Astronomy 501: Radiative Processes

Lecture 41

Dec 5, 2022

Announcements:

- **Take-Home Final Exam – Tuesday Dec 13.**
  info on Canvas
- Gala meme exhibition next class!

last time: began effects of EM propagation in plasmas

Q: what's a plasma?

Q: what astrophysical EM signals pass through plasmas?

Plasmas

roughly speaking a **plasma** is a

- *globally neutral*
- *partially or completely ionized gas*

electrons and ions in plasma are not *free*
but have Coulomb interactions with each other
and can interact with static and propagating EM fields

the interstellar and intergalactic medium has varying ionization
but essentially *all* of the ISM and IGM
is at last partially ionized
lesson: *essentially all astrophysical signals pass through plasmas*

So: must understand plasma effects on EM propagation!

Plasma Frequency

on average (*globally*) the plasma is *neutral*:

$$\langle n_e \rangle = \sum Z_i \langle n_i \rangle \tag{1}$$

with n_e the electron density
and n_i the density of ion species i of atomic number Z_i

but *locally* the unbound charges can move
fluctuations can create small separation between e and ions

electric field due to plasma charge separation
restores charges back to equilibrium position
$\rightarrow$ a stable equilibrium!

charges *oscillate* with **plasma frequency**

ω

$$\omega_{\mathrm{p}}^2 = \frac{4\pi \; e^2 n_e}{m_e} \tag{2}$$

Electromagnetic Waves in a Plasma

Till now: assumed EM propagation in *vacuum*
but astrophysically, almost always in plasma!

must *revisit Maxwell equations*, now allowing for
- electron charge density $\rho_q = -en_e$
- current density (charge flux!) $\vec{j} = \rho_q \vec{v}_e = -en_e \vec{v}_e$

method: look for **wavelike solutions**
- assume sinusoidal variations in all quantities $n_e, v_e, \vec{E}, \vec{B} \propto e^{i(\vec{k}\cdot\vec{r}-\omega t)}$
- see what this requires of the solutions

Searching for Wavelike Solutions in Plasma

Maxwell eqs in plasma with $e^{i(\vec{k}\cdot\vec{r}-\omega t)}$ variations:

$$\nabla \cdot \vec{E} = i\vec{k} \cdot \vec{E} = -4\pi e n_e \qquad \nabla \cdot \vec{B} = i\vec{k} \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = i\vec{k} \times \vec{E} = i\frac{\omega}{c}\vec{B} \qquad \nabla \times \vec{B} = i\vec{k} \times \vec{B} = -4\pi e n_e \frac{\vec{v}}{c} - i\frac{\omega}{c}\vec{E}$$

electron velocity governed by fluid equation ($F = ma$)

$$m_e \dot{\vec{v}} = -e(\vec{E} + \vec{v}/c \times \vec{B}) \approx -e\vec{E} \tag{3}$$

and thus

$$\vec{v} = \frac{e\vec{E}}{i\omega m_e} \tag{4}$$

note that e *velocity* $\propto$ *electric field*

$$\vec{v} = \frac{e\vec{E}}{i\omega m_e} \tag{5}$$

and thus $\vec{j} = -en_e\vec{v} = \sigma\vec{E}$ with *conductivity*

$$\sigma = \frac{ie^2 n_e}{\omega m_e} \tag{6}$$

continuity equation: $i\omega e n_e = ien_e\vec{k}\cdot\vec{v} = \sigma\vec{k}\cdot\vec{E}$

using this, can rewrite Maxwell as

$$\begin{array}{cc} i\left(1 - \frac{4\pi\sigma}{i\omega}\right)\vec{k}\cdot\vec{E} = 0 & i\vec{k}\cdot\vec{B} = 0 \\ i\vec{k}\times\vec{E} = i\frac{\omega}{c}\vec{B} & i\vec{k}\times\vec{B} = -i\left(1 - \frac{4\pi\sigma}{i\omega}\right)\frac{\omega}{c}\vec{E} \end{array} \tag{7}$$

 a miracle! *Q: why? Hint–what changes when* $\sigma \neq 0$*?*

have recast Maxwell in plasmas into "source-free" form
so still have:

- wavelike solutions
- $\vec{k}$, $\vec{E}$, $\vec{B}$ mutually orthogonal

but now have new **dispersion relation**

$$c^2k^2 = \epsilon \ \omega^2 \tag{8}$$

with the **dielectric constant**

$$\epsilon = 1 - \frac{4\pi\sigma}{i\omega} = 1 - \frac{4\pi e^2 n_e}{\omega^2 m_e} \tag{9}$$

and thus we have

$$\omega^2 = \omega_{\rm p}^2 + c^2k^2 \tag{10}$$

Q: implications for EM propagation in plasmas?

Plasma Dispersion Relation

vacuum relation $\omega = ck$ replaced by

$$\omega^2 = \omega_{\rm p}^2 + c^2 k^2 \qquad (11)$$

where $\omega_{\rm p}^2 = 4\pi e^2 n_e / m_e$

if $\omega < \omega_{\rm p}$, then $k^2 < 0$!
$\rightarrow$ wavenumber imaginary!

$$k = \frac{i}{c}\sqrt{\omega_{\rm p}^2 - \omega^2} \qquad (12)$$

wave amplitude *damped* as e^{-kr}
$\rightarrow$ low frequency waves do not propagate! "cutoff" in spectrum
e.g., Earth ionosphere damps waves with $\nu < 1$ MHz

characteristic *damping scale* $2\pi c/\omega_{\rm p}$

Group and Phase Velocity

in the other limit $\omega > \omega_{\mathrm{p}}$
waves do propagate without damping

waves move according to $e^{i\phi}$, with phase
$\phi = \vec{k} \cdot \vec{x} - \omega t = k\hat{n} \cdot (\vec{x} - \omega/k \; t \; \hat{n})$
$\rightarrow$ wavefronts propagate with *phase velocity*

$$v_\phi = \frac{\omega}{k} = \frac{c}{n_r} \tag{13}$$

where the *index of refraction* is

$$n_r \equiv \sqrt{\epsilon} = \sqrt{1 - \left(\frac{\omega_{\mathrm{p}}^2}{\omega^2}\right)} \tag{14}$$

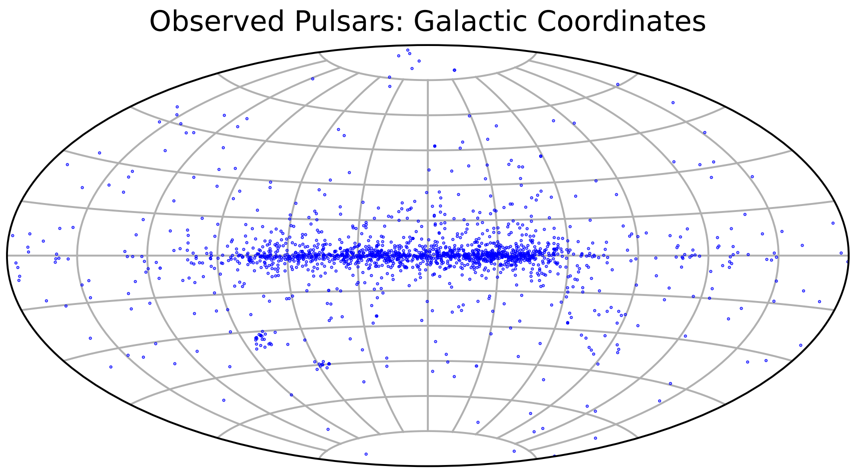
but *signals* move with *group velocity*

$$v_{\mathrm{g}} \equiv \frac{\partial \omega}{\partial k} = c \sqrt{1 - \left(\frac{\omega_{\mathrm{p}}^2}{\omega^2}\right)} \tag{15}$$

Group Velocity Awesome Example: Pulsars

Pulsars: spinning, magnetized neutron stars
pulsed emission with period = spin period
pulsed $\rightarrow$ narrow in time $\rightarrow$ broadband in frequency

`www: pulsar signals in audio`



Q: why this pattern?
Q: east-west asymmetry?

Pulsar Signal Dispersion Over Time

pulsar signals propagate through interstellar medium–a plasma!
every small band of frequencies propagates with different $v_{\rm g}(\omega)$
$\rightarrow$ pulses *dispersed*
arrival time spread with ν

if pulsar distance is r
then arrival time at Earth at each frequency ω is

$$t_{\rm pulsar}(\omega) = \int_0^r \frac{ds}{v_{\rm g}(\omega)} \tag{16}$$

Q: how should arrive time depend on ω?

pulsar at d has arrival time

$$t_{\mathrm{pulsar}}(\omega) = \int_0^r \frac{ds}{v_{\mathrm{g}}(\omega)} \tag{17}$$

frequency dependence set by

$$\frac{1}{v_{\mathrm{g}}} = \frac{1}{c}\left(1 - \frac{\omega_{\mathrm{p}}^2}{\omega^2}\right)^{-1/2} \approx \frac{1}{c}\left(1 + \frac{\omega_{\mathrm{p}}^2}{2\omega^2}\right) \tag{18}$$

where we used $\omega \gg \omega_{\mathrm{p}} \sim$ kHz, and so

$$t_{\mathrm{pulsar}}(\omega) \approx \frac{d}{c} + \frac{1}{2c\omega^2}\int_0^r \omega_{\mathrm{p}}^2 \, ds = \frac{d}{c} + \frac{1}{2c\omega^2} \, \mathcal{D} \tag{19}$$

Q: implications? how can we be sure dispersion is real?

pulsar time delay

$$t_{\rm pulsar}(\omega) \approx \frac{d}{c} + \frac{1}{2c\omega^2} \int_0^r \omega_{\rm p}^2 \, ds = \frac{d}{c} + \frac{1}{2c\omega^2} \, \mathcal{D} \qquad (20)$$

- depends on frequency: $\delta t \propto \nu^{-2} \propto \lambda_{\rm obs}^2$
- free electron column: **dispersion measure** $\mathcal{D} = \int_0^r n_e \, ds$

to test whether dispersion is real:
should obey correct frequency dependence
$\rightarrow$ this isolates dispersion measure

- if have estimate of electron density n_e

$\rightarrow$ get *distance* to pulsar!

- if have idea of pulsar distance
  can use pulsar ensemble to *map free electron density* n_e!
  $\rightarrow$ reveals Galactic spiral arm pattern!
  `www: Taylor & Cordes 1993`

Q: applications for Sgr A?*

Sgr A*: our very own neighborhood black hole
a laboratory for study of General Relativity

so far: black hole properties studied via orbits
of closely approaching stars
available closest approach distances still $\gg GM/c^2$
$\rightarrow$ GR effects too small to detect

the great hope: find a *pulsar* around Sgr A*
not crazy! many supernova remnants near Galactic center!

- good news: hyperaccurate pulsar timing $\rightarrow$ GR probe
- bad news: surrounding free e "screen" will disperse signal
  limit the strength of GR probe...

The Mystery of Fast Radio Bursts

A 21st Century phenomenon!
see reviews by Petroff, Hessels, & Lormier (2019, 2022)

starting 2007: *radio surveys* pulsar searches discovered

- single radio pulses `www: data`
- pulse duration $\Delta t \sim few$ ms
- sky distribution consistent with isotropic
- long dispersive delay, requiring large dispersion measure exceeds that from MW disk electrons: $\mathcal{D}_{\rm obs} \gtrsim 5\ \mathcal{D}_{\rm MW}$
- some have recurred on sec to yr timescales
- radio emission is often linearly polarized

Q: What does this suggest–where are the sources?

Fast Radio Bursts: Where Are They?

- *isotropic* sky distribution:
  either *very local*: typical distance $r_{\mathsf{FRB}} \ll h_{\mathsf{disk}} \sim 100$ pc
  or *cosmological*: $r_{\mathsf{FRB}} \gtrsim d_{\mathsf{H}} \sim 4$ Gpc

- dispersion measure $\mathcal{D}_{\mathsf{obs}} > D_{\mathsf{MW}}$ MW disk:
  must pass through more plasma
  not nearby: extragalactic $\rightarrow$ **cosmological distances**

- well-localized events seen to coincide with galaxies,
  in star-forming regions!

- one seen in globular cluster hosted by M81 galaxy

Q: So what are they?

Fast Radio Bursts: What Are They?

- large flux at large distance:
- huge (isotropic) radio luminosity $L_{\text{FRB}} = 4\pi r_{\text{FRB}}^2 F$ requires large energy reservoir (less if beamed)
- small duration: causality requires small emitting region $\ell_{\text{emit}} < c\Delta t \sim 300$ km tiny!
- some sources repeat: sources not destroyed upon emission
- suggests compact sources: neutron stars, accreting black holes
- Milky Way event seen from a *magnetar* highly magnetized neutron star!

New field! Origin is one of *many open questions*.
Opportunities for young astro-folk!

Gamma Rays

MeV Gamma Rays

consider photons with $E_\gamma \sim 0.5 - 10$ MeV
these have been observed astrophysically

Q: what physical processes can make MeV gammas?
hint: some we have discussed already, some we have not...

Q: what are possible astrophysical sites for these processes

MeV Gamma Rays: Emission Processes

MeV photons are high energy
can be made by nonthermal processes we have already seen

- nonthermal bremsstrahlung from cosmic-ray electrons
- inverse Compton of starlight by cosmic-ray electrons

But the MeV scale has other charms

- $m_e c^2 = 0.511$ MeV
  positron annihilation $e^{\pm} \rightarrow \gamma\gamma$
  emits back-to-back 511 keV photons (in rest frame)
- *atomic nuclei are quantum bound states*
  with energy level spacings ~ 1 MeV
  `www: nuclear energy level diagram`

Astrophysical sources?

- positrons $e^{+} \rightarrow 511$ keV photons
- excited nuclei $\rightarrow$ MeV lines

Q: expected sky distribution for each?

The Postitronic Sky

The 511 keV Sky `www: sky map`

line emission seen!

- concentrated in Galactic center, but not point source
- a faint disk component present

this requires huge numbers of positrons!

an open question where they came from

decay of radioactive nucleosynthesis products? cosmic rays? dark matter?

The Radioactive Sky

The Sky at 1.8 MeV

aluminum isotope ^{26}Al is unstable: $t_{1/2} = 1.5$ Myr

decays to excited state: $^{26}\mathrm{Al} \to {}^{26}\mathrm{Mg}^* \to {}^{26}\mathrm{Mg}^{\mathrm{g.s.}} + \gamma$

each decay produces 1.8 MeV line

`www:  1.8 MeV line sky map`

Q: implications of line detection/existence?

Q: features of map? origin?

Aluminum-26 Gamma-Rays: Mapping Element Production

emission seen across Galactic plane (*CGRO*/COMPTEL, *INTEGRAL*/SPI)

- strongest towards Galactic center: longest sightline
- features in plane: spiral arm tangents, star-forming regions
- beware! angular resolution $\sim 1^\circ$! "impressionist" view

Presence of 1.8 MeV line: decays ongoing
$\rightarrow$ sources are ^{26}Al made in last $\sim t_{1/2} = 1.5 Myr$
$\ll$ Galaxy age: fresh!
$\rightarrow$ nucleosynthesis is ongoing in the Galaxy
$\rightarrow$ line intensity measures total recent ^{26}Al production
and also Milky Way supernova rate!

Director's Cut Extras

EM Propagation Along A Magnetic Field

the interstellar medium (ISM) contains not only plasma but also *magnetic fields*

thus we are obliged to understand EM propagation in a magnetized plasma

consider idealized case:
a fixed, uniform external field $\vec{B}_0$
in a nonrelativistic plasma: $v_T \ll c \rightarrow kT \ll m_e c^2 \rightarrow T \ll 10^{10}$ K

Q: effect on plasma electrons?
Q: effect on EM waves propagating $\perp \vec{B}_0$*?*
Q: effect on EM waves propagating along $\vec{B}_0$*?*

in a fixed uniform external field $\vec{B}_0$
(non-relativistic) electrons move in Larmor orbits
and new frequency/timescale introduced: Larmor/gyro-frequency

$$\omega_B = \frac{eB_0}{m_e c} = 17 \text{ Hz } \left(\frac{B_0}{1 \ \mu\text{Gauss}} \right) \tag{21}$$

magnetic field introduces a special direction and thus anisotropy
which affects EM propagation $\rightarrow$ dielectric constant anisotropic

that is:

- electrons orbit around field lines
- for *waves* $\parallel$ *field*: $\hat{k} = \hat{B}_0$
  e motion due to $\vec{E}_{\text{wave}}$ in Larmor orbit plane
  $\rightarrow$ expect B_0 to change wave propagation
- for *waves* $\perp$ *field*: $\hat{k} \cdot \hat{B}_0 = 0$
  e motion due to $\vec{E}_{\text{wave}}$ is orthogonal to orbit
  $\rightarrow$ expect no/less change in EM propagation

Electron Motion in a Magnetized Plasma

if $B_0 \gg B_{\rm wave}$, then e equation of motion

$$m_e \dot{\vec{v}} \approx -e\vec{E} - e\frac{\vec{v}}{c} \times \vec{B}_0 \qquad (22)$$

assume a propagating, sinusoidal, *circularly polarized* EM wave:

$$E(t) = E\ e^{i\omega t} (\hat{\epsilon}_1 \mp \hat{\epsilon}_2) \qquad (23)$$

where $\mp \ \leftrightarrow$ right/left circular polarization

also assume propagation is *along the field*

$$\vec{B}_0 = B_0\ \hat{\epsilon}_3 \qquad (24)$$

solutions with $v(t) \propto e^{i\omega t}$ have

$$\vec{v}(t) = -\frac{ie}{m_e(\omega \pm \omega_B)} \vec{E}(t) \qquad (25)$$

electron velocity has

$$\vec{v}(t) = -\frac{ie}{m_e(\omega \pm \omega_B)}\vec{E}(t) \qquad (26)$$

so still have Ohm's law current density $\vec{j} = -en_e\vec{v} = \sigma\vec{E}$
but now with $\sigma = ie^2 n_e/m_e(\omega \pm \omega_B)$

and so now the dielectric constant is

$$\epsilon_{\mathrm{R,L}} = 1 - \frac{\omega_{\mathrm{p}}^2}{\omega(\omega \pm \omega_B)} \qquad (27)$$

- right(+) and left(-) circular waves travel with different speeds
- speed difference sense is $v_{\mathrm{R}} > v_{\mathrm{L}}$

Q: effect of sending circularly polarized radiation thru a plasma?

Q: effect of sending linearly polarized radiation thru a plasma?

Faraday Rotation

for EM waves *along* magnetic field, dielectric constant is

$$\epsilon_{\mathrm{R,L}} = 1 - \frac{\omega_{\mathrm{p}}^2}{\omega(\omega \pm \omega_B)} \tag{28}$$

if incident radiation is *circularly* polarized (either R or L)
then will encounter different dispersion than unmagnetized case
but still remain circularly polarized

if incident radiation is *linearly* polarized
then it has equal superposition of R and L components
$\rightarrow$ these components dispersed differently $\rightarrow$ tblue*changes phase*
$\rightarrow$ *polarization rotated due to magnetic field*
$\Rightarrow$ **Faraday rotation**

Faraday Rotation

after wave propagates distance $\vec{d}$, phase is $\vec{k} \cdot \vec{d}$
but if k nonuniform in space, then

$$\phi_{\mathrm{R,L}} = \int_0^d k_{\mathrm{R,L}} \; ds \tag{29}$$

with $ck_{\mathrm{R,L}} = \omega\sqrt{\epsilon_{\mathrm{R,L}}}$

if $\omega \gg \omega_{\mathrm{p}}$ and $\omega \gg \omega_B$ then

$$k_{\mathrm{R,L}} \approx \frac{\omega}{c}\left[1 - \frac{\omega_{\mathrm{p}}^2}{2\omega^2}\left(1 \mp \frac{\omega_B}{\omega}\right)\right] \tag{30}$$

and thus *polarization plane rotates* through angle

$$\Delta\theta = \frac{\Delta\phi}{2} = \frac{1}{2}\int_0^d (k_{\mathrm{R}} - k_{\mathrm{L}}) ds = \frac{1}{2}\int_0^d \frac{\omega_{\mathrm{p}}^2 \omega_B \; ds}{c\omega^2} \tag{31}$$

Faraday rotation of linear polarization angle is therefore

$$\Delta\theta = \frac{2\pi e^3}{m_e^2 c^2 \omega^2} \int_0^d n_e \; B_{\parallel} \; ds \tag{32}$$

Q: how can we be sure Faraday rotation really has occurred?

Q: what does Faraday rotation directly tell us? with other information?

Q: what if field changes along line of sight?

Astrophysics of Faraday Rotation

effect occurs when *linearly polarized* radiation passes through a *magnetized plasma*

But we don't know initial polarization angle!
true, but $\Delta\theta \propto \nu^{-2} \propto \lambda^2$
$\rightarrow$ use this dependence to confirm effect

if Faraday rotation observed:

- immediately know $B_{\parallel} \neq 0$: existence of interstellar magnetism
- if know n_e and d, then *measure* $B_{\parallel}$
- if field direction changes, then $B > B_{\parallel}$:
  Faraday gives *lower limit* to true field strength

Q: what astrophysical situation needed to observe this? examples?

to observe Faraday rotation, need both

- polarized background source and
- foreground plasma

typical example:

- AGN have (partially) linearly polarized emission and are cosmological $\rightarrow$ isotropically distributed on sky
- if you are lucky, one is behind your source!

Awesome Example I: our Galaxy
find rotation for many AGN across the sky
plot *rotation measure* $\Delta\theta = \mathrm{RM}\ \lambda^2$

$$\mathrm{RM} = \frac{1}{2\pi} \frac{e^3}{m_e^2 c^4} \int n_e \ B_{\parallel} \ ds \tag{33}$$

`www: results` *Q: implications?*

Results:

- Faraday rotation detected! *the Galaxy is magnetized!*
- largest signal in plane $\rightarrow$ fields associated with ISM
- typical strength $B_{\text{ism}} \sim few\ \mu\text{Gauss}$

Awesome Example II: supernova remnants

recall: supernovae are mighty particle accelerators

the engines of cosmic-ray acceleration

$\rightarrow$ supernova remnants are very bright in synchrotron
from electrons accelerated in the remnant
and this radiation is polarized

$\rightarrow$ so can measure Faraday rotation in the remnant
using its own synchrotron!