Astronomy 501: Radiative Processes

Lecture 38

Nov 28, 2022

Announcements:

- **Final Exam – Tuesday Dec 13.**
  take home. Questions posted 1:30pm, due by 10pm.
  designed to take < 3 hours. More info to come on Canvas
  open book, open notes. No internet, no collaboration.
- Some rest for the weary – no more problem sets!
- Submit an **AstroMeme**! Oustanding entries so far.

before break:

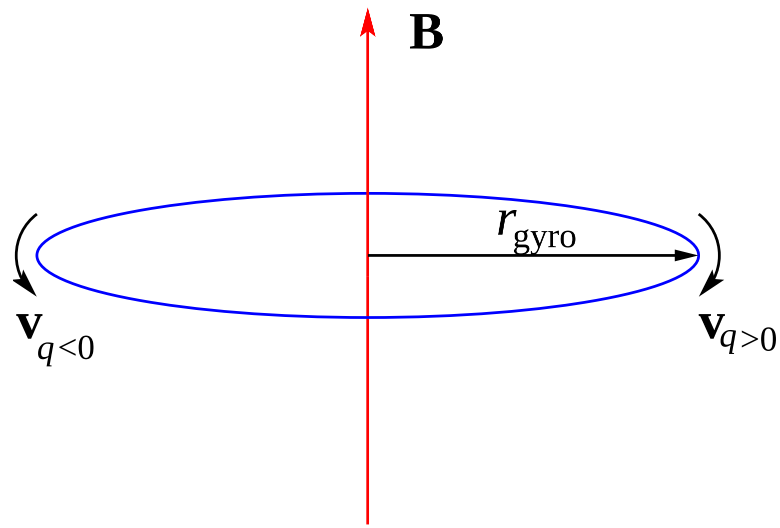
almost finished synchrotron radiation

Q: what is synchrotron radiation?

Q: what physical conditions are needed?

charged particle in uniform $\vec{B}$

$$\begin{aligned} v_{\parallel} &= \text{const} \\ \frac{d\vec{v_{\perp}}}{dt} &= \vec{v} \times \vec{\omega}_B \\ v^2 &= v_{\parallel}^2 + v_{\perp}^2 = \text{const} \\ \gamma &= \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{E_{\text{tot}}}{mc^2} = \text{const} \end{aligned}$$



- *uniform velocity* $v_{\parallel}$ *along* $\hat{B}$
- *uniform circular motion orthogonal to* $\hat{B}$

  gyrofrequency $\omega_B = qB/\gamma mc$

  gyroradius $r_{\text{gyro}} = v_{\perp}/\omega_B = mc\gamma v_{\perp}/qB = cp_{\perp}/qB$
- net motion: **spiral around field line**

curved path →acceleration →radiation!

- non-relativistic particles: cyclotron radiation
- ultra-relativistic particles: **synchrotron radiation**

Synchrotron Emission and Absorption

depends on energy spectrum of the relativistic electrons
for power-law electron spectrum $dN/d\gamma = C\gamma^{-p}$:

emission coefficient

$$4\pi j_{\rm tot}(\omega) = \frac{\sqrt{3}q^3 CB\sin\alpha}{2(p+1)\pi mc^2}\,\Gamma\left(\frac{p}{4}+\frac{9}{12}\right)\,\Gamma\left(\frac{p}{4}-\frac{1}{12}\right)\,\left(\frac{mc\omega}{3qB\sin\alpha}\right)^{-(p-1)/2} \quad (1)$$

with $\Gamma(x)$ the gamma function, with $\Gamma(x+1) = x\,\Gamma(x)$

absorption coefficient

$$\alpha_\nu = \frac{\sqrt{3}}{8\pi}\Gamma\left(\frac{3p+2}{12}\right)\Gamma\left(\frac{3p+22}{12}\right) \left(\frac{3q}{2\pi m^3c^5}\right)^{p/2}\left(\frac{q^3C}{m}\right)\,(B\sin\alpha)^{(p+2)/2}\,\nu^{-(p+4)/2}$$

Synchrotron Radiation: the Big Picture

for relativistic electrons with power-law energy distribution

emission coefficient

$$j_\nu \propto \nu^{-(p-1)/2} \tag{2}$$

absorption coefficient (see Directors' cut extras from last time)

$$\alpha_\nu \propto \nu^{-(p+4)/2} \tag{3}$$

source function (note nonthermal character!)

$$S_\nu \propto \nu^{5/2} \tag{4}$$

Q: optical depth vs ν? implications?
Q: spectrum of a synchrotron emitter?

`www: awesome example: pulsar wind nebulae`

young pulsars are spinning down
much of rotational energy goes into relativistic wind
which collides with the supernova ejecta an emits synchrotron

Build Your Toolbox–Synchrotron Radiation

emission physics: matter-radiation interactions

Q: physical conditions for synchrotron emission? absorption?

Q: physical nature of sources?

Q: spectrum characteristics?

Q: frequency range?

real/expected astrophysical sources of synchrotron radiation

Q: what do we expect to emit synchrotron? absorb?

Q: relevant temperatures? EM bands?

Toolbox: Synchrotorn Radiation

emission physics

- physical conditions: relativistic charged particles in magnetic field
- physical sources: relativistic electrons dominate
- spectrum: for electron energy distribution $dN_e/dE_e \propto E_e^{-p}$ synchrotron emission is continuum with *power law* $j_\nu \sim \nu^{-(p-1)/2}$ spectrum and source function $S_\nu \sim \nu^{5/2}$

astrophysical sources of synchrotron

- emitters: relativistic electrons: cosmic rays in galaxies or in jets
- temperatures: trick question! sources are nonthermal!
- EM bands: max synch energy depends on max γ and magnetic field, can go from radio to X-ray!

Astrophysical Context: Blazars

we met radio galaxies in the context of synchrotron radiation
but there are many beasts in the active galaxy zoo

Blazars

- seen as luminous nuclear region
  at center of giant elliptical galaxies
  `www: optical blazar images (`R`-band)`
- but *do not* show the elongated jets seen in radio galaxies
- flux shows rapid and large-amplitude time variability
- subclasses: BL Lacertae objects–weak radio emission
  optically violent variables (OVV)–strong radio emission
- demographics: many fewer blazars than other AGN
  e.g., Seyfert galaxies
  `www: AGN demographics plot (INTEGRAL)`
- blazar emission spans radio to TeV gamma rays

Q: what does this suggest about the nature of blazars?

Blazars: Staring Down the Jet

AGN "Unification Model" `www: unification cartoon`
idea: all active galaxies have similar physical conditions

- a supermassive black hole (SMBH)
  possibly actively accreting matter
- a surrounding accretion disk, and dusty torus
- a relativistic jet, if SMBH is actively accreting

in unification picture: *blazar = jet pointing directly at us!*
"looking down the barrel of the gun"
emission from small region of jet "tip" $\rightarrow$ highly variable

blazar spectra `www: example`
over full EM range, two large features

- power-law rise from radio, peaks near optical
- falls to X-rays, then peak and power-law fall at gamma-ray

Q: what could be going on?

Blazar Spectra

Power-law rise from radio to ~optical

- nonthermal
- similarity with radio galaxies suggests *synchrotron origin* from relativistic electrons in jet

Peak and power-law fall in gamma rays

- in non-flare ("quiescent") state, gamma-ray energy content similar to synchrotron
- suggests similar origin
  → perhaps a *reprocessing* of the synchrotron photos
- reprocessed how? *by the relativistic electrons themselves!*

Thus: we want to understand how relativistic electrons interact with photons *Q: the name for which is...?*

Note: blazar neutrinos seen! implies proton emission $pp \to \pi^0 \to \gamma\gamma$!

Compton Scattering

Thomson Scattering Revisited

We already discussed the scattering of light by electrons in the context of *Thomson scattering*

Thomson highlights:
energies of incident photon ϵ and scattered photon ϵ_1 related by

$$\epsilon_1 = \epsilon \tag{5}$$

differential cross section, with $\hat{k} \cdot \hat{k}_1 = \cos\theta$

$$\frac{d\sigma_{\mathrm{T}}}{d\Omega} = \frac{1}{2} r_0^2 \left(1 + \cos^2\theta\right) \tag{6}$$

total cross section, with $r_0 = e^2/m_e c^2$

$$\sigma_{\mathrm{T}} = \frac{8\pi}{3} r_0^2 \tag{7}$$

Q: what assumptions went into this? When will they fail?

Enter the Quantum: Compton Scattering

Thomson scattering derived for classical EM wave

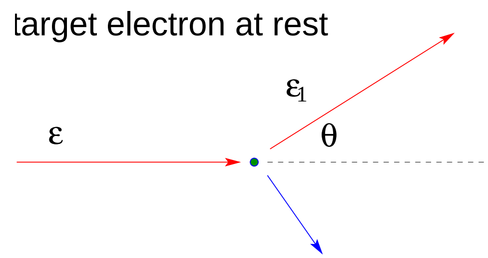
- $\nu_1 = \nu$ classically
- carrying this to photon picture: $h\nu_1 = h\nu$

$\rightarrow$ coherent or elastic scattering but really: photon quanta carry momentum and energy

$\rightarrow$ and electron will *recoil* and carry away energy

$\rightarrow$ expect scattered photon to have less energy, and to move in a different direction

Compton: treat light as massless particle



for photon incident on electron *at rest*
conservation of energy and momentum implies

$$\epsilon_1 = \frac{\epsilon}{1 + (\epsilon/m_e c^2)(1 - \cos\theta)} \tag{8}$$

scattered photon energy is lower, and direction different

so the wavelength shifts by

$$\lambda_1 - \lambda = \lambda_C (1 - \cos\theta) \tag{9}$$

where the electron *Compton wavelength* $\lambda_C = h/m_e c = 0.02426$ Å

Q: what energy does a photon with λ_C have?
Q: What region of the spectrum is this?

Q: when is the wavelength shift important? negligible?

Cross Section for Compton Scattering

Compton wavelength shift is $\Delta\lambda \sim \lambda_{\rm C}$

- *small* if $\lambda \gg \lambda_{\rm C}$ i.e., $h\nu \ll m_e c^2$
  i.e., radio through soft X-rays
- *large* if $h\nu \gg m_e c^2$: hard X-rays, gamma rays

differential cross section: **Klein-Nishina** formula

$$\frac{d\sigma}{d\Omega} = \frac{r_0^2}{2}\,\frac{\epsilon_1^2}{\epsilon^2}\,\left(\frac{\epsilon}{\epsilon_1} + \frac{\epsilon_1}{\epsilon} - \sin^2\theta\right) \tag{10}$$

- classical Thomson expression recovered when $\epsilon \sim \epsilon_1$
- main effect: *smaller* cross section at high energy
- total cross section, with $x = h\nu/m_e c^2$ in e rest frame

$$\sigma \to \begin{cases} \sigma_{\rm T}\ (1 - 2x + \cdots) & x \ll 1 \\ 3\sigma_{\rm T}/8\ x^{-1}(\ln 2x + \cdots) & x \gg 1 \end{cases} \tag{11}$$

recall: to understand blazars, we are interested in

- high-energy electrons interacting with ambient photons

Q: why can't *we just use the Compton scattering formulae?*

Q: how can *we use the formulae?*

Inverse Compton Scattering

the usual Compton scattering expressions
assume the electron is initially *at rest*
and the *photon loses energy* in scattering
$\rightarrow$ "ordinary kinematics"
but this is not the case we are interested in!

in a frame where the electron is relativistic

- then there can be a momentum and energy transfer and the photon *gains energy*
- "upscattered" to higher frequencies

$\rightarrow$ "inverse kinematics" – **inverse Compton scattering**

New Dance Craze: Inverse Compton Style

Step 0: plant your feet = consider *lab/observer frame*:
- relativistic electrons with $E = \gamma m_e c^2$
- isotropic photon distribution, energies ϵ

Step 1: jump (boost) to *electron rest frame*
Ask ourselves: what does the electron "see"?
Q: incident photon angular distribution? typical energy ϵ'?

for simplicity: let $\gamma\epsilon \ll m_e c^2$
$\rightarrow$ Thompson approximation good in e frame K'
Q: then what is angular distribution of scattered photons in K'?
Q: scattered photon energy lab frame, roughly?

Step 2: jump (boost) again, return to *lab frame*
Q: what is angular distribution of scattered photons?
Q: scattered photon energy in e rest frame, roughly?

Inverse Compton and Beaming

Recall: a photon distribution isotropic in frame K
is *beamed* into angle $\theta \sim 1/\gamma$ in highly boosted frame K'

so in *electron rest frame* K'
most lab-frame photons "seen" in head-on beam
with energy $\epsilon' \sim \gamma\epsilon$

if rest-frame energies in Thompson regime:

- scattered photon directions $\propto d\sigma/d\Omega \propto 1 + \cos^2\theta$
  $\rightarrow$ isotropic + quadrupole piece
- scattered energy $\epsilon'_1 \sim \epsilon' \sim \gamma\epsilon$

back in lab frame

- boost $\rightarrow$ scattered photons beamed forward
- scattered photon energy *boosted* to $\epsilon_1 \sim \gamma\epsilon'_1 \sim \gamma^2\epsilon$

Q: implications for blazar spectra?

Inverse Compton Power for Single-Electron Scattering

Consider a relativistic electron (γ, β)
incident on an isotropic distribution of ambient photons

Order of magnitude estimate of *power* into inverse Compton

- if typical ambient photon energy is ϵ
  then typical *upscattered energy* is $\epsilon_1 \sim \gamma^2 \epsilon$
- if ambient photon number density is $n_{\rm ph}$
  then *scattering rate per electron* is $\Gamma = n_{\rm ph} \sigma_{\rm T} c$ *Q: why?*

thus expect power = rate of energy into inverse Compton

$$\frac{dE_{1,\rm upscatter}}{dt} \sim \Gamma \ \epsilon_1 \sim \gamma^2 \epsilon n_{\rm ph} \sigma_{\rm T} c \sim \gamma^2 \sigma_{\rm T} c u_{\rm ph} \tag{12}$$

where $u_{\rm ph} = \langle \epsilon \rangle \, n_{\rm ph}$ is the ambient photon
energy density in the lab (observer) frame

Q: but what about scattering "removal" of incident photons?

upscattering "removes" some photons from ambient distribution
removal rate is scattering rate per electron: $\Gamma = n_{\rm ph} \sigma_{\rm T} c$
and thus rate of energy "removal" per electron is

$$\frac{dE_{1,\rm init}}{dt} = -\Gamma \langle \epsilon \rangle = -\sigma_{\rm T}\ c\ \langle \epsilon \rangle\, n_{\rm ph} = -\sigma_{\rm T}\ c\ u_{\rm ph} \tag{13}$$

because $\langle \epsilon \rangle \equiv u_{\rm ph}/n_{\rm ph}$

Note that

$$\frac{dE_{1,\rm upscatter}}{dt} \simeq \gamma^2 \left| \frac{dE_{1,\rm init}}{dt} \right| \tag{14}$$

$\rightarrow$ for $\gamma \gg 1$, large net energy gain!

net inverse Compton power per electron, when done carefully:

$$P_{\rm Compt} = \frac{4}{3} \sigma_{\rm T}\ c\ \gamma^2\ \beta^2\ u_{\rm ph} \tag{15}$$

Q: note any family resemblances?

Synchrotron vs Compton Power

We found the single-electron inverse Compton power to be

$$P_{\mathrm{Compt}} = \frac{4}{3}\sigma_{\mathrm{T}}\ c\ \gamma^2\ \beta^2\ u_{\mathrm{ph}} \tag{16}$$

but recall *synchrotron power*

$$P_{\mathrm{synch}} = \frac{4}{3}\sigma_{\mathrm{T}}\ c\ \gamma^2\ \beta^2\ u_B \tag{17}$$

formally identical! and note that

$$\frac{P_{\mathrm{synch}}}{P_{\mathrm{Compt}}} = \frac{u_B}{u_{\mathrm{ph}}} \tag{18}$$

for *any* electron velocity as long as $\gamma\epsilon \ll m_e c^2$

we turn next to spectra: good time to ask
Q: what is conserved in Compton scattering? implications?

Inverse Compton Spectra: Monoenergetic Case

in Compton scattering, the *number of photons is conserved*
i.e., ambient photons given new energies, momenta
but neither created nor destroyed

thus: the photon *number* emission coefficient $\mathcal{J}(\epsilon_1)$
must have $4\pi \int \mathcal{J}(\epsilon_1)\ d\epsilon_1 =$ number of scatterings per unit volume
and $4\pi \int (\epsilon_1 - \epsilon)\ \mathcal{J}(\epsilon_1)\ d\epsilon_1 =$ net Compton power

detailed derivation appears in RL: answer is

$$j(\epsilon_1; \epsilon, \gamma) = \frac{3}{4} N(\gamma)\ \sigma_{\mathrm{T}}\ c\ \frac{du_{\mathrm{ph}}}{d\epsilon}(\epsilon)\ x\ f(x) \tag{19}$$

with $N(\gamma) = dN/d\gamma$ the electron flux at γ
and $du_{\mathrm{ph}}/d\epsilon$ the ambient photon energy density at ϵ
and $f(x) = 2x \ln x + 1 + x - 2x^2$, with $x = \epsilon_1/(4\gamma^2\epsilon)$

Inverse Compton Scattering: Power-Law Electrons

as usual, assume power-law electron spectrum $N(\gamma) = C\ \gamma^{-p}$

still for a single ambient photon energy
integrate emission coefficient over all electron energies

$$j(\epsilon_1; \epsilon) = \int j(\epsilon_1; \epsilon, \gamma) d\gamma \tag{20}$$

$$= \frac{3}{4}\sigma_{\mathrm{T}}\ c\ \frac{du_{\mathrm{ph}}}{d\epsilon}(\epsilon) \int\ x\ f(x)\ N(\gamma) d\gamma \tag{21}$$

with $x = \epsilon_1/(4\gamma^2\epsilon)$
and where $x\ f(x)$ is peaked, with max at $x = 0.611$

Q: notice a family resemblance?

Q: strategies for doing integral?

Q: anticipated result?

for both IC and synchrotron: spectrum is integral of form

$$j(\epsilon_1;\epsilon) \propto \int G\left(\frac{\epsilon_1}{\gamma^2\epsilon_0}\right)\ \gamma^{-p}\ d\gamma \tag{22}$$

strategy is to change variables to $x = \epsilon_1/(\gamma^2\epsilon_0)$

result factorizes into product of
- dimensionless integral, times
- power law $j \propto (\epsilon_1/\epsilon)^{-(p-1)/2}$

so once again:
peaked emission spectrum for single-energy electron
smoothed to power-law emission spectrum, index $s = (p-1)/2$
for power-law electron energy distribution

full result in RL, guts are (up to numerical factors)

$$4\pi\ j(\epsilon_1;\epsilon) \sim \sigma_{\mathrm{T}}\ c\ C\ \epsilon_1^{-(p-1)/2}\ \epsilon^{(p-1)/2}\ \frac{du_{\mathrm{ph}}}{d\epsilon}(\epsilon) \tag{23}$$

Q: interesting choice of ambient photon distribution?

emission coefficient is

$$4\pi \; j(\epsilon_1;\epsilon) \sim \sigma_{\mathsf{T}} \; c \; C \; \epsilon_1^{-(p-1)/2} \; \epsilon^{(p-1)/2} \; \frac{du_{\mathsf{ph}}}{d\epsilon}(\epsilon) \qquad (24)$$

depends on background photon distribution via $du_{\mathsf{ph}}/d\epsilon$

for a thermal (Planck) photon distribution:

- $du/d\epsilon \sim T^4/T \sim T^3$, and
- $\epsilon^{(p-1)/2} \sim T^{(p-1)/2}$ and so

expect temperature scaling $j \sim T^{3+(p-1)/2} = T^{(p+5)/2}$

in fact:

$$4\pi \; j(\epsilon_1) = 4\pi \int j(\epsilon_1;\epsilon) \sim \frac{\sigma_{\mathsf{T}} \; C}{h^3 c^2} (kT)^{(p+5)/2} \; \epsilon_1^{-(p-1)/2} \qquad (25)$$

Awesome Examples

`www: Fermi sky movie: mystery object`

Q: what strikes you?

Q: how does the mystery object radiate > 100 MeV *photons?*

`www: WMAP Haze`

Q: what strikes you?

haze spectrum: $\propto \nu^{-0.5}$, flatter than usual synchrotron

Q: what electron index would this imply?

Q: if electrons continue to high E*, what should we see?*

`www: Fermi search for that feature`

Inverse Compton: Non-Relativistic Electrons

if electrons are nonrelativistic
but still on average more energetic than the photons
we have $\beta = v/c \ll 1$
and $\gamma \approx 1 + \beta^2/2 + \cdots$, so that

$$P_{\mathsf{Compt}} = \frac{4}{3}\sigma_{\mathsf{T}}\ c\ \gamma^2\ \beta^2\ u_{\mathsf{ph}} \approx \frac{4}{3}\sigma_{\mathsf{T}}\ c\ \beta^2\ u_{\mathsf{ph}}\ +\ \vartheta(\beta^4) \qquad (26)$$

if electrons has a **thermal velocity distribution** at T_e
then velocities have Maxwell-Boltzmann distribution $e^{-v^2/2v_T^2} v^2\, dv$
with $v_T^2 = kT_e/m_e$, and so averaging, we get

$$\left\langle v^2 \right\rangle = 3v_T^2 = 3\frac{kT_e}{m_e} \qquad (27)$$

and thus

$$\left\langle P_{\mathsf{Compt}} \right\rangle = 4\sigma_{\mathsf{T}}\ c\ \frac{kT_e}{m_e c^2}\ u_{\mathsf{ph}} \qquad (28)$$