Astronomy 501: Radiative Processes

Lecture 35

Nov 14, 2022

Announcements:

- **Problem Set 11–final one!–due Friday**

  Q1 wordy but not much to calculate!
- **Radiative Meme Submission on Canvas**

Last time: radiation from relativistic matter

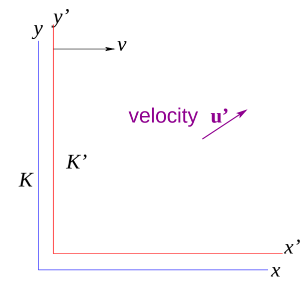
Q: condition for relativistic vs non?

Q: what are the known astrophysical relativistic sources?

Q: special relativity: events? spacetime? boosts?

Addition of Velocities

consider an object moving wrt to frame K'
as seen in frame K':
What is speed in frame K (speed v wrt K)?



decompose $\vec{u}' = \vec{u}'_{\parallel} + \vec{u}'_{\perp}$ where $\parallel$ is along $K - K'$ motion:

$$u_{\parallel} = \frac{u'_{\parallel} + v}{1 + u_{\parallel} v/c^2} \tag{1}$$

$$u_{\perp} = \frac{u'_{\perp}}{\gamma(1 + u_{\parallel} v/c^2)} \tag{2}$$

boost **changes in velocity direction** angle θ wrt $\vec{v}_{\rm frame}$

$$\tan\theta = \frac{u_{\perp}}{u_{\parallel}} = \frac{u' \sin\theta'}{\gamma(u' \cos\theta' + v)} \tag{3}$$

Boosted Light: Moving Emitter

consider the case of a moving emitter

- emitter speed $v = \beta c$ relative to observer
- emit **light**: speed $u' = c$, velocity $\vec{u}'$ in moving frame
- photon angle θ' in moving frame wrt v

in observer frame, photon angle wrt v is

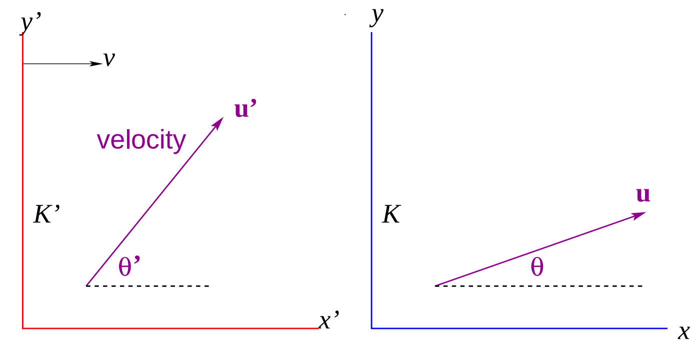
$$\tan\theta = \frac{\sin\theta'}{\gamma(\cos\theta' + \beta)} \qquad (4)$$

$$\cos\theta = \frac{\cos\theta' + \beta}{1 + \beta\cos\theta'} \qquad (5)$$

angular shift is the **aberration of light**

a light signal emitted in K'
at angle θ
is seen in K at angle

$$\cos\theta = \frac{\cos\theta' + v/c}{1 + v/c \ \cos\theta'}$$



Q: what if $\theta' = 0$? π?
Q: how can we understand this physically?

Q: what if $\theta' = \pi/2$?
Q: how can we understand this physically?

consider photons emitted *isotropically* in K'
with v/c not small
Q: what is angular pattern in K? *implications?*

c

Relativistic Beaming

for light emitted in K' at $\theta' = \pi/2$
observed angle after boosting is

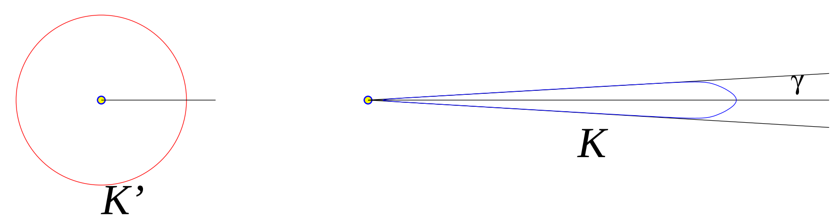
$$\tan\theta = \frac{1}{\gamma v/c} \tag{6}$$

and thus

$$\sin\theta = \frac{1}{\gamma} \tag{7}$$

if emitted K' is highly relativistic,
then $\gamma \gg 1$, and

$$\theta \to \frac{1}{\gamma}$$

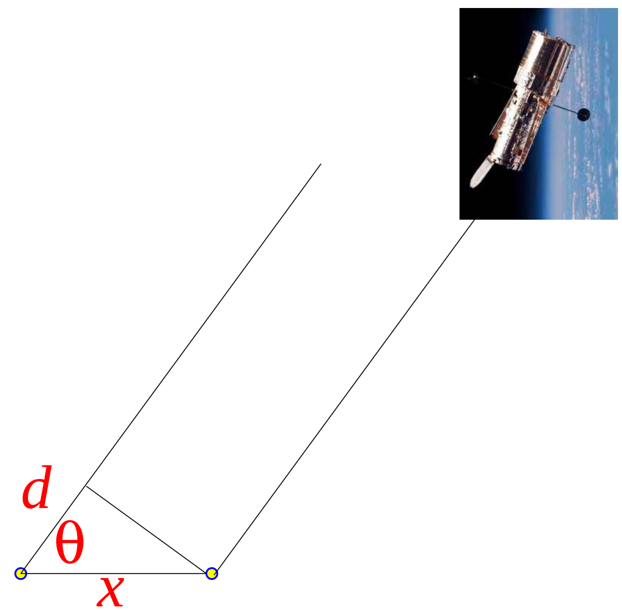


i.e., a small forward angle!
a highly relativistic emitter gives a **beamed radiation pattern**
strongly concentrated ahead of emitter direction

Relativistic Doppler Effect

emitter moves with speed v wrt observer

in emitter frame K':
light has (rest) frequency ω'
first wave crest emitted at $t' = 0$
second wave crest emitted at $t' = 2\pi\omega'$



in observer frame K:
observe light at angle θ
second wave crest after emitter travels distance $x = vt$

difference in observed light arrival times is

$$\delta t = t - d/c = (1 - v\cos\theta/c)t \tag{8}$$

difference in observed light arrival times is

$$\delta t = t - d/c = (1 - v \cos\theta/c)t \tag{9}$$

and since $t = t'/\gamma$, we have

$$\omega' = \left(1 - \frac{v}{c}\cos\theta\right)\frac{\omega}{\gamma} \tag{10}$$

so: light emitted at rest frequency $\omega' = \omega_{\text{emit}}$
is observed at angle θ to have frequency

$$\omega = \left(1 - \frac{v}{c}\cos\theta\right)\frac{\omega'}{\gamma} \tag{11}$$

and thus

$$\omega_{\text{obs}} = \gamma\left(1 - \frac{v}{c}\cos\theta\right)\omega_{\text{emit}} \tag{12}$$

relativistic Doppler formula

Awesome Example: Relativistic Jets

Consider back-to-back jets ejected from a black hole
- moving fast: $\gamma \gg 1$
- at some inclination angle i relative to the sightline

Q: appearance for $i = 90^\circ$*?* 0°*? intermediate inclination?*

`www: M87 jet`

The Mystery of the Ionizing Radiation

Early history: $\sim 1900 - 1912$
pioneers of radioactivity studies (M. Curie et al) found :
some elements (isotopes) emit α, β, or γ-rays
powerful ionizing agents
with different ranges in matter = "penetrating power"
Q: what are α, β, γ rays? why emitted?
Q: for, e.g., $\sim few$ MeV, which is most, least penetrating?

But soon realized that even *without* radioactive samples
ionization gauges give nonzero signal!
$\Rightarrow$ *"background" radiation*

Q: possible sources?
Q: how would you design an experiment to
discriminate among then using 1912 technology?

Victor Hess and the Discovery of Cosmic Rays

possible background ionization sources:

- **terrestrial:** from radioactive isotopes in Earth's crust
  e.g., uranium, thorium
- **extraterrestrial:** from Sun?

Victor Hess, 1912: take ionization detectors on hot-air balloon

- ionization signal first goes *down*,
  but by $h \sim 5$ km goes *up* to $\sim few\times$ sea level rate!

$\Rightarrow$ terrestrial ionization sources dominant at ground
...but extraterrestrial sources exist!

- survive passage thru atmosphere $\Rightarrow$ very penetrating: γ rays?

Q: how to tell is cosmic rays are solar or extrasolar?

Hess repeated balloon experiment during solar eclipse:

• no reduction in signal

$\Rightarrow$ radiation does not come from Sun!

$\Rightarrow$ *"cosmic radiation"* = **cosmic rays**

`www: 1936 Nobel Prize in Physics`

Cosmic Rays: Vital Statistics

Cosmic rays: population of particles which are

- **electrically charged**
- **energetic** ($\gtrsim 1$ MeV)
- **nonthermal** *Q: meaning?*

Cosmic Ray Sources:

- *solar activity:* ~ 0.1 MeV to ~ 1 GeV, typically few MeV
  `www: Solar Flares`
- but most cosmic rays: *extrasolar*

`www: real-time satellite data`

Cosmic Ray Composition

composition: mostly nuclei (fully stripped of e^-)

- **nuclear ("hadronic") component**
  90% are *protons*
  of remainder, 90% are α
  elements up to Se detected
  `www: proton flux`

- **electron/positron ("leptonic") component**
  mostly e^-, some e^+
  at fixed energy, electron flux $\mathcal{I}_E(e) \sim \mathcal{I}_E(p)/100$ of protons

angular distribution:
isotropic over most of energy range

cosmic rays often annoy non-CR astronomers *Q: why?*

Observed Electron Component

Experimental techniques:

- balloons
- space missions
- ground-based (high-energy): atm Čerenkov, air shower arrays

flux at top of atmosphere depends on location *Q: why?*
and on time

anti-correlation between CR flux at Earth and solar activity

$\Rightarrow$ solar "modulation" of CR

- excludes $\lesssim 100$ MeV particles
- reduces $\lesssim 1$ GeV flux

must correct for solar effects ("demodulate") to infer interstellar spectra

Cosmic Ray Spectrum

specific *number* intensity usually expressed in units of energy

$$\mathcal{I}_E = \frac{d\mathcal{N}}{dA\,dt\,d\Omega\,dE} \tag{13}$$

$$= v(E)\frac{d\mathcal{N}}{dV\,d\Omega\,dE} \tag{14}$$

or units of Lorentz $\gamma = E_{\rm tot}/m_e c^2$

`www: CR electron spectrum`

Q: what is number intensity spectrum $\mathcal{I}_E$? energy intensity spectrum I_E?

`www: CR proton spectrum`

Cosmic-Ray Electron Spectrum

cosmic ray spectrum clearly **nonthermal**
i.e., not a Fermi-Dirac form appropriate for thermal fermions
rather: a succession of *power laws*

- observed CR electrons: *number spectrum* roughly

$$\mathcal{I}_E(e) \propto E^{-3} \tag{15}$$

and thus *usual energy intensity* $I_E(e) = E\, \mathcal{I}_E(e) \propto E^{-2}$

Q: will protons be the same? different?

Director's Cut Extras

Cosmic-Ray Proton Spectrum

- protons w/ 1 GeV $\lesssim E \lesssim$ 300 TeV:

$$I_E(p) \simeq 1.4 \left(\frac{E}{\text{GeV}}\right)^{-s} \text{ protons cm}^{-2}\text{ s}^{-1}\text{ sr}^{-1}\text{GeV}^{-1} \qquad (16)$$

  where spectral index ("slope") $s \simeq 2.7$
- beyond "knee' at $E_{\text{knee}} \sim 10^{15}$ eV
  power law index steepens to $s \sim 3$
- then beyond "ankle" at $E_{\text{anlke}} \simeq 10^{18}$ eV, flattens again

Q: Tevatron energy? LHC? implications?

historically: many particles first discovered via CRs

Q: in which regime are most CR particles? most CR energy?

What's typical?

cosmic-ray number flux
$\Phi(>E) = 4\pi \int I(E)\ dE = 4\pi \int E\ I(E)\ d\ln E$
per log energy interval, number distribution is
$d\Phi/d\ln E \sim E\ I(E) \sim E^{-(s-1)}$
$\rightarrow$ *number peaks at smallest (but still relativistic) energies*
typical proton: $E \sim 1$ GeV

cosmic-ray energy proton flux $F(>E) = 4\pi \int E\ I(E)\ dE$
per log energy interval, $dF/d\ln E \sim E^2\ I(E) \sim E^{-(s-2)}$
$\Rightarrow$ since $s > 2$, *energy also peaks at low energies*

ensemble of cosmic rays acts as *mildly relativistic gas*

spectrum poses questions:
- origin(s) of the power-law behavior?
- what leads to the different regimes?