z$ : event  $\rightarrow (t, x, y, z)$   
physics asks (and answers) what is the relationship  
between two events, e.g.,  $(t_1, x_1, y_1, z_1)$  and  $(t_2, x_2, y_2, z_2)$

Note: more on spacetime in Director's Cut Extras to today's notes

# Galilean Relativity

consider *two laboratories*

(same apparatus, funding, required courses, vending machines)

*move at constant velocity* with respect to each other

Galileo:

no experiment done in either lab (without looking outside)

can answer the question “which lab is moving”

→ *no absolute motion*, only relative velocity

Newton: laws of mechanics invariant

for observers moving at const  $\vec{v}$

“*inertial observers*”

Implications for spacetime

no absolute motion → *no absolute space*

(but still no reason to abandon absolute time)



## Galilean Frames

each inertial obs has own personal frame:

obs (“Angelina”) at rest in own frame:  $(x, y, z)$  same for all  $t$

but to another obs (“Brad”) in relative motion  $\vec{v} = v\hat{x}$

B sees A’s frame as time-dependent:

$$x_{\text{A as seen by B}} = x' = x - vt$$



but still absolute time:  $t' = t$

Newton’s laws (and Newtonian Gravity) hold in both frames

can show:  $d^2\vec{x}/dt^2 = \vec{F}(\vec{x}) \Rightarrow d^2\vec{x}'/dt'^2 = \vec{F}(\vec{x}')$

“Galilean invariance”

## Trouble for Galileo

Maxwell: equations govern light

very successful, but:

- predicts unique (constant) *light speed*  $c$ —relative to whom?
- Maxwell eqs **not** Galilean invariant

Lorentz: Maxwell eqs invariant when

$$t' = \gamma(t - vx/c^2) \quad (2)$$

$$x' = \gamma(x - vt) \quad (3)$$

$$y' = y \quad (4)$$

$$z' = z \quad (5)$$

with Lorentz factor  $\gamma = 1/\sqrt{1 - v^2/c^2}$

Einstein:

Lorentz transformation not just a trick

but correct relationship between inertial frames!

⇒ this is the way the world is

# Einstein: Special Relativity

consider two *nearby events*

$(t, x, y, z)$  and  $(t + \Delta t, x + \Delta x, y + \Delta y, z + \Delta z)$

different inertial obs *disagree* about  $\Delta t$  and  $\Delta \vec{x}$   
but all *agree* on the value of the **interval**

$$\Delta s^2 \equiv (c\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2 \quad (6)$$

$$= (c\Delta t)^2 - (\Delta \ell)^2 \quad (7)$$

Note: interval can have  $\Delta s^2 > 0, < 0, = 0$

quantities agreed upon by all observers: *Lorentz invariants*

Light pulse:  $\Delta \ell = c\Delta t$

→  $\Delta s_{\text{light}} = 0$

→ light moves at  $c$  in all frames!

Motion and time:

Consider two events, at rest in one frame:

$\Delta \vec{x}_{\text{rest}} = 0$  in rest frame, so

$\Delta s = c\Delta t_{\text{rest}}$ :  $c \times$  elapsed time in rest frame

In another inertial frame, relative speed  $v$ :

events separated in space by  $\Delta x' = v\Delta t'$

$$\Delta s = \sqrt{c^2 \Delta t'^2 - \Delta x'^2} = \sqrt{c^2 - v^2} \Delta t' = \frac{1}{\gamma} c \Delta t' \quad (8)$$

since  $\Delta s$  same: infer  $\Delta t' = \gamma \Delta t_{\text{rest}} > \Delta t_{\text{rest}}$

$\Rightarrow$  moving clocks appear to run slow

(special) relativistic **time dilation**

$\Rightarrow$  no absolute time (and no absolute space)

H. Minkowski:

“Henceforth, space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality.”

# Lorentz Transformations

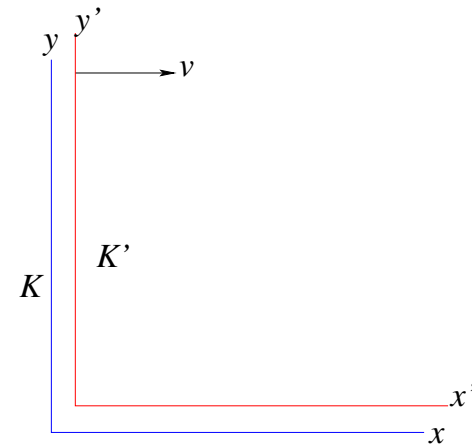
consider two coordinate systems  $K, K'$   
moving with *relative* speed  $\vec{v} = v\hat{x}$

$$t' = \gamma(t - vx/c^2)$$

$$x' = \gamma(x - vt)$$

$$y' = y$$

$$z' = z$$



- **boost** from one frame to another
  - truly mix space and time  $\rightarrow$  *spacetime*
  - look like rotations, but 4-dimensional
- $\rightarrow$  should express laws in terms of 4-D vectors:  
“4-vectors,”  $t, x$  components transform via Lorentz

## Length Contraction

consider a *standard ruler*: measure length at factory

- ruler is at rest wrt observer
- measure both ends *at same time*  $\delta t = t_2 - t_1 = 0$
- ends are  $x_1 = 0$ ,  $x_2 = L \rightarrow$  length  $L = \delta x = x_2 - x_1$

observer flying by a speed  $\vec{v} = v\hat{x}$ , makes measurement

$$\delta x' = \gamma(\delta x - v\delta t) = \gamma(L - v\delta t/c^2) \quad (9)$$

$$\delta t' = \gamma(\delta t - v\delta x/c^2) = \gamma(\delta t - vL/c^2) \quad (10)$$

but length measurement is done at same time

$\rightarrow \delta t' = 0 \rightarrow \delta t = vL/c^2$  Q: implications?

and thus length found is  $L' = \delta x' = \gamma(1 - v^2/c^2)L$

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$\Rightarrow L' = L/\gamma$  **length contraction**

Q: what if the observer were moving in  $\hat{y}$ ?

## Addition of Velocities

consider an object moving wrt to frame  $K'$   
as seen by  $K'$ : in time interval  $dt$ , moves distance  $dx$  → has  
velocity  $u' = dx'/dt'$

What is speed in frame  $K$  (speed  $v$  wrt  $K$ )?

$$dt = \gamma(dt' + v dx'/c^2) \quad (11)$$

$$dx = \gamma(dx' + v dt') \quad (12)$$

$$dy = dy' \quad dz = dz' \quad (13)$$

and thus

$$u_x = dx/dt = \frac{u'_x + v}{1 + u'_x v/c^2} \quad (14)$$

$$u_y = dy/dt = \frac{u'_y}{\gamma(1 + u'_x v/c^2)} \quad (15)$$

$$u_z = dz/dt = \frac{u'_z}{\gamma(1 + u'_x v/c^2)} \quad (16)$$



If object is moving with arbitrary velocity  $\vec{u}'$  in  $K'$   
 decompose  $\vec{u}' = \vec{u}'_{\parallel} + \vec{u}'_{\perp}$  where  $\parallel$  is along  $K - K'$  motion:

$$u_{\parallel} = \frac{u'_{\parallel} + v}{1 + u'_{\parallel}v/c^2} \quad (17)$$

$$u_{\perp} = \frac{u'_{\perp}}{\gamma(1 + u'_{\parallel}v/c^2)} \quad (18)$$

boost causes **change in velocity direction**

$$\tan \theta = \frac{u_{\perp}}{u_{\parallel}} = \frac{u' \sin \theta'}{\gamma(u' \cos \theta' + v)} \quad (19)$$

and consider **the case where**  $u' = c$

$$\tan \theta = \frac{\sin \theta'}{\gamma(\cos \theta' + \beta)} \quad (20)$$

$$\cos \theta = \frac{\cos \theta' + \beta}{1 + \beta \cos \theta'} \quad (21)$$

angular shift is the **aberration of light**

a light signal emitted in  $K'$  at angle  $\theta'$   
is seen in  $K$  at angle  $\theta$

$$\cos \theta = \frac{\cos \theta' + v/c}{1 + v/c \cos \theta'} \quad (22)$$

Q: what if  $\theta' = 0$ ?  $\pi$ ?

Q: how can we understand this physically?

Q: what if  $\theta' = \pi/2$ ?

Q: how can we understand this physically?

consider photons emitted *isotropically* in  $K'$   
with  $v/c$  not small

26 Q: what is angular pattern in  $K$ ? implications?

## Relativistic Beaming

for light emitted in  $K'$  at  $\theta' = \pi/2$   
observed angle after boosting is

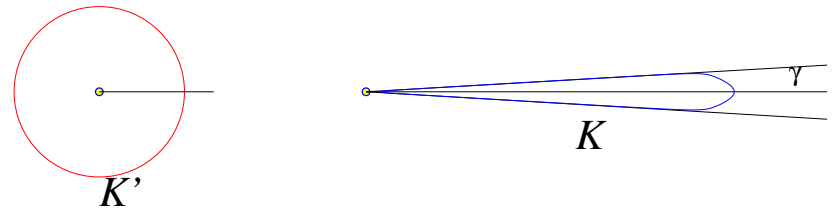
$$\tan \theta = \frac{1}{\gamma v/c} \quad (23)$$

and thus

$$\sin \theta = \frac{1}{\gamma} \quad (24)$$

if emitted  $K'$  is highly relativistic,  
then  $\gamma \gg 1$ , and

$$\theta \rightarrow \frac{1}{\gamma}$$



27 i.e., a small forward angle!  
a highly relativistic emitter gives a **beamed radiation pattern**  
strongly concentrated ahead of emitter direction

# Director's Cut Extras

## Pre-Relativity: Aristotle

$x, y, z$  Cartesian (Euclidean geometry)  
spatial distance  $\ell$  between events is:

$$\ell^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 \quad (25)$$

and is independent of time

elapsed time between events is:  $t_2 - t_1$

and is independent of space

“absolute space” and “absolute time”

Is a particle at rest?  $\Leftrightarrow$  do  $(x, y, z)$  change?

$\rightarrow$  “absolute rest, absolute motion”

∞ *Diagram: Aristotelian spacetime*  
unique “stacking” of “time slices”

## Life According to Aristotle

Note: even in absolute space

event location indep of coordinate description

e.g., two observers choose coordinates different by a rotation:

$(x, y)$  and  $(x', y') = (x \cos \theta - y \sin \theta, y \cos \theta + x \sin \theta)$

preserves distance from origin:  $x^2 + y^2 = (x')^2 + (y')^2$

objects (observers) at rest:

same  $x, y, z$  always,  $t$  ticks forward

geometrically, a line in spacetime: **“world line”**

if at rest: world line vertical

constant speed:  $x = vt$ : diagonal line light: moves at “speed of light”  $c$

→ well-defined, since motion absolute

in spacetime: light pulse at origin  $(t, x, y, z) = (0, 0, 0, 0)$

moves so that distance  $\ell = \sqrt{x^2 + y^2 + z^2} = ct$

geometrically: **light cone**

## Galilean Transformations: Spacetime Picture

Geometrically:

different inertial frames  $\rightarrow$  transformation of spacetime

slide the “space slices” at each time

(picture “shear,” or beveling a deck of cards)

## Spacetime and Relativity

Pre-Relativity: space and time separate and independent but *rotations* mix *space* coords, e.g.,

$$x' = x \cos \theta - y \sin \theta \quad ; \quad y' = y \cos \theta + x \sin \theta \quad (26)$$

without changing underlying vector (rotation of coords only)

transform rule holds not only for  $\vec{x}$

but all other physical directed quantities: e.g.,  $\vec{v}, \vec{a}, \vec{p}, \vec{g}, \vec{E}$

all transform under rotations following same rule, e.g.,

$$E'_x = E_x \cos \theta - E_y \sin \theta \quad ; \quad E'_y = E_y \cos \theta + E_x \sin \theta \quad (27)$$

Lesson: express & guarantee underlying rotational invariance by writing physical law in vector form

∞ e.g.,  $\vec{F} = m\vec{a}$  gives same physics for any coord rotation