Astronomy 501: Radiative Processes

Lecture 40

Dec 2, 2022

Announcements:

- **Final Exam – Tuesday Dec 13.**
  take home. Questions posted 1:30pm, due by 10pm.
  designed to take < 3 hours. More info to come on Canvas
  open book, open notes. No internet, no collaboration.
- Some rest for the weary – no more problem sets!

last time: **Inverse Compton scattering**

Q: physical ingredients?

Q: effect on photons? electrons?

Sunyaev-Zel'dovich Effect

The CMB Reprocessed: Hot Intracluster Gas

CMB is cosmic photosphere: "as far as the eye can see"
CMB created long ago, comes from far away

- all other observable cosmic objects are in *foreground*
- CMB passes through all of the observable universe

Sunyaev & Zel'dovich:
what happens when CMB passes through hot gas?

- *Q: hot gas examples?*
- *Q: observable effect on CMB?*

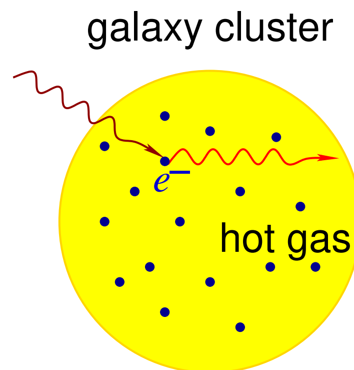
Sunyaev-Zeldovich Effect

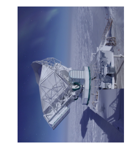
Sunyaev-Zel'dovich:

CMB photons upscattered in hot gas

in galaxy clusters: **intracluster medium**

- changes ν of scattered photons
- CMB spectral distortion towards cloud





consider gas of electrons at temperature $T_e \gg T_{\rm cmb}$
but where $kT_e \ll m_e c^2$ *Q: how good an approximation is this?*

Q: what's probability for scattering of CMB photon with ν?

CMB Scattering by Intracluster Gas

mean free path is that for Thompson scattering:
$\ell_\nu^{-1} = \alpha_\nu = n_e \sigma_{\mathrm{T}}$ independent of frequency
and thus optical depth is integral over cloud sightline

$$\tau_\nu = \int \alpha_\nu \, ds = \sigma_{\mathrm{T}} \int n_e \, ds \tag{1}$$

thus transmission probability is $e^{-\tau_\nu}$, and so
absorption probability is $1 - e^{-\tau_\nu}$

but for galaxy clusters: $\tau < 10^{-3} \ll 1$,
and so *absorption probability* is just $\approx \tau$
Q: implications?
Q: effect of scattering if electrons cold, scattering is elastic?
Q: what if electrons are hot?

if electrons are hot, they transfer energy to CMB photons
change temperature pattern, in frequency-dependent way

What is net change in energy?
initial photon energy density is $u_0 = u_{\rm cmb} = 4\pi B(T_{\rm cmb})/c$
power transfer per electron is $P_{\rm Compt} = 4(kT_e/m_e c^2)\sigma_{\rm T} c\ u_0$, so

$$\frac{\partial u}{\partial t} = P_{\rm Compt}\ n_e = 4\frac{kT_e}{m_e c^2}\sigma_{\rm T} c\ u_0\ n_e \tag{2}$$

and thus net energy density change

$$\Delta u = 4\sigma_{\rm T}\ u_0 \int \frac{n_e\ kT_e}{m_e c^2} ds = 4\frac{kT_e}{m_e c^2}\tau\ u_0 \tag{3}$$

Q: implications?

CMB energy density change through cluster

$$\Delta u = 4\sigma_{\rm T}\ u_0 \int \frac{n_e\ kT_e}{m_e c^2} ds = 4 \frac{kT_e}{m_e c^2} \tau\ u_0 \equiv 4y\ u_0 \tag{4}$$

- dimensionless **Compton-y parameter**

$$y \equiv \sigma_{\rm T} \int \frac{n_e\ kT_e}{m_e c^2} ds \simeq \tau \frac{kT_e}{m_e c^2} \simeq 3\tau\beta^2 \tag{5}$$

- note $n_e kT_e = P_e$ electron pressure
  $\rightarrow y$ set by line-of-sight pressure

fractional change in (integrated) energy density $\Delta u/u_0 = 4y$
- positive change $\rightarrow$ (small) net heating of CMB photons
- since $u \propto I$, this also means

$$\frac{\Delta I_{\rm cmb}}{I_{\rm cmb}} = 4y \tag{6}$$

cluster generated net CMB "hotspot"

Q: expected frequency dependence?

SZ Effect: Frequency Dependence

on average, we expect photons to gain energy
adding intensity at high ν, at the expense of low ν `www: images`

but note that in isotropic electron population
- some scatterings will reduce energy
- while others will increase it

detailed derivation is involved:
- allow for ordinary and stimulated emission
- include effects of electron energy distribution
- allow for Compton shift in energy
- use Thomson (Klein-Nishina) angular distribution

∞ in Director's Cut Extras:
full equation (Kompaneets and generalization)
describes *"diffusion" in energy (frequency) space*

SZ Effect: Cosmological Applications

- *SZ identifies all clusters without redshift bias!*
  $\rightarrow$ SZ can be used to discover high-z clusters

- SZ + X-ray gives cluster size, gas mass, T_e
  if cluster physics well-understood (Ricker, Vijayaraghavan)
  $\rightarrow$ *cluster mass*

- cluster number density ("abundance") and mass vs z
  i.e., cluster *mass function* a sensitive probe of cosmology

today: clusters are the *largest bound objects*; in early U: rare number and mass vs time sensitive to *cosmic acceleration* that competes with *structure growth via gravitational instability*
$\Rightarrow$ clusters probe this competition

Q: so how to find clusters, measure redshifts?

note that SZ redshift independence also means
SZ does not give cluster redshift

Dark Energy Survey key project:
optical images, redshifts of clusters
compare with SZ survey by South Pole Telescope

`www: SPT survey image`

Build Your Toolbox–Inverse Compton Scattering

emission physics: matter-radiation interactions

Q: physical conditions for inverse Compton emission? absorption?

Q: physical nature of sources?

Q: spectrum characteristics?

Q: frequency range?

real/expected astrophysical sources of synchrotron radiation

Q: what do we expect to emit inverse Compton?

Q: relevant temperatures? EM bands?

Toolbox: Inverse Compton Scattering

emission physics

- physical conditions: energetic charged particles upscattering ambient photons
- physical sources: electrons dominate
- spectrum: relativistic e with $dN_e/dE_e \propto E_e^{-p}$ gives *power law* $j_\nu \sim \nu^{-(p-1)/2}$ spectrum non-relativistic e: Sunyaev-Zel'dovich perturbation

astrophysical sources of inverse Compton

- emitters: relativistic electrons: cosmic rays and jets and SZ from hot intracluster gas
- temperatures: nonthermal CR, or $T_e \gg \langle \epsilon_\gamma \rangle$
- EM bands: max IC energy depends on max γ and ambient photons, can go from radio to gamma-ray!

Plasma Effects

Plasmas

roughly speaking a **plasma** is a

- *globally neutral*
- *partially or completely ionized gas*

more quantitatively:
ionization $\rightarrow$ (at least some) particles have $E_{\mathrm{thermal}} > E_{\mathrm{binding}}$

"a little ionization goes a long way"

- electrons and ions in plasma are not *free*
  but have Coulomb interactions with each other
  and can interact with static and propagating EM fields
- gas does *not* need to be fully ionized to show plasma effects

Plasma Frequency

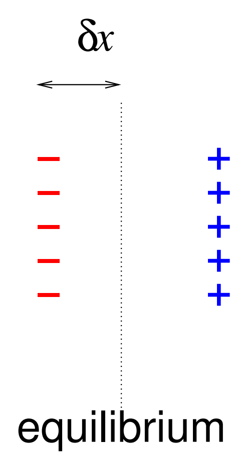
on average (*globally*) the plasma is *neutral*:

$$\langle n_e \rangle = \sum Z_i \langle n_i \rangle \tag{7}$$

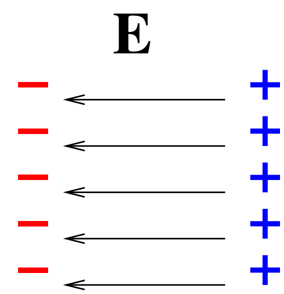
with n_e the electron density
and n_i the density of ion species i of atomic number Z_i

but *locally* the unbound charges can move
fluctuations can create small separation between e and ions

consider idealized picture:
"walls" of electrons and ions
both *displaced from equilibrium*
Q: effect of charge distribution?
Q: response of particles?



Charge separation → capacitor
electric field $\vec{E}$ between "walls"



find electric field given electron density n_e
equilibrium separation δx, and wall area A:
Gauss box around electrons: $EA = 4\pi Q_{\text{enc}} = 4\pi e n_e A \ \delta x$
$\rightarrow$ $E = -4\pi e n_e \ \delta x$: note area A drops out!

electron equation of motion

$$
\begin{aligned}
m_e \ddot{\delta x} &= -eE = -4\pi \ e^2 n_e \ \delta x \qquad (8) \\
\ddot{\delta x} &= -\frac{4\pi \ e^2 n_e}{m_e} \ \delta x \qquad (9)
\end{aligned}
$$

Q: and so? fundamental scales?

electric field due to plasma charge separation
restores charges back to equilibrium position
→ a stable equilibrium!

charges *oscillate*

$$\ddot{\delta x} = -\omega_{\rm p}^2 \; \delta x \tag{10}$$

with **plasma frequency** $\omega_{\rm p}^2 = 4\pi \; e^2 n_e / m_e$ and so

$$\nu_{\rm p} = \frac{\omega_{\rm p}}{2\pi} = \sqrt{\frac{4\pi \; e^2 n_e}{m_e}} = 8.97 \text{ kHz } \left(\frac{n_e}{1 \; cm^{-3}} \right)^{1/2} \tag{11}$$

sets fundamental plasma timescale $\tau = 1/\omega_{\rm p}$
e thermal speed is $v_T \sim \sqrt{kT/m_e}$
→ fundamental lengthscale: *Debye length* $\lambda_{\rm D} = v_T \tau = \sqrt{kT/4\pi e^2 n}$
→ plasma like behavior on timescales $\gg \tau$, on lengthscales $\gg \lambda_{\rm D}$

Electromagnetic Waves in a Plasma

Till now: assumed EM propagation in *vacuum*
but astrophysically, almost always in plasma!

must revisit Maxwell equations, now allowing for
- electron charge density $\rho_q = -en_e$
- current density (charge flux!) $\vec{j} = \rho_q \vec{v}_e = -en_e \vec{v}_e$

look for wavelike solutions: all quantities $\propto e^{i(\vec{k}\cdot\vec{r}-\omega t)}$

$$\begin{array}{cc} i\vec{k}\cdot\vec{E} = -4\pi e n_e & i\vec{k}\cdot\vec{B} = 0 \\ i\vec{k}\times\vec{E} = i\frac{\omega}{c}\vec{B} & i\vec{k}\times\vec{B} = -4\pi e n_e \frac{\vec{v}}{c} - i\frac{\omega}{c}\vec{E} \end{array} \tag{12}$$

electron velocity governed by $m_e \dot{\vec{v}} = -e(\vec{E} + \vec{v}/c \times \vec{B}) \approx -e\vec{E}$, so

$$\vec{v} = \frac{e\vec{E}}{i\omega m_e} \tag{13}$$

note that e *velocity* $\propto$ *electric field*

$$\vec{v} = \frac{e\vec{E}}{i\omega m_e} \tag{14}$$

and thus $\vec{j} = \sigma\vec{E}$ with *conductivity*

$$\sigma = \frac{ie^2 n_e}{\omega m_e} \tag{15}$$

continuity equation: $i\omega e n_e = i e n_e \vec{k}\cdot\vec{v} = \sigma\vec{k}\cdot\vec{E}$

using this, can rewrite Maxwell as

$$\begin{array}{cc} i\left(1 - \frac{4\pi\sigma}{i\omega}\right)\vec{k}\cdot\vec{E} = 0 & i\vec{k}\cdot\vec{B} = 0 \\ i\vec{k}\times\vec{E} = i\frac{\omega}{c}\vec{B} & i\vec{k}\times\vec{B} = -i\left(1 - \frac{4\pi\sigma}{i\omega}\right)\frac{\omega}{c}\vec{E} \end{array} \tag{16}$$

a miracle! *Q: why?*

have recast Maxwell in plasmas into "source-free" form
so still have:

- wavelike solutions
- $\vec{k}$, $\vec{E}$, $\vec{B}$ mutually orthogonal

but now have new **dispersion relation**

$$c^2 k^2 = \epsilon \, \omega^2 \tag{17}$$

with the **dielectric constant**

$$\epsilon = 1 - \frac{4\pi\sigma}{i\omega} = 1 - \frac{4\pi e^2 n_e}{\omega^2 m_e} \tag{18}$$

and thus we have

$$\omega^2 = \omega_{\mathrm{p}}^2 + c^2 k^2 \tag{19}$$

Q: implications for EM propagation in plasmas?

Plasma Dispersion Relation

vacuum relation $\omega = ck$ replaced by

$$\omega^2 = \omega_{\rm p}^2 + c^2 k^2 \tag{20}$$

where $\omega_{\rm p}^2 = 4\pi e^2 n_e / m_e$

if $\omega < \omega_{\rm p}$, then $k^2 < 0$!

$\rightarrow$ wavenumber imaginary!

$$k = \frac{i}{c}\sqrt{\omega_{\rm p}^2 - \omega^2} \tag{21}$$

wave amplitude *damped* as e^{-kr}

$\rightarrow$ low frequency waves do not propagate! "cutoff" in spectrum

e.g., Earth ionosphere damps waves with $\nu < 1$ MHz

characteristic *damping scale* $2\pi c/\omega_{\rm p}$

Group and Phase Velocity

in the other limit $\omega > \omega_\mathrm{p}$
waves do propagate without damping

waves move according to $e^{i\phi}$, with phase
$\phi = \vec{k} \cdot \vec{x} - \omega t = k\hat{n} \cdot (\vec{x} - \omega/k \; t \; \hat{n})$
$\rightarrow$ wavefronts propagate with *phase velocity*

$$v_\phi = \frac{\omega}{k} = \frac{c}{n_r} \tag{22}$$

where the *index of refraction* is

$$n_r \equiv \sqrt{\epsilon} = \sqrt{1 - \left(\frac{\omega_\mathrm{p}^2}{\omega^2}\right)} \tag{23}$$

but *signals* move with *group velocity* (PS8)

$$v_\mathrm{g} \equiv \frac{\partial \omega}{\partial k} = c \sqrt{1 - \left(\frac{\omega_\mathrm{p}^2}{\omega^2}\right)} \tag{24}$$

Group Velocity Awesome Example: Pulsars

Pulsars: spinning, magnetized neutron stars
pulsed emission with period = spin period
pulsed $\rightarrow$ narrow in time $\rightarrow$ broadband in frequency

`www: pulsar signals in audio`
`www: pulsar sky distribution`

pulsar signals propagate through interstellar medium–a plasma!
every small band of frequencies propagates with different $v_{\rm g}(\omega)$
$\rightarrow$ pulses *dispersed*, arrive with spread time

if pulsar distance is d
then arrival time at Earth at each frequency ω is

$$t_{\rm pulsar}(\omega) = \int_0^d \frac{ds}{v_{\rm g}(\omega)} \tag{25}$$

Q: how should arrive time depend on ω?

pulsar at d has arrival time

$$t_{\mathrm{pulsar}}(\omega) = \int_0^d \frac{ds}{v_{\mathrm{g}}(\omega)} \tag{26}$$

frequency dependence set by

$$\frac{1}{v_{\mathrm{g}}} = \frac{1}{c}\left(1 - \frac{\omega_{\mathrm{p}}^2}{\omega^2}\right)^{-1/2} \approx \frac{1}{c}\left(1 + \frac{\omega_{\mathrm{p}}^2}{2\omega^2}\right) \tag{27}$$

where we used $\omega \gg \omega_{\mathrm{p}} \sim$ kHz, and so

$$t_{\mathrm{pulsar}}(\omega) \approx \frac{d}{c} + \frac{1}{2c\omega^2}\int_0^d \omega_{\mathrm{p}}^2 \; ds = \frac{d}{c} + \frac{1}{2c\omega^2}\; \mathcal{D} \tag{28}$$

Q: implications? how can we be sure dispersion is real?

pulsar time delay

$$t_{\rm pulsar}(\omega) \approx \frac{d}{c} + \frac{1}{2c\omega^2} \int_0^d \omega_{\rm p}^2 \, ds = \frac{d}{c} + \frac{1}{2c\omega^2} \, \mathcal{D} \qquad (29)$$

- depends on frequency: $\delta t \propto \nu^{-2} \propto \lambda_{\rm obs}^2$
- free electron column: **dispersion measure** $\mathcal{D} = \int_0^d n_e \, ds$

to test whether dispersion is real:
should obey correct frequency dependence
$\rightarrow$ this isolates dispersion measure

- if have estimate of electron density n_e

$\rightarrow$ get *distance* to pulsar!

- if have idea of pulsar distance
  can use pulsar ensemble to *map free electron density* n_e!
  $\rightarrow$ reveals Galactic spiral arm pattern!
  `www: Taylor & Cordes 1993`

Q: applications for Sgr A?*

Sgr A*: our very own neighborhood black hole
a laboratory for study of General Relativity

so far: black hole properties studied via orbits
of closely approaching stars
available closest approach distances still $\gg GM/c^2$
$\rightarrow$ GR effects too small to detect

the great hope: find a *pulsar* around Sgr A*
not crazy! many supernova remnants near Galactic center!

- good news: hyperaccurate pulsar timing $\rightarrow$ GR probe
- bad news: surrounding free e "screen" will disperse signal
  limit the strength of GR probe...

Director's Cut Extras

EM Propagation Along A Magnetic Field

the interstellar medium (ISM) contains not only plasma
but also *magnetic fields*

thus we are obliged to understand
EM propagation in a magnetized plasma

consider idealized case:
a fixed, uniform external field $\vec{B}_0$
in a nonrelativistic plasma: $v_T \ll c \rightarrow kT \ll m_e c^2 \rightarrow T \ll 10^{10}$ K

Q: effect on plasma electrons?
Q: effect on EM waves propagating $\perp \vec{B}_0$*?*
Q: effect on EM waves propagating along $\vec{B}_0$*?*

in a fixed uniform external field $\vec{B}_0$
(non-relativistic) electrons move in Larmor orbits
and new frequency/timescale introduced: Larmor/gyro-frequency

$$\omega_B = \frac{eB_0}{m_e c} = 17 \text{ Hz} \left(\frac{B_0}{1 \ \mu\text{Gauss}} \right) \tag{30}$$

magnetic field introduces a special direction and thus anisotropy
which affects EM propagation → dielectric constant anisotropic

that is:

- electrons orbit around field lines
- for *waves* $\parallel$ *field*: $\hat{k} = \hat{B}_0$
  e motion due to $\vec{E}_{\text{wave}}$ in Larmor orbit plane
  → expect B_0 to change wave propagation
- for *waves* $\perp$ *field*: $\hat{k} \cdot \hat{B}_0 = 0$
  e motion due to $\vec{E}_{\text{wave}}$ is orthogonal to orbit
  → expect no/less change in EM propagation

Electron Motion in a Magnetized Plasma

if $B_0 \gg B_{\text{wave}}$, then e equation of motion

$$m_e \dot{\vec{v}} \approx -e\vec{E} - e\frac{\vec{v}}{c} \times \vec{B}_0 \tag{31}$$

assume a propagating, sinusoidal, *circularly polarized* EM wave:

$$E(t) = E\ e^{i\omega t}\,(\hat{\epsilon}_1 \mp \hat{\epsilon}_2) \tag{32}$$

where $\mp\ \leftrightarrow$ right/left circular polarization

also assume propagation is *along the field*

$$\vec{B}_0 = B_0\ \hat{\epsilon}_3 \tag{33}$$

solutions with $v(t) \propto e^{i\omega t}$ have

$$\vec{v}(t) = -\frac{ie}{m_e(\omega \pm \omega_B)}\vec{E}(t) \tag{34}$$

electron velocity has

$$\vec{v}(t) = -\frac{ie}{m_e(\omega \pm \omega_B)}\vec{E}(t) \tag{35}$$

so still have Ohm's law current density $\vec{j} = -en_e\vec{v} = \sigma\vec{E}$
but now with $\sigma = ie^2 n_e / m_e(\omega \pm \omega_B)$

and so now the dielectric constant is

$$\epsilon_{\mathrm{R,L}} = 1 - \frac{\omega_{\mathrm{p}}^2}{\omega(\omega \pm \omega_B)} \tag{36}$$

- right(+) and left(-) circular waves travel with different speeds
- speed difference sense is $v_{\mathrm{R}} > v_{\mathrm{L}}$

Q: effect of sending circularly polarized radiation thru a plasma?

Q: effect of sending linearly polarized radiation thru a plasma?

Faraday Rotation

for EM waves *along* magnetic field, dielectric constant is

$$\epsilon_{\mathrm{R,L}} = 1 - \frac{\omega_{\mathrm{p}}^2}{\omega(\omega \pm \omega_B)} \tag{37}$$

if incident radiation is *circularly* polarized (either R or L)
then will encounter different dispersion than unmagnetized case
but still remain circularly polarized

if incident radiation is *linearly* polarized
then it has equal superposition of R and L components
$\rightarrow$ these components dispersed differently $\rightarrow$ tblue*changes phase*
$\rightarrow$ *polarization rotated due to magnetic field*
$\Rightarrow$ **Faraday rotation**

Faraday Rotation

after wave propagates distance $\vec{d}$, phase is $\vec{k} \cdot \vec{d}$
but if k nonuniform in space, then

$$\phi_{\mathrm{R,L}} = \int_0^d k_{\mathrm{R,L}} \; ds \tag{38}$$

with $ck_{\mathrm{R,L}} = \omega \sqrt{\epsilon_{\mathrm{R,L}}}$

if $\omega \gg \omega_{\mathrm{p}}$ and $\omega \gg \omega_B$ then

$$k_{\mathrm{R,L}} \approx \frac{\omega}{c} \left[1 - \frac{\omega_{\mathrm{p}}^2}{2\omega^2} \left(1 \mp \frac{\omega_B}{\omega} \right) \right] \tag{39}$$

and thus *polarization plane rotates* through angle

$$\Delta\theta = \frac{\Delta\phi}{2} = \frac{1}{2} \int_0^d (k_{\mathrm{R}} - k_{\mathrm{L}}) ds = \frac{1}{2} \int_0^d \frac{\omega_{\mathrm{p}}^2 \omega_B \; ds}{c\omega^2} \tag{40}$$

Faraday rotation of linear polarization angle is therefore

$$\Delta\theta = \frac{2\pi e^3}{m_e^2 c^2 \omega^2} \int_0^d n_e \, B_\parallel \, ds \tag{41}$$

Q: how can we be sure Faraday rotation really has occurred?

Q: what does Faraday rotation directly tell us? with other information?

Q: what if field changes along line of sight?

Astrophysics of Faraday Rotation

effect occurs when *linearly polarized* radiation passes through a *magnetized plasma*

But we don't know initial polarization angle!
true, but $\Delta\theta \propto \nu^{-2} \propto \lambda^2$
$\rightarrow$ use this dependence to confirm effect

if Faraday rotation observed:

- immediately know $B_{\parallel} \neq 0$: existence of interstellar magnetism
- if know n_e and d, then *measure* $B_{\parallel}$
- if field direction changes, then $B > B_{\parallel}$: Faraday gives *lower limit* to true field strength

Q: what astrophysical situation needed to observe this? examples?

to observe Faraday rotation, need both

- polarized background source and
- foreground plasma

typical example:

- AGN have (partially) linearly polarized emission and are cosmological $\rightarrow$ isotropically distributed on sky
- if you are lucky, one is behind your source!

Awesome Example I: our Galaxy
find rotation for many AGN across the sky
plot *rotation measure* $\Delta\theta = \mathrm{RM}\ \lambda^2$

$$\mathrm{RM} = \frac{1}{2\pi} \frac{e^3}{m_e^2 c^4} \int n_e\ B_{\parallel}\ ds \qquad (42)$$

`www: results` *Q: implications?*

Results:

- Faraday rotation detected! *the Galaxy is magnetized!*
- largest signal in plane $\rightarrow$ fields associated with ISM
- typical strength $B_{\text{ism}} \sim few\ \mu\text{Gauss}$

Awesome Example II: supernova remnants

recall: supernovae are mighty particle accelerators

the engines of cosmic-ray acceleration

$\rightarrow$ supernova remnants are very bright in synchrotron
from electrons accelerated in the remnant
and this radiation is polarized

$\rightarrow$ so can measure Faraday rotation in the remnant
using its own synchrotron!