Astronomy 501: Radiative Processes

Lecture 7

Sept 7, 2022

Announcements:

- **Problem Set 2 due this Friday Sept 9**
  - Office hours: BDF after class or by appointment

  TA: tomorrow 11:30 am - 12:30 pm
- Great to see grads and hear grad questions at Colloquium!

  Q: key speaker point straight out of A501?

Last time: blackbody radiation

Q: blackbody total intensity $B(T)$*? flux* F*? energy density* $u(T)$*?*

Q: for what ν *is* $B_\nu = 0$*? but...?*

blackbody specific and total/integrated values

$$
\begin{aligned}
\text{intensity } B_\nu(T) &= \frac{2h}{c^2}\frac{\nu^3}{e^{h\nu/kT}-1} && (1)\\
B(T) &= \frac{2\pi^4}{15}\frac{k^4T^4}{c^3h^3} = \frac{\sigma}{\pi}T^4 = \frac{c}{4\pi}a\ T^4 && (2)\\
\text{flux } F_\nu(T) &= \pi B_\nu(T) = \frac{2\pi h}{c^2}\frac{\nu^3}{e^{h\nu/kT}-1} && (3)\\
F(T) &= \pi B(T) = \frac{2\pi^5}{15}\frac{k^4T^4}{c^2h^3} \equiv \sigma\ T^4 && (4)\\
\text{energy density } u_\nu(T) &= \frac{4\pi B_\nu(T)}{c} = \frac{8\pi h}{c^3}\frac{\nu^3}{e^{h\nu/kT}-1} && (5)\\
u(T) &= \frac{8\pi^5}{15}\frac{k^4}{c^3h^3}\ T^4 \equiv a\ T^4 = \frac{4\sigma}{c}T^4 && (6)
\end{aligned}
$$

Q: to order of magnitude: total number density?

hint: $[hc] = [\text{energy} \times \text{length}]$

mean number density: dimensions $[n] = [\text{length}^{-3}]$

can only depend on T, and physical constants h, c, k

can form only one length: $[hc/kT] = [\text{length}]$

$\rightarrow$ expect $n \sim (hc/kT)^3$

photon number density

$$n_\nu(T) = \frac{4\pi B_\nu(T)}{hc\nu} = \frac{8\pi}{c^3}\frac{\nu^2}{e^{h\nu/kT} - 1} \quad (7)$$

$$n(T) = 16\pi\zeta(3)\left(\frac{kT}{hc}\right)^3 \quad (8)$$

where $\zeta(3) = 1 + 1/2^3 + 1/3^3 + 1/4^3 + \cdots = 1.2020569\ldots$

Q: implications?

blackbody photon number density

$$n(T) = 16\pi\zeta(3)\left(\frac{kT}{hc}\right)^3 \tag{9}$$

i.e., $n \propto T^3$

So if temperatures changes, photon number changes

blackbody photon number is not conserved

photons massless $\rightarrow$ can always make more!

if heat up, photon number increases

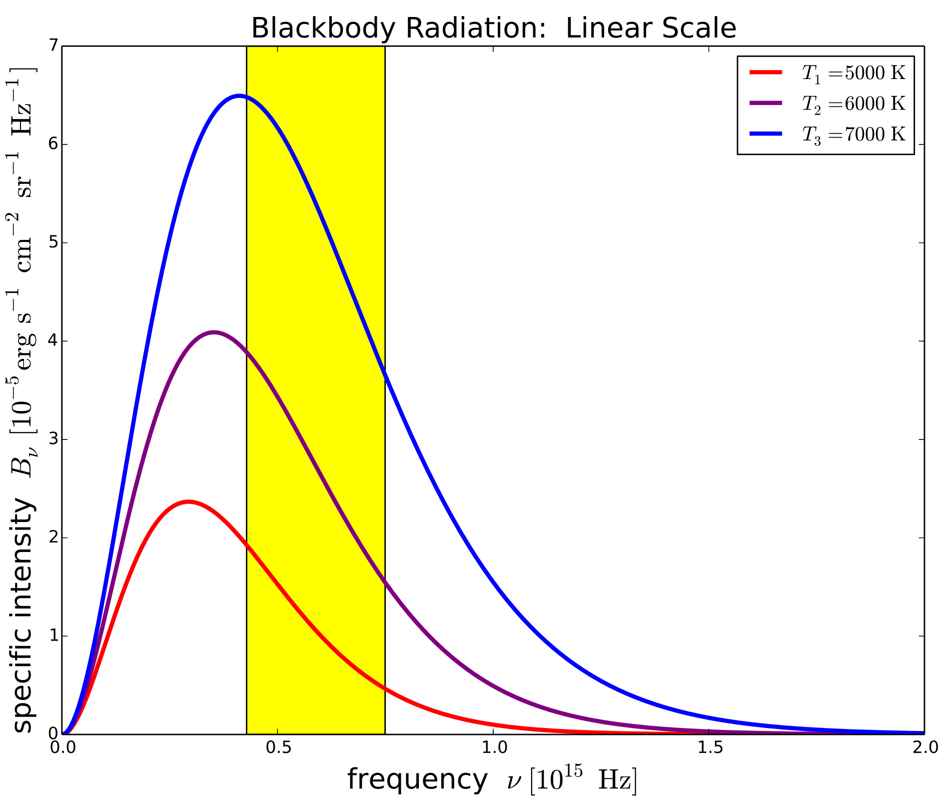
and spectrum relaxes to blackbody form

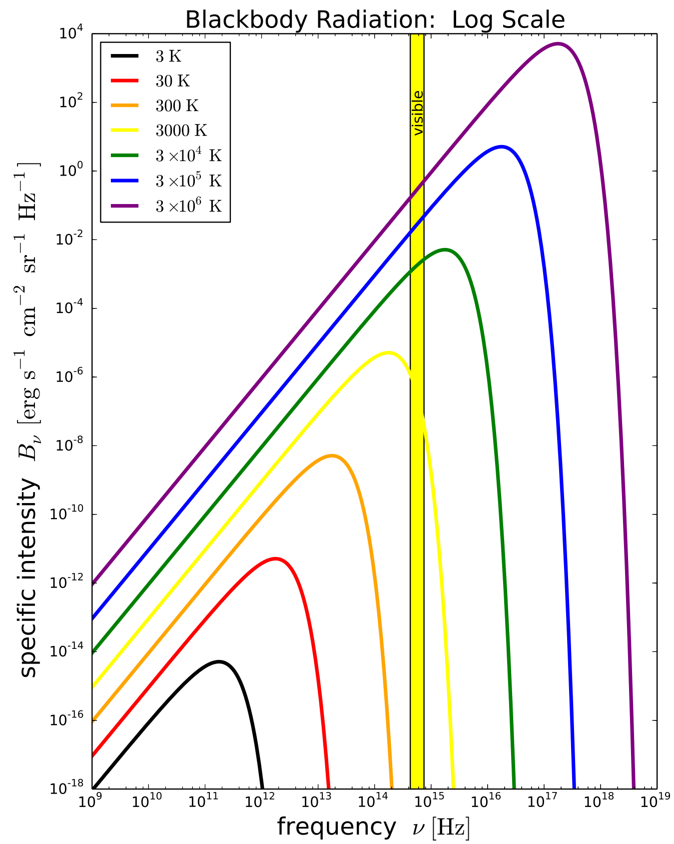
alternatively: given energy density $u \sim T^4$

and mean photon energy $\langle E \rangle \sim kT$

number density must be $n \sim T^3$

Blackbody Spectral Properties





plots of B_ν vs ν *Q: what strikes you?*

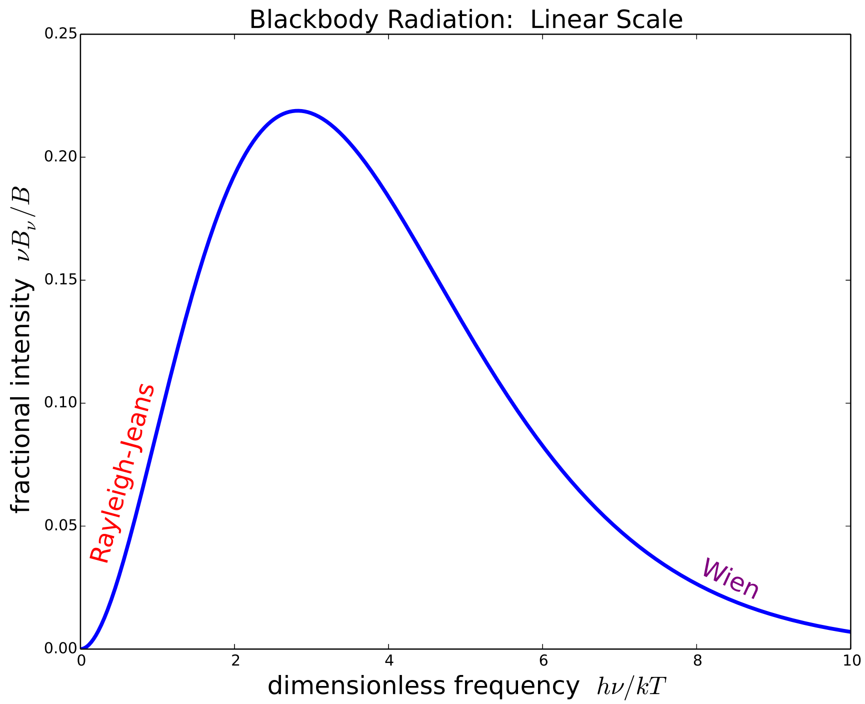
Blackbody Spectral Properties

at fixed ν, occupation number $\partial_T f(\nu, T) > 0$
$\rightarrow$ at larger T: larger f and so more photons
$\rightarrow$ also more specific intensity, flux, energy density
$\rightarrow$ slogan: *"blackbody spectra at different T never cross"*

natural energy scale kT, sets two limits

Rayleigh-Jeans limit $h\nu \ll kT$
occupation number $f(\nu) \rightarrow kT/h\nu \gg 1$
many photons, expect classical behavior
specific intensity $I_\nu = 2h/c^2 \; \nu^3 f \rightarrow 2kT \, \nu^2/c^2$

- $I_\nu \propto \nu^2$: power-law scaling
- h does not appear in I_ν: classical behavior!



Wien limit $h\nu \gg kT$

occupation number $f(\nu) \to e^{-h\nu/kT} \ll 1$

photon starved: thermal bath cannot "pay energy cost"

specific intensity $I_\nu \to 2h\,\nu^3/c^2\ e^{-h\nu/kT}$

- exponentially damped due to quantum effects

Brightness Temperature

at each ν, blackbody $B_\nu(T)$ unique for each T
$\rightarrow$ for resolved blackbody, measured B_ν gives T!

invert to define **brightness / antenna temperature**

$$B_\nu(T_{\rm b}) = I_\nu \tag{10}$$

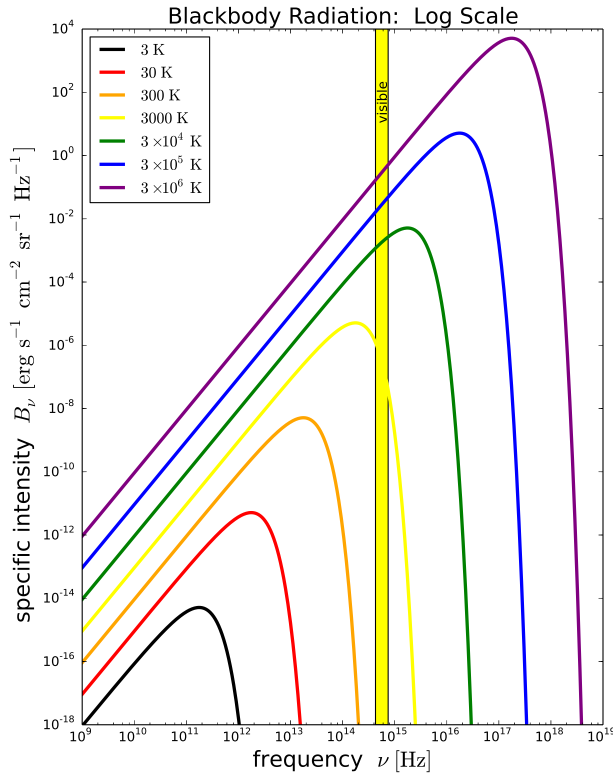
$$T_{\rm b}(\nu) = \frac{h\nu/k}{\ln\left(\frac{2h\nu^3}{c^2 I_\nu} + 1\right)} \tag{11}$$

$$\xrightarrow{h\nu \ll kT} \frac{c^2}{2k}\,\frac{I_\nu}{\nu^2} \tag{12}$$

note: *defined for all I_ν even if not blackbody!*
∞ another way of characterizing intensity

brightness temperature

$B_\nu(T_b) = I_\nu$



Q: why is this useful?

Q: what does it mean for a nonthermal source?

`www: famous discovery`

`www: modern` T_b `results plotted`

Wien's Displacement Laws

for blackbodies,
specific intensity, and flux, and energy density have

$$I_\nu \propto F_\nu \propto u_\nu \propto \frac{\nu^3}{e^{h\nu/kT} - 1} \quad (13)$$

at fixed T, these *spectra all peak at same frequency*

maximum when $x = h\nu/kT$ satisfies $x = 3(1 - e^{-x})$
$\rightarrow x_{\rm max} = 2.821439\ldots$, which gives

$$\frac{\nu_{\rm max}}{T} = x_{\rm max}\frac{k}{h} = 5.88 \times 10^{10} \text{ Hz K}^{-1} \quad (14)$$

i.e., $\nu_{\rm max} \propto T$, as expected from dimensional analysis in wavelength space, $I_\lambda \propto \lambda^{-5}/(e^{hc/\lambda kT} - 1)$

maximum when $y = hc/\lambda kT$ satisfies $y = 5(1 - e^{-y})$
$\rightarrow y_{\rm max} = 4.9651\ldots$, which gives

$$\lambda_{\rm max}\, T = \frac{1}{y_{\rm max}} \frac{hc}{k} = 0.290 \text{ cm K} \tag{15}$$

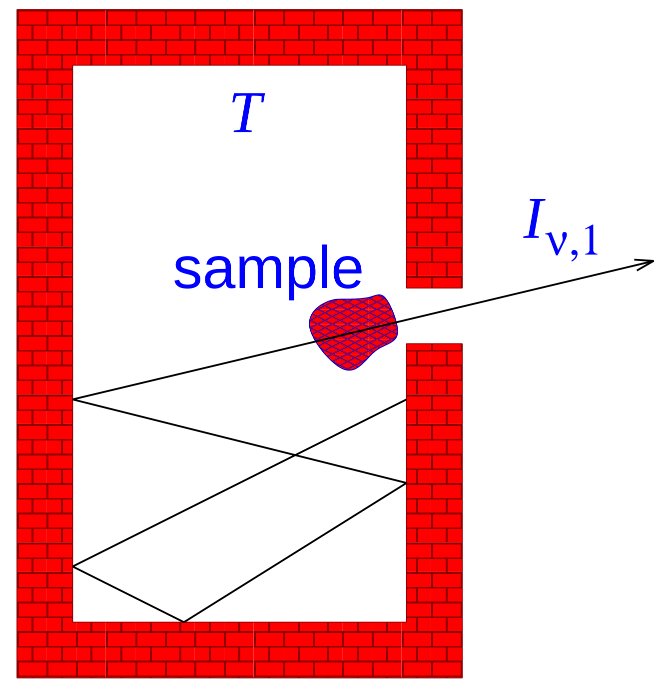
i.e., $\lambda_{\rm max} \propto 1/T$, as expected from dimensional analysis

both versions of Wien's Law measure T: **color temperature**

crucial gotcha: **beware!** $\lambda_{\rm max} \neq c/\nu_{\rm max}$
Q: why?

Thermal Radiation Transfer

Consider a cavity in thermodynamic equilibrium



place *small sample of matter* in cavity
along peephole sightline

note: "sample," most generally:

- *not* necessarily large
- *nor* optically thick!
- and might contain free ions, and/or bound states (atoms and molecules)

allow system to come into *thermodynamic equilibrium* at T

Q: effect on I_ν emerging from peephole?
Q: implications?

matter sample will emit and absorb radiation
according to its detailed composition,
and according to its state at T (i.e., ion, atom, molecule)
$\Rightarrow$ absorption $\alpha_\nu(T)$ and emission $j_\nu(T)$
both characteristic of the matter, *not universal!*

But in cavity+sample system, there is thermal equilibrium
$\rightarrow$ must still be blackbody emitter!
$\rightarrow$ *emitted radiation must have* $I_{\nu,\text{out}} = B_\nu(T)$

Yet radiation *incident* on sample in cavity
already was blackbody: $I_{\nu,\text{in}} = B_\nu(T)$

specific intensity change through sample

$$\frac{dI_\nu}{ds} = -\alpha_\nu (I_\nu - S_\nu) \tag{16}$$

Q: so what condition required for ray to maintain
$I_{\nu,\text{in}} = I_{\nu,\text{out}} = B_\nu(T)$ *through sample?*

Kirchhoff's Law for Thermal Emission

blackbody radiation $B_\nu(T)$ must remain unchanged when traversing arbitrary matter with temperature T
$\Rightarrow$ demands that *all matter* at T *has source function*

$$S_\nu(T) = B_\nu(T) \tag{17}$$

Kirchhoff's Law for thermal emission

Physically: equilibrium forces *emission–absorption relation*:

$$j_\nu = \alpha_\nu \; B_\nu(T) \tag{18}$$

Consequences:

- a good (poor) emitter is a good (poor) absorber
- *thermal* emission has $S_\nu = B_\nu(T)$ and has

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + B_\nu \tag{19}$$

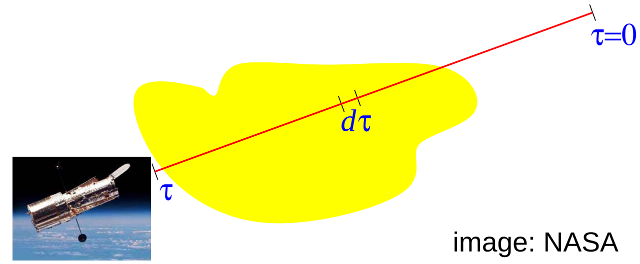
Q: so when will thermal *source have* blackbody *spectrum?*

thermal source has $S_\nu(T) = B_\nu(T)$

as seen in the PS1 Olber's problem!

and so transfer equation becomes

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + B_\nu$$



but since $B_\nu(T)$ is homogeneous, isotropic

if no background source, then $I_\nu(0) = 0$, and solution is

$$I_\nu(s) = \left(1 - e^{-\tau_\nu(s)}\right) B_\nu(T) \tag{20}$$

- if *optically thin*, then observe $I_\nu \approx \tau_\nu B_\nu(T) = j_\nu(T)\ \Delta s$
  intensity *not blackbody, and depends on emitter physics*
  `www: Cas A X-ray spectrum (Chandra)` *Q: what is Cas A?*

- if *optically thick*, then $I_\nu \to B_\nu$ as seen in PS1
  intensity relaxes to emitter-independent *blackbody* radiation

Build Your Toolbox: Imaging

Questions to ask about resolved astronomical images:

Q: what does it mean to be resolved?

Q: what questions to ask?

Q: how to answer?

Toolbox: Imaging

resolved images:

- *angular size > angular resolution*
- surface brightness I_ν observable

Questions to ask

- What wavebands are in image?
- is the source optically thin or thick at these ν?

  optically thick: image shows surface

  $I_\nu \to S_\nu = j_\nu \ell_{\rm mfp,\nu}$ at surface $\to B_\nu(T_{\rm surf})$ if thermal equilib

  optically thin: measure volume in projection

  $I_\nu \approx j_\nu \, \delta s$

Q: How to tell if thermal object is optically thick/thin?

Thermal Emitter: Thick or Thin?

How to tell if a thermal emitter is optically thick/thin?
Or: *is* $I_\nu = S_\nu = B_\nu(T)$ *for some* T*?*

If resolved: measure I_ν, find brightness temperature $T_\mathrm{b}(\nu)$
over multiple ν or colors
if blackbody: uniform $T_\mathrm{b} = T$ across colors
gives thermodynamic temperature of surface
otherwise: spectrum nonthermal,
not all emission optically thick

If unresolved: non-trivial! Measure F_ν^{obs}

- use spectral info: is $F_\nu^{\mathrm{obs}} \propto F_\nu(T)$ for some T?
- if so: dominated by optically thick emission at one T

Resolved Blackbody: Temperature Map

Imagine we *are* observing an optically thick thermal object

- That is: we observe blackbody radiation
- but make broadband observations – not a spectrum

if the object is **resolved**:

- *total intensity* $B(T) = \sigma T^4/\pi$ uniquely fixed by T
- a map of surface brightness is a map of temperature!

  `www: canine and lunar examples`

Now imagine the object is **unresolved**

Q: how to determine T with F_ν spectrum?

Q: how to determine T with based on total (bolometric) luminosity?

Other Temperature Measures: Unresolved Sources

unresolved: measure spectrum or just broadband/total flux

with spectrum $F_\nu^{\rm obs}$ vs ν: color temperature

- better: fit to blackbody $F_\nu(T)$ for different T
  best fit gives $T_{\rm obs}$
  quick and dirty: find peak of spectrum
  use Wien's law to go from $\nu_{\rm max}$ to T

with total flux $F^{\rm obs}$: effective temperature

- with distance to object, and infer luminosity $L = 4\pi r^2 F$
- from this and radius R infer surface flux $F_{\rm surf} = L/4\pi R^2 \sigma$
- effective temperature: $F_{\rm surf} = \sigma T_{\rm eff}^4$, so

$$T_{\rm eff} = \left(\frac{r}{R}\right)^{1/2} \left(\frac{F_{\rm obs}}{\sigma}\right)^{1/4}$$

Build Your Toolbox: Blackbody Radiation

emission physics: matter-radiation interactions

Q: physical conditions for blackbody radiation?

Q: physical nature of sources?

Q: spectrum characteristics?

Q: frequency range?

real/expected astrophysical sources of blackbody radiation

Q: what do we expect to emit blackbody radiation?

Q: relevant temperatures? EM bands?

Toolbox: Blackbody Radiation

emission physics

- physical conditions: *optically thick* objects *in thermodynamic equilibrium*
- physical sources: *solids, dense liquids or gasses*
- spectrum: continuum. nonzero at all frequencies
  single peak at $h\nu = 2.82kT$ (Wien's law)
  power law $I_\nu \sim \nu^2 T$ for $h\nu \ll kT$
  exponentical cutoff $I_\nu \sim \nu^3 e^{-h\nu/kT}$ $h\nu \gg kT$

astrophysical sources of blackbody radiation

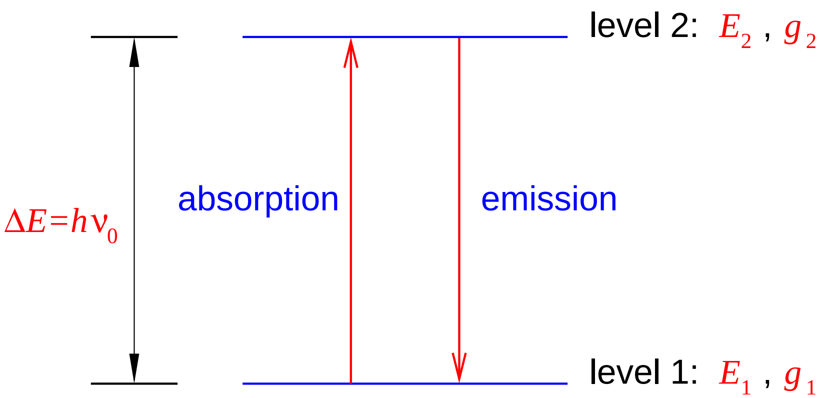
- *emitters: solids: dust, asteroids, planets, compact objects stars, some disks, the Universe*
- temperatures: *spans* 3K *to at least* 10^9 K
- EM bands: *microwave to MeV (and above?)*

Lines: Two-Level Systems

Two-Level Systems in Radiative Equilibrium

consider an ensemble of systems ("atoms") with

- **two discrete energy levels** E_1, E_2
- and degeneracies g_1, g_2
  i.e., a number g_1 of distinct states have energy E_1



in thermodynamic equilibrium at T, emission and absorption exchange energy with photon field

Q: when & why emit? absorb?
Q: connection between emission, absorption rates in ensemble?

Spontaneous Emission

in general, atoms in states with higher energy E_2
will *decay* to lower level E_1
photon of energy $\Delta E = E_2 - E_1 = h\nu_0$ will be emitted

transition can occur without influence of other atoms, photons
spontaneous emission: $X_2 \to X_1 + h\nu$

spontaneous emission rate per atom is
$E_2 \to E_1$ *transition rate per atom*:

$$\text{transition probability per unit time per atom} = \boxed{A_{21}} \qquad (21)$$

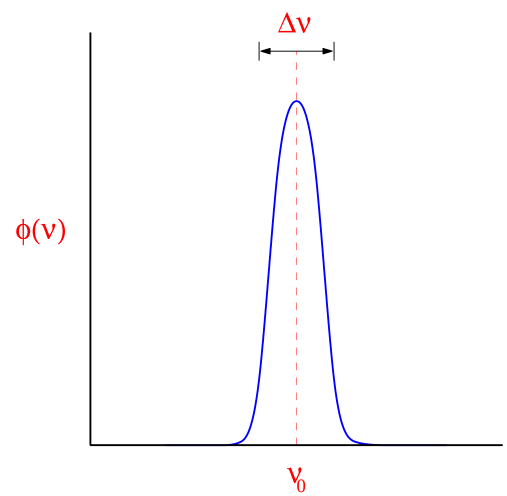
- "**Einstein** A" coefficient
- units $[A_{21}] = [\text{sec}^{-1}]$
- spontaneous: A_{21} independent of T
- but A_{21} *does depend on detailed atom properties*

Absorption

atoms in lower state E_1 only promoted to state E_2
by absorbing a photon of energy ΔE
$h\nu + X_1 \rightarrow X_2$

if levels were perfectly sharp, absorb only at $\Delta E = h\nu_0$
but in general, energy levels have *finite width*
i.e., *line energies "smeared out"* by some amount $h\,\Delta nu$
so transitions can be made by photons with frequencies
$\nu_0 - \Delta\nu \;\lesssim\; \nu \;\lesssim \nu_0 + \Delta\nu$

useful to define **line profile function** $\phi(\nu)$
with normalization $\int \phi(\nu)\; d\nu = 1$
e.g., Gaussian, Lorentzian, Voight functions



limiting case of sharp levels $\Delta\nu \rightarrow 0$:
$\phi(\nu) \rightarrow \delta(\nu - \nu_0)$

absorptions require ambient photons

thus absorption rate per atom *depends on photon field*
and ensemble *average absorption rate*
depends on average intensity

$$\bar{J} \equiv \int \phi(\nu)\ J_\nu\ d\nu \tag{22}$$

limiting case of sharp levels: $\bar{J} \to J_{\nu_0}$

thus write average absorption rate as

$$\text{transition probability per time per atom} \quad = \boxed{B_{12}\bar{J}} \tag{23}$$

- "**Einstein B coefficient**"
- B_{12} is probability per time per intensity
- depends on atom and state details
- but *does not depend on T*

Stimulated Emission

Einstein postulated a new emission mechanism:
driven by photons with transition energy $\nu_0 - \Delta\nu \lesssim \nu \lesssim \nu_0 + \Delta\nu$
$h\nu + X_2 \rightarrow X_1 + 2h\nu$

I.e., the presence of transition photons creates "peer pressure"
"encourages" atoms in higher state to make transition
faster than they would spontaneously: **stimulated emission**
plausible? yes–photons interact with and perturb atoms

if stimulated emission exists, should also depend on $\bar{J}$
$\rightarrow$ rate per atom is

$$\text{transition probability per time per atom} = B_{21}\bar{J} \tag{24}$$

- note stimulated emission coefficient B_{21}
  can be *different from* absorption coefficient B_{12}
- if stimulated emission doesn't happen, would find $B_{21} = 0$