Astronomy 501: Radiative Processes

Lecture 37

Nov 18, 2022

Announcements:

- **Problem Set 11–final one!–due today**
- Enjoy your break, consider submitting a meme!

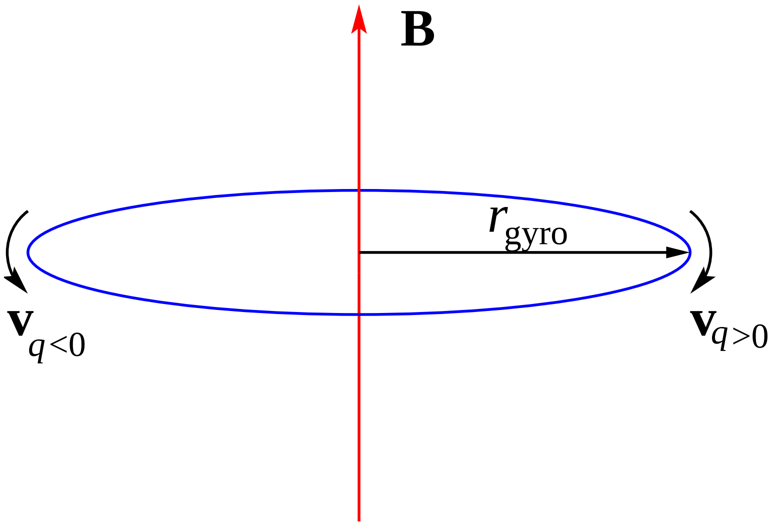
last time:

synchrotron radiation from cosmic rays

- *Q: how do cosmic rays propagte in and beyond the Galaxy?*
- *Q: motion of a charged particle in a uniform magnetic field?*
- *Q: why will this lead to radiation? affect on the particle?*

charged particle in uniform $\vec{B}$

$$\begin{aligned} v_{\parallel} &= \text{const} \\ \frac{d\vec{v}_{\perp}}{dt} &= \vec{v} \times \vec{\omega}_B \\ v^2 &= v_{\parallel}^2 + v_{\perp}^2 = \text{const} \\ \gamma &= \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{E_{\text{tot}}}{mc^2} = \text{const} \end{aligned}$$



- *uniform velocity $v_{\parallel}$ along $\hat{B}$*
- *uniform circular motion orthogonal to $\hat{B}$*
  gyrofrequency $\omega_B = qB/\gamma mc$
  gyroradius $r_{\text{gyro}} = v_{\perp}/\omega_B = mc\gamma v_{\perp}/qB = cp_{\perp}/qB$
- net motion: **spiral around field line**

curved path →acceleration →radiation!

- non-relativistic particles: cyclotron radiation
- ultra-relativistic particles: **synchrotron radiation**

Synchrotron Radiation: Total Power

for isotropic electron population
average emitted power per electron:

$$P_e = \left|\frac{dE_e}{dt}\right| = \left(\frac{2}{3}\right)^2 r_0^2 \ c \ \gamma^2 \beta B^2 = \frac{4}{3} \sigma_\mathrm{T} \ c \ \beta^2 \gamma^2 \ u_B \tag{1}$$

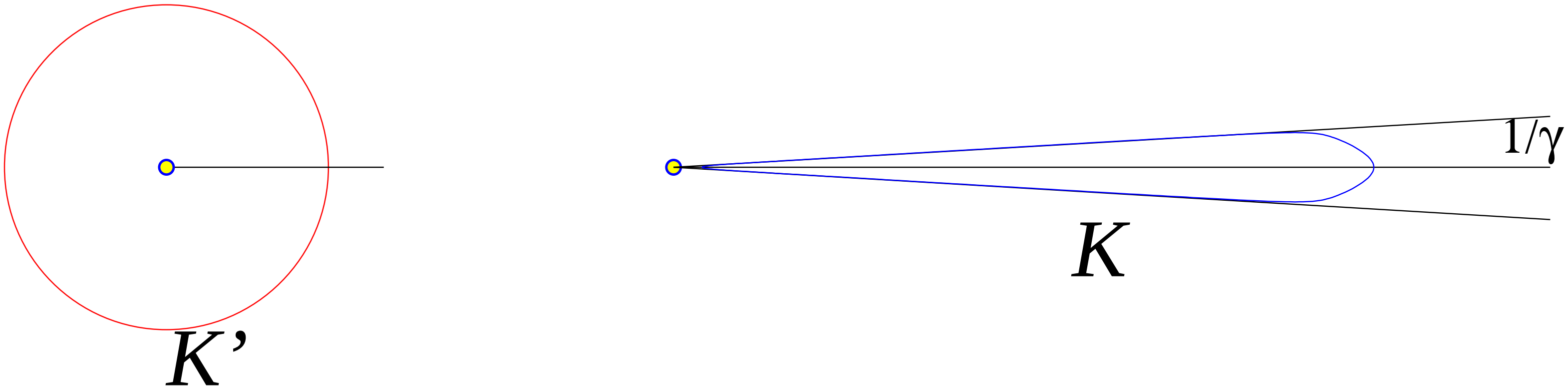
where $\sigma_\mathrm{T} = 8\pi r_0^2/3$ and $u_B = B^2/8\pi$

Q: energy dependence for non-relativistic electrons?
Q: energy dependence for ultra-relativistic electrons?
Q: stopping timescale for ultra-relativistic electrons?

Spectrum of Synchrotron Radiation: Order of Magnitude

key issue:

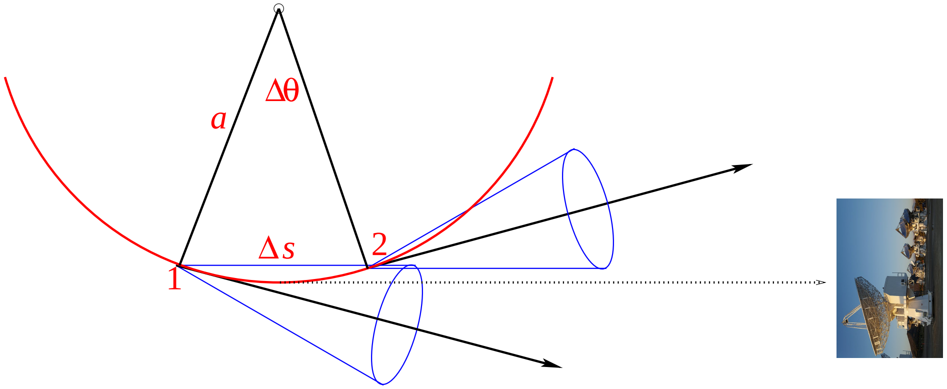
radiation from a relativistic accelerated particle is *beamed* into forward cone of opening angle $\theta_{\text{beam}} \sim 1/\gamma$



so observer receives pulses or “flashes” of radiation spread over narrow timescale $\ll 2\pi/\omega_B$

sharply peaked signal in time domain

$\Rightarrow$ *broad signal in frequency domain* consider relativistic charge moving in circle of radius a



observer only sees emission over angular range

$$\Delta\theta \simeq 2\theta_{\mathrm{beam}} \simeq \frac{2}{\gamma} \tag{2}$$

representing a path length

$$\Delta s = a \ \Delta\theta = \frac{2a}{\gamma} \tag{3}$$

curvature radius $a = v/\omega_B \sin\alpha$, with $\sin\alpha = v_\perp/v$ so

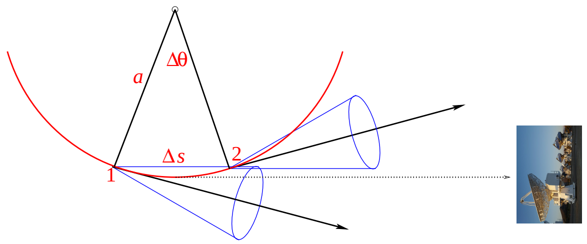
$$\Delta s \simeq \frac{2v}{\gamma\omega_B \sin\alpha} \tag{4}$$

if the particle *passes point 1* at t_1 and point 2 at t_2
$\Delta s = v(t_2 - t_1)$, and

$$\Delta t = t_2 - t_1 \simeq \frac{2}{\gamma\omega_B \sin\alpha} \tag{5}$$

what is *arrival time* of radiation?
note that point 2 is closer than point 1 by $\approx \Delta s$

$$\begin{aligned} \Delta t^{\rm arr} = t_2^{\rm arr} - t_1^{\rm arr} &= \Delta t - \frac{\Delta s}{c} \\ &= \Delta t \left(1 - \frac{v}{c}\right) \\ &= \frac{2}{\gamma\omega_B \sin\alpha}\left(1 - \frac{v}{c}\right) \end{aligned}$$



radiation arrive time duration

$$\Delta t^{\mathrm{arr}} = \frac{2}{\gamma \omega_B \sin \alpha} \left(1 - \frac{v}{c} \right) \tag{6}$$

but note that $1 - v/c \approx 1/2\gamma^2$ for relativistic motion *Q:why?*

and thus radiation arrives in pulse of duration

$$\Delta t^{\mathrm{arr}} \approx \frac{1}{\gamma^3 \omega_B \sin \alpha} \tag{7}$$

shorter than ω_B^{-1} by factor γ^3!

define **critical frequency**

$$\omega_{\mathrm{c}} \equiv \frac{3}{2}\gamma^3 \omega_B \sin \alpha = \frac{3}{2}\gamma^2 \; \frac{qB \sin \alpha}{mc} = \frac{3}{2}\gamma^2 \; \omega_{\mathrm{g}} \sin \alpha \tag{8}$$

$$\nu_{\mathrm{c}} = \frac{\omega_{\mathrm{c}}}{2\pi} = \frac{3}{4\pi}\gamma^3 \omega_B \sin \alpha \tag{9}$$

Q: will radiation spectrum cut off above or below ω_{c}*?*

critical frequency

$$\nu_{\mathrm{c}} = \frac{3}{4\pi}\gamma^3 \omega_B \sin\alpha \sim \frac{1}{\Delta t^{\mathrm{arr}}} \tag{10}$$

Fourier transform of pulse Δt^{arr} broad up to ν_{c}
and should cut off above this

numerically:

$$\nu_{\mathrm{c}} = 25\ \mathrm{MHz}\ \left(\frac{E_e}{1\ \mathrm{GeV}}\right)^2\ \left(\frac{B}{1\ \mu\mathrm{Gauss}}\right)\ \sin\alpha \tag{11}$$

Q: lessons? irony?

critical = characteristic frequency $\nu_c \sim 25\ \mathrm{MHz}\ (E_e/1\ \mathrm{GeV})^2$
typical cosmic-ray electrons emit in the observable *radio*
→ *high-energy* electrons can emit *low-frequency* radiation!

expect synchrotron power of form $P(\omega) \sim P/\omega_{\rm c}\ F(\omega/\omega_{\rm c})$
with dimensionless function $F(x)$

- should be peaked at $x \sim 1$, then drop sharply
- can only be gotten from an honest calculation!

note: $P \propto \gamma^2$ but $\omega_{\rm c} \propto \gamma^2 \rightarrow P/\omega_{\rm c}$ indep of γ

for a particle with a fixed v and γ,
conventional to define synchrotron spectrum as

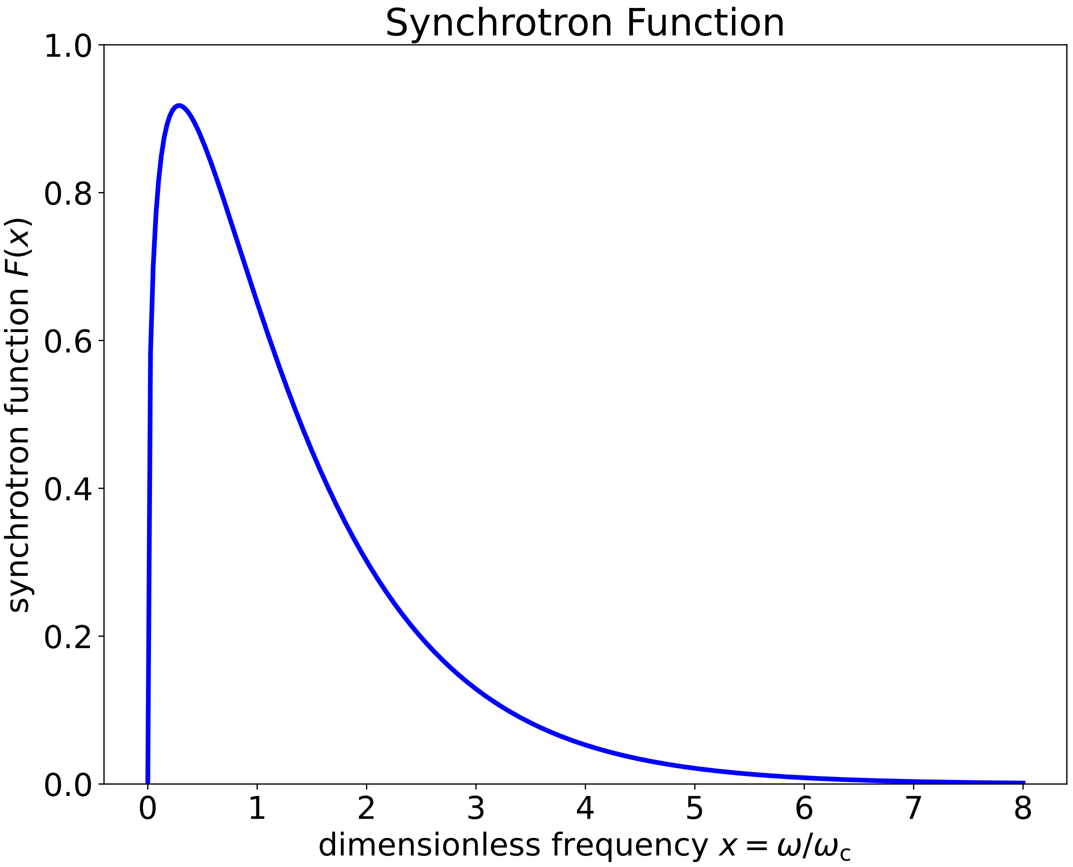
$$\frac{dP}{d\omega} = P(\omega) = \frac{\sqrt{3}}{2\pi}\frac{q^3 B \sin\alpha}{mc^2}\; F\left(\frac{\omega}{\omega_{\rm c}}\right) \tag{12}$$

with $\omega_{\rm c} \propto \gamma^2$

where the *synchrotron function* (derived in RL) is

$$F(x) = x\int_x^\infty K_{5/3}(t)\; dt \longrightarrow \begin{cases} \frac{4\pi}{\sqrt{3}\Gamma(1/3)}\left(\frac{x}{2}\right)^{1/3} & x \ll 1 \\ \left(\frac{\pi}{2}\right)^{1/2} e^{-x} x^{1/2} & x \gg 1 \end{cases} \tag{13}$$

with $K_{5/3}(x)$ the modified Bessel function of order 5/3
→ *sharply peaked* at $\omega_{\rm max} = x_{\rm max}\omega_{\rm c} = 0.29\omega_{\rm c}$



Q: so is this the spectrum we would see for real CR es?

for a **single** electron γ
emission spectrum is synchrotron function $F(\omega/\omega_{\rm c})$
sharply peaked near $\omega_{\rm c} \propto \omega_{\rm g}\gamma^2$

but the *population* of cosmic-ray electrons
has a *spectrum* of energies and thus of γ

resulting synchrotron spectrum is

- *superposition* of peaks $\propto \gamma^2$,
- *weighted by electron energy spectrum*

Q: what if CRs had two energies? N energies?

Q: what does the real spectrum look like?

Q: what's the synchrotron spectral shape for the ensemble of all electron energies?

recall: cosmic-ray electron spectrum well-fit by *power law*
so number of particles with energy in $(E, E + dE)$ is

$$N(E)\ dE = C\ E^{-p}\ dE \tag{14}$$

and so

$$N(\gamma)\ d\gamma = C'\ \gamma^{-p}\ d\gamma \tag{15}$$

note that for a single electron v and γ
$P(\omega) \propto F(\omega/\omega_{\mathrm{c}})$ and $\omega_{\mathrm{c}} = \omega_{\mathrm{g}}\gamma^2$

so integrating over full CR spectrum means

$$\begin{aligned}
\langle P(\omega) \rangle &= \int P(\omega)\ N(\gamma)\ d\gamma && (16)\\
&= C' \int P(\omega)\ \gamma^{-p}\ d\gamma && (17)\\
&\propto \int F\left(\frac{\omega}{\omega_{\mathrm{g}}\gamma^2}\right)\ \gamma^{-p}\ d\gamma && (18)
\end{aligned}$$

Q: strategy?

$$\langle P(\omega)\rangle \propto \int F\left(\frac{\omega}{\omega_{\rm g}\gamma^2}\right)\ \gamma^{-p}\ d\gamma \tag{19}$$

change integration variable to $x = \omega/\omega_{\rm c} = \gamma^{-2}\omega/\omega_{\rm g}$
$\to \gamma = (\omega x/\omega_{\rm g})^{-1/2}$, and $d\gamma = -(\omega/\omega_{\rm g})^{-1/2} x^{-3/2} dx$

$$\langle P(\omega)\rangle \propto \left(\frac{\omega}{\omega_{\rm g}}\right)^{-(p-1)/2} \int F(x)\ x^{(p-3)/2}\ dx \tag{20}$$

and so

$$\langle P(\omega)\rangle \propto \omega^{-(p-1)/2} = \omega^{-s} \tag{21}$$

with **spectral index** $s = (p-1)/2$

even though each electron energy → peaked emission
average over power-law electron distribution
→ power-law synchrotron emission

full expression for power-law electron spectrum
of the form $dN/d\gamma = C\gamma^{-p}$

$$4\pi j_{\rm tot}(\omega) = \frac{\sqrt{3}q^3 CB\sin\alpha}{2(p+1)\pi mc^2}\,\Gamma\left(\frac{p}{4}+\frac{9}{12}\right)\,\Gamma\left(\frac{p}{4}-\frac{1}{12}\right)\,\left(\frac{mc\omega}{3qB\sin\alpha}\right)^{-(p-1)/2} \tag{22}$$

with $\Gamma(x)$ the gamma function, with $\Gamma(x+1) = x\,\Gamma(x)$

Q: overall dependence on B? does this make sense?

Q: expected spectral index?

Q: do you expect the signal to be polarized? how?

Source Function

source function

$$S_\nu = \frac{j_\nu}{\alpha_\nu} \propto \frac{\nu^{-(p-1)/2}}{\nu^{-(p+4)/2}} = \nu^{5/2} \tag{23}$$

to see this, recall that

$$j_\nu \sim \int dE\ N(E)\ P(\nu) \tag{24}$$

$$\alpha_\nu \sim \nu^{-2} \int dE\ \frac{N(E)}{E}\ P(\nu) \tag{25}$$

thus source function has

$$S_\nu \sim \nu^2 \bar{E} \tag{26}$$

with typical electron energy $\bar{E} = m\bar{\gamma}$ for freq ν
but $\nu(E) \approx \nu_{\mathrm{c}}(E) \sim E^2$, so $\bar{E} \propto \nu^{1/2}$
and thus $S_\nu \sim \nu^{5/2}$ *independent of electron spectral index*

Synchrotron Radiation: the Big Picture

for relativistic electrons with power-law energy distribution

emission coefficient

$$j_\nu \propto \nu^{-(p-1)/2} \tag{27}$$

absorption coefficient (see Directors' cut extras from last time)

$$\alpha_\nu \propto \nu^{-(p+4)/2} \tag{28}$$

source function (note nonthermal character!)

$$S_\nu \propto \nu^{5/2} \tag{29}$$

Q: optical depth vs ν? implications?
Q: spectrum of a synchrotron emitter?

`www: awesome example: pulsar wind nebulae`
young pulsars are spinning down
much of rotational energy goes into relativistic wind
which collides with the supernova ejecta an emits synchrotron

Build Your Toolbox–Synchrotron Radiation

emission physics: matter-radiation interactions

Q: physical conditions for synchrotron emission? absorption?

Q: physical nature of sources?

Q: spectrum characteristics?

Q: frequency range?

real/expected astrophysical sources of synchrotron radiation

Q: what do we expect to emit synchrotron? absorb?

Q: relevant temperatures? EM bands?

Toolbox: Synchrotorn Radiation

emission physics

- physical conditions: relativistic charged particles in magnetic field
- physical sources: relativistic electrons dominate
- spectrum: for electron energy distribution $dN_e/dE_e \propto E_e^{-p}$ synchrotron emission is continuum with *power law* $j_\nu \sim \nu^{-(p-1)/2}$ spectrum and source function $S_\nu \sim \nu^{5/2}$

astrophysical sources of synchrotron

- emitters: relativistic electrons: cosmic rays in galaxies or in jets
- temperatures: trick question! sources are nonthermal!
- EM bands: max synch energy depends on max γ and magnetic field, can go from radio to X-ray!

Astrophysical Context: Blazars

we met radio galaxies in the context of synchrotron radiation
but there are many beasts in the active galaxy zoo

Blazars

- seen as luminous nuclear region
  at center of giant elliptical galaxies
  `www: optical blazar images (`R`-band)`
- but *do not* show the elongated jets seen in radio galaxies
- flux shows rapid and large-amplitude time variability
- subclasses: BL Lacertae objects–weak radio emission
  optically violent variables (OVV)–strong radio emission
- demographics: many fewer blazars than other AGN
  e.g., Seyfert galaxies
  `www: AGN demographics plot (INTEGRAL)`
- blazar emission spans radio to TeV gamma rays

Q: what does this suggest about the nature of blazars?

Blazars: Staring Down the Jet

AGN "Unification Model" `www: unification cartoon`
idea: all active galaxies have similar physical conditions

- a supermassive black hole (SMBH)
  possibly actively accreting matter
- a surrounding accretion disk, and dusty torus
- a relativistic jet, if SMBH is actively accreting

in unification picture: *blazar = jet pointing directly at us!*
"looking down the barrel of the gun"
emission from small region of jet "tip" → highly variable

blazar spectra `www: example`
over full EM range, two large features

- power-law rise from radio, peaks near optical
- falls to X-rays, then peak and power-law fall at gamma-ray

Q: what could be going on?

Blazar Spectra

Power-law rise from radio to ~optical
- nonthermal
- similarity with radio galaxies suggests *synchrotron origin* from relativistic electrons in jet

Peak and power-law fall in gamma rays
- in non-flare (“quiescent”) state, gamm-ray energy content similar to synchrotron
- suggests similar origin
  → perhaps a *reprocessing* of the synchrotron photos
- reprocessed how? *by the relativistic electrons themselves!*

Thus: we want to understand how relativistic electrons interact with photons *Q: the name for which is...?*

Note: blazar neutrinos seen! → imply proton emission $pp \to \pi^0 \to \gamma\gamma$!